ICONOGRAPHIC
ENCYCLOPÆDIA
OF
SCIENCE, LITERATURE, AND ART.

SYSTEMATICALLY ARRANGED

BY

J. G. HECK.

TRANSLATED FROM THE GERMAN, WITH ADDITIONS,

AND EDITED BY

SPENCER F. BAIRD, A.M., M.D.,
PROFESSOR OF NATURAL SCIENCES IN DICKINSON COLLEGE, CARLISLE, PA.

ILLUSTRATED BY FIVE HUNDRED STEEL I
CONTAINING UPWARDS OF TWELVE THOUSAND ENGRAVING

IN FOUR VOLUMES.

VOL. I:

MATHEMATICS AND ASTRONOMY,
PHYSICS AND METEOROLOGY, CHEMISTRY, MINERALOGY,
GEOGNOSY AND GEOLOGY.

NEW YORK:
D. APPLETON AND COMPANY,
346 & 348 BROADWAY.
M.DCCC.LX.

PREFACE.

THE text of the work which is now presented to the American public is based upon the well known "BILDER ATLAS ZUM CONVERSATIONS LEXICON," just published in Leipsic, by F. A. BROCKHAUS, and edited by Mr. JOHN G. HECK. The engravings are impressions from the original steel plates.

The object steadily kept in view in preparing the ICONOGRAPHIC ENCYCLOPÆDIA has been to furnish a book to which the general reader may apply, for an explanation of the principal physical facts which come under his notice. To do this satisfactorily, pictorial representation is necessary, which it is hoped the five hundred quarto plates, with their 12,000 figures, will abundantly furnish.

Much of the utility of an Encyclopædia depends on its arrangement. The method which the Editor's experience of works of this kind has shown to be most convenient, is that of a systematic grouping of distinct treatises, according to their natural affinities. The work thus becomes, as it were, a series of text-books, capable of being used as such, and to which recourse may be had for all the general information required on a given subject.

To enable the reader, however, to refer readily to any individual fact a copious alphabetical index, or series of indexes, is indispensable. By including numerous cross references, it will be possible to furnish all the facilities of a strictly alphabetical arrangement, without any of its disadvantages.

This, then, is the plan which has been adopted in the arrangement of the Iconographic Encyclopædia. Each article falling within its scope has been treated of independently, and, as far as it goes, is complete in itself. It will not be expected that in the extensive range of subjects involved, even with the exclusion of Biography, Speculative Philosophy, and all abstract sciences in general, any one can be treated in its fullest extent. All that has been aimed at, and indeed all that could have been looked for, was to present a general view of each subject, essentially popular in character, and fitted, more particularly, for those who wish to have the principal facts of numerous works condensed in a single one. Nevertheless, it will be found, on examination, that many of the subdivisions of this Encyclopædia are much fuller in their details than most of the text-books or popular treatises of the day.

Tables of Contents and Indexes have been prepared for each volume, and no pains have been spared to make these more than usually accurate. The indexes do not refer to words merely, but to facts and ideas, so that the text can be readily consulted upon any given topic. The lists of the figures on the plates will be found under the contents of the text which they are intended to elucidate, with references to the pages in the letter-press where explanations may be looked for. They furnish an immediate explanation of any figure that may arrest the eye. A glossary of the German terms and phrases used in a few of the plates is also added to these lists. It would undoubtedly have been more convenient if the few plates which have caused the necessity of such translations, had been re-engraved in English; but the expense of doing so would have more than doubled the price of the work, whose unparalleled cheapness could only be secured by a liberal contract for impressions from the excellent German plates.

To Mr. Heck belongs exclusively the credit of the conception and execution of the original work; and whether we regard its magnitude, or the regularity and efficiency of its performance, it is one that has rarely, if ever, been excelled.

In undertaking an English version of the Iconographic Encyclopædia it was soon found that a literal translation of the original

would not satisfy the wants of the American public. Written in and for Germany, the different subjects were treated of much more fully in relation to that country than to the rest of the world. In some articles, too, owing to the lapse of time or other causes, certain omissions of data occurred, which did not allow of their being considered as representing the present state of science, or as suiting the wants of the United States. This, therefore, has rendered it necessary to make copious additions, alterations, and abridgments in the respective translations; while, in some instances, it has been thought proper to re-write entire articles. Several of these original papers have been prepared by the Editor, and the remainder kindly furnished by some of his friends. Some of these again have relieved him of the burden of translating, and have added much to the merit of their work by judicious alterations and additions; while others have revised his MSS. and enriched them with important suggestions. The authority and value of the assistance thus obtained will be sufficiently evident from the names of those who have so kindly rendered it. To all he here takes the opportunity of returning his warmest acknowledgments.

The second volume, or the one containing Botany, Zoology, and Anthropology, has been entirely re-written. The articles in it not prepared by the Editor are *Invertebrate Zoology*, by Prof. S. S. Haldeman; *Ornithology*, by John Cassin, Esq.; and *Mammalia*, by Charles Girard, Esq.

The friends to whom he is indebted for careful revision of his MSS. are, Prof. Wolcott Gibbs (*Chemistry*); Prof. J. D. Dana (*Mineralogy*); Prof. L. Agassiz (*Geognosy and Geology*); Dr. Asa Gray (*Botany*); Dr. T. G. Wormley (*Anatomy*); and Herman Ludewig, Esq. (*Geography*).

Those who have assisted him by translating and editing entire articles are, Wm. M. Baird, Esq. (*Ethnology of the Present Day*); Major C. H. Larned, U. S. Army (*Military and Naval Sciences*); F. A. Petersen, Esq. (*Architecture*); Prof. Chas. E. Blumenthal (*Mythology and Religious Rites*); Prof. Wm. Turner (*Fine Arts*); and Samuel Cooper, Esq. (*Technology*).

The Editor is likewise under very great obligations to the

Publisher, not only for affording him every facility in the prosecution of his task, but for unwearied and invaluable assistance in the discharge of his editorial duties. He here also takes occasion to acknowledge his indebtedness to Mr. Wm. H. Smith for revision of the proof-sheets and preparation of the Alphabetical Indexes; and also to Mr. Robert Craighead for the care which he has displayed in the typographical execution.

S. F. Baird.

Washington City, D. C., April, 1851.

CONTENTS

OF

MATHEMATICS AND ASTRONOMY.

[The numbers refer to the bottom paging of the text.]

CONTENTS OF THE PLATES (I. Nos. 1—15)

TO

MATHEMATICS AND ASTRONOMY.

[Including the translation of German words and phrases in the same.]

*** The references for explanations of the subjects are to the bottom paging of the text.

GLOSSARY.

a = der Projection des Bergstriches, a = Projection of the mountain tract.
b = der Höhe der horizontalen Schicht, b = Height of the horizontal stratum.
c = wahre Länge des Bergstriches, c = True length of the mountain tract.

GLOSSARY.

Abend, Evening ; — *stern*, Evening star.
Abnehmender Mond, Decreasing moon.
Aphelium, Aphelion.
Apsidenlinie, Line of Apsides.
Aufsteigender Knoten, Ascending node.
Austritt beim Aufgang der Sonne, Emersion at sunrise ; — *beim Untergang der Sonne*, Emersion at sunset.
Axe der Ekliptik, Axis of the Ecliptic.
Bahn des Merkur, Orbit of Mercury ; — *der Venus*, Orbit of Venus.

GLOSSARY—(*Continued.*)

Colur des Frühlings-Æquinoxiums, Colure of the vernal equinox ; — *des Herbst-Æquinoxiums,* Col. of the Autumnal equinox ; — *der Nachtgleichen,* Col. of the equinoxes ; — *des Sommersolstitiums,* Col. of the Summer solstice ; — *des Wintersolstitiums,* Col. of the Winter solstice.
Comet von 1811 *vor seiner Erscheinung. Projectirt auf die Ekliptik am* 25 *März* 1811. *Neigung der Bahn* 75° 5′, Comet of 1811 before its appearance. Projected on the ecliptic March 25, 1811, Inclination of the orbit 75° 5′. — *von* 1811 *nach seiner Erscheinung b. am* 1 *März* 1812, Comet of 1811 after its appearance until March 1, 1812.
Dauer der längsten Nächte, Duration of the longest nights ; — *des längsten Tages,* Duration of the longest day.
Dichteres Medium, Denser medium.
Drei Uhr Morgens, 3 o'clock A. M. ; — *Nachmittag,* 3 o'clock P. M.
Dritter Octant, Third octant.
Dünneres Medium, Rarer medium.
Ebene der Pallas, d. Juno, &c., Planes of the orbits of Pallas, Juno, &c.
Eintritt beim Aufgang der Sonne, Entrance at sunrise ; — *beim Untergang d. Sonne,* Entrance at sunset.
Ekliptik, Ecliptic.
Erdaxe, Axis of the earth.
Erdbahn, Orbit of the earth.
Erde, Earth.
Erster Octant, First octant.
Erstes Viertel, First quarter.
Excentricität, Eccentricity.
Frühling, Spring.
Frühlingsnachtgleiche, Vernal equinox.
Gemässigte Zone, Temperate zone.
Grosse Axe, Axis of the heavens.
Halley'scher Comet v. 1759 *u.* 1835, *Neigung seiner Bahn,* Halley's Comet of 1759 and 1835, inclination of its orbit.
Heisse Zone, Torrid zone.
Herbst, Autumn.
Herbstnachtgleiche, Autumnal equinox.
Horizont, Horizon.
Kalte Zone, Frigid zone.
Kometenbahn, Orbit of a comet.
Letztes Viertel, Last quarter.
Millionen Meilen, Millions of miles.
Mittag, Noon.
Mittelkraft, Mean force.
Mitternacht, Midnight.
Monate, Months.
Mond, Moon ; — *-bahn,* Orbit of the moon ; — *-finsterniss,* Eclipse of the moon.
Morgen, Morning ; — *-stern,* Morning star.
Neumond, New moon.
Neun Uhr Abends, 9 P.M. ; — *Morgens,* 9 A.M.
Niedersteigender Knoten, Descending node.
Nord, North ; — *-pol,* Northpole.
Nördliche Declination der Sonne, Northern declination of the sun.
Nördlicher Polarkreis, Arctic circle.
Obere Conjunction, Superior conjunction.
Œstliche Digression, East digression.
Ost, East.
Perihelium, Perihelion.
Polarstern, Polar star.
Polhöhe über dem Horizont, Elevation of the pole above the horizon.

GLOSSARY—(*Continued.*)

Richtung des Schattens um Mittag, Direction of the shadow at noon.
Rotationsaxe, Axis of rotation.
Scheinbarer Himmelsbogen, Apparent arch of the heavens ; — *Horizont,* visible horizon.
Sechs Uhr Abends, 6 P.M. ; — *Morgens,* 6 A.M.
Solstitial oder Wendepunktlinie, Solstitial colure.
Sommer, Summer ; — *-Sonnenwende,* Summer Solstice.
Sonne, Sun ; — *Sonnen-Æquator,* Sun's equator ; — *-finsterniss,* Eclipse of the sun ; — *-scheibe im Grössenverhältniss zu den Planeten,* The sun's disk ; its size compared to the diameters of the planets.
Stunden, Hours ; — *Entfernung,* Hours' distance ; — *-ring,* Hour-circle.
Süd, South ; — *-pol,* South pole.
Südliche Declination der Sonne, Southern declination of the sun.
Südlicher Polarkreis, Antarctic circle.
Südwestlicher Sonnenrand, Southwestern edge of the sun.
Trabanten des Jupiter ; — des Saturn ; — des Uranus, Satellites of Jupiter, Saturn, and Uranus.
Untere Conjunction, inferior conjunction.
Vierter Octant, Fourth octant.
Vollmond, Full moon.
Vom Pol bis zum Zenith, From pole to zenith ; — *Zenith biz zum Æquator,* From zenith to equator.
Wahrer Horizont, True horizon.
Wendekreis des Krebses, Tropic of Cancer ; — *des Steinbocks,* Tropic of Capricorn.
Westliche Digression, West digression.
Winter-Sonnenwende, Winter solstice.
Zoll, Digit, inch.
Zunehmender Mond, Increasing moon.
Zweites Octant, Second octant.

☞ For names of Constellations of the Zodiac, see p. 90.

PLATE 11.

PLATES 12, 13.

PLATE 14.

GLOSSARY.

Anfang bei Sonnenaufgang, Beginning at sunrise; — *bei Sonnenuntergang,* beginning at sunset.
Arabien, Arabia.
Atlantischer Ocean, Atlantic Ocean.
Azorische Inseln, The Azores.
Berberey, Barbary.
Berührung des Sonnen-, und Mondrandes, Contact of the edges of sun and moon.
Canarische Inseln, Canary Islands.
Capverdische Inseln, Cape Verd Islands.
Centrale oder totale Verfinsterung, Central or total eclipse.
Drei Zoll Verfinsterung, Three digits eclipsed.
Ende bei Sonnenaufgang, End at sunrise; — *bei Sonnenuntergang,* End at sunset.
Grönland, Greenland.
Grossbritanien, Great Britain.
Grosser Ocean, Pacific Ocean.
Indisches Meer, Indian Sea.
Island, Iceland,
Mittel bie Sonnenaufgang, Middle at sunrise; — *bei Sonnenuntergang,* Middle at sunset.
Mittelländisches Meer, Mediterranean Sea.
Mongolei, Mongolia.
Neun Zoll Verfinsterung Nine digits eclipsed.
Nordpol, North pole.
Norwegen, Norway.
Nubien, Nubia.
Ost Indien, East Indies.

GLOSSARY—(*Continued.*)

Russland, Russia.
Sechs Zoll Verfinsterung, Six digits eclipsed.
Sibirien, Siberia.

PLATE 15.

Astronomical Instruments.

CONTENTS

OF

PHYSICS AND METEOROLOGY.

[The numbers refer to the bottom paging of the text.]

CONTENTS OF THE PLATES (I. Nos. 16—29)

TO

PHYSICS AND METEOROLOGY.

[Including the translation of German words and phrases in the same.]

*** The references for explanations of the subjects are to the bottom paging of the text.

GLOSSARY.

Februar, February.
Fische, Pisces.
Frühling 90 *Tage*, Spring 90 days.
Heisseste Temperatur, Hottest temperature.
Herbst 90 *Tage*, Autumn 90 days.
Januar, January.
Juli, July.
Jungfrau, Virgo.
Juni, June.
Kälteste Temperatur, Coldest temperature.
Krebs, Cancer.
Löwe, Leo.
Mai, May.
März, March.
Mittlere Temperatur, Medium temperature.
Schütze, Sagittarius.
Skorpion, Scorpio.
Sommer 93 *Tage*, Summer 93 days.
Steinbock, Capricornus.
Stier, Taurus.
Waage, Libra.
Wassermann, Aquarius.
Widder, Aries.
Winter 93 *Tage*, Winter 93 days.
Zwillinge, Gemini.

GLOSSARY.

Ægypten, Egypt.
Æquator-Grenze des Schneefalls, Equatorial boundary of the snow region.
Aleuten, Aleutian Islands.
Arabien, Arabia.
Asien, Asia.
Barometer steigt,—fällt, Barometer rises,— falls.
Berberei, Barbary.
Beständiger Regen, Perpetual rain.
C. der guten Hoffnung, Cape of Good Hope.
Canarische Inseln, Canary Islands.
Capverdische Inseln, Cape Verd Islands.
Deutschland, Germany.
Erd-Æquator, Terrestrial equator.
Europa, Europe.
Frankreich, France.
Freundschafts Inseln, Friendly Islands.
Gebiet der Monsun Regen, Region of the Monsoons.
Gesellschafts Inseln, Society Islands.
Grönland, Greenland.
Grossbritanien, Great Britain.
Grosser Ocean, Pacific Ocean.

GLOSSARY—(*Continued.*)

Herbst und Winter Regen, Autumn and winter rains.
Indisches Meer, Indian Sea.
Italien, Italy.
Kopenhagen, Copenhagen.
Kurve von Leith, Br., Curve of Leith, latitude.
Kurve von Padua, Br., Curve of Padua, latitude.
Linie ohne Inclination, Line without inclination.
Magnetischer Æquator, Magnetic equator.
Manschurei, Mandshoo territory.
Maximum der magnet. Kraft, Maximum of magnetic power.
Mittlere Tageswärme, Medium daily temperature.
Mongolei, Mongolia.
N. O., N. E.
Neufundland, Newfoundland.
Neu Guinea, New Guinea.
Neu Sealand, New Zealand.
Niedrige Inseln, Low Islands.
Nord Amerika, North America.
Nördlicher Gürtel der beständigen Niederschläge, Northern zone of perpetual deposits.
Nördliche Hemisphäre, Northern Hemisphere.
Nördlicher Polarkreis, Arctic Circle.
Nördliches Eismeer, Arctic Sea.
Nordpol, North pole.
Nord See, North sea.
Norwegen, Norway.
Nubien, Nubia.
Oceanien, Oceania.
Ost Indien, The East Indies.
Ostseite, East side.
Persien, Persia.
Provinz des Herbstregens; — des Sommerregens; — des Winterregens, Region of autumnal, of summer, and of winter rains.
Regenloses Gebiet, Rainless region.
Russland, Russia.
Schwacher Sommerregen, Light summer rain.
Sibirien, Siberia.
S. O., S. E.
S. O. Monsun im Apr.–Oct., N. W. Monsun im Oct.–Apr., S. E. Monsoon from April to October, N. W. Monsoon from October to April.
Spanien, Spain.
Süd Amerika, South America.
Südpol, South pole.
Südlicher Continent, Southern continent.
Südlicher Gürtel der beständigen Niederschläge, Southern zone of perpetual deposits.
Südliche Hemisphäre, Southern Hemisphere.
S. W. Monsun im Apr.–Oct., N. O. Monsun im Oct.–Apr., S. W. Monsoon from April to October, N. E. Monsoon from October to April.
Thermometer steigt, — fällt, Thermometer rises, — falls.
Turkei, Turkey.
Vereinigte Staaten, United States.
Wärme-Æquator, Equator of heat.
Wendekreis des Krebses; — des Steinbocks, Tropic of Cancer; — of Capricorn.
West Indien, The West Indies.
Westseite, West side.
Winterregen, Winter rains.
Wüste Schamo oder Gobi, Desert of Shamo, or Gobi.
Zone häufiger, fast beständiger Niederschläge, stets mit electrischen Explosionen, Zone of frequent, nearly perpetual deposits, always accompanied by electrical explosions.

CONTENTS

OF

CHEMISTRY, MINERALOGY, AND GEOLOGY.

[The numbers refer to the bottom paging of the text.]

CONTENTS OF THE PLATES (I. Nos. 30—53)

TO

CHEMISTRY, MINERALOGY, AND GEOLOGY.

[Including the translation of German words and phrases in the same.]

*** The references for explanations of the subjects are to the bottom paging of the text.

PAGE

Plate 33—(*Continued.*)

PLATE 34.

Minerals and their Crystalline Forms.

PAGE

PLATE 34—(*Continued.*)

PLATE 35.

Minerals and their Crystalline Forms.

PLATE 36.

Minerals and their Crystalline Forms.

GLOSSARY.

Basalt Boden, Basaltic soil.
Bugt, Bay.
Busen von Neapel, — *von Salerno*, Bay of Naples, — of Salerno.
Cyclopen Inseln, Cyclops Islands.
Die Flegreischen Felder, The Phlegræan fields.
Fiorden, The inlet.
Island, Iceland.
Längenthal im Trachyt, Long valley in the trachyte formation.
Lava Ströme, Lava streams.
Neapel, Naples.
Nord Cap, North Cape.
Œ, Island.
Passhöhe, Height of the pass.
Polarkreis, Arctic circle.
See, Lake.
Vesuv, Vesuvius.

GLOSSARY.

Aufgeschw. Land, Deposited ground.
Braunschweig, Brunswick.
Bunter Sandstein, Variegated sandstone.
Donau, Danube.
Ebbe Geb., Ebbe Mountains.
Egge Geb., Egge Mountains.
Geb. v. Charolais, Mountains of Charolais.
Harz Gebirge, Hartz Mountains; — *und Thüringer Wald*, Hartz and Thuringian Mountains.
Hochwald, High forest (mountain tract).
Jura Gebirg, Jura Mountains; — *-kalk*, Jurassic Limestone.
Kohlengebirge, Carboniferous formation.
Kohlenkalk, Carboniferous limestone.
Kohlensandstein, Carboniferous sandstone.
Kreide, Chalk.
Laacher See, Lake Laach.
Porphyr, Porphyry.
Quadersandstein, Freestone.
Rhein, Rhine.
Rheinisches Uebergangsgebirg, Rhenish transition rock.
Rothhaar Geb., Rothhaar Mountains.
Schuttland, Conglomerate.
Steink. Gebilde, Carboniferous formation.
Tertiär, Tertiary.
Teutoburger Wald, Teutoburg forest.
Todtliegendes, Red sandstone.
Trias, Rock salt formation.
Trier, Treves.
Uebergangskalk, Transition limestone.
Vogel Geb., Vogel Mountains.

GLOSSARY—(*Continued.*)

Vogesen sandstein, Vosges sandstone.
Vulcan Gebilde; — *Gerölle*, Volcanic formation; — rubble.
Weser Geb., Weser Mountains.

GLOSSARY.

Ægatische In., Ægatian Islands.
Atlantischer Ocean, Atlantic Ocean.
Canal von Malta, Maltese Channel.
Deutschland, Germany.
Frankreich, France.
Grosse Antillen, The larger West India Islands.
Indischer Ocean, Indian Ocean.
Kleine Antillen, The smaller West India Islands.
Liparische In., Liparian Islands.
Lissabon, Lisbon.
Meerb. v. Taranto, Bay of Taranto.
Mittelländisches Meer, Mediterranean Sea.
Mozambique Kanal, Mozambique Channel.
Neapel, Naples.
Nord Amerika, North America.
Sicilien, Sicily.
Spanien, Spain.
Str. von Messina, Strait of Messina.
Türkei, Turkey.
Tyrrhenisches Meer, Tyrrhenian Sea.
Venedig, Venice.
Vesuv, Vesuvius.
Wien, Vienna.

ERRATUM.

Page 220 (Physics, p. 46). Under the head of *Velocity of Efflux* the following paragraph was omitted; it ought to be inserted after the description of Plate 17, Fig. 33.

The Swimmer of Prony (fig. 35) is an apparatus employed for obtaining a constant height of pressure of water. C is a box floating in the vessel, D, and supporting, by means of the rods A, B, a second box, G (below the aperture, E, of the vessel, D), into which all the water flowing from D is received through the funnel, F. The height of pressure in D will thereby remain unchanged, as the weight of the box, G, increased at the rate of the efflux from D, will draw down the box, C, so as to replace the water which has passed into G.

MATHEMATICS.

PLATES 1, 2, 3, 4, 5.

MATHEMATICS is the science which treats of quantity and of the various forms and combinations of magnitudes. The idea of magnitude applies to everything which either actually or abstractly admits of increase or diminution.

Mathematics is divided into Pure and Applied. The former derives all its ideas and conclusions directly through the understanding, without requiring the least assistance from the experience and knowledge obtained through the senses, while the latter applies the deductions of the former to the various objects of experience and of the external world. Pure Mathematics distinguishes two kinds of magnitudes,—continuous, and interrupted or discrete. A magnitude is said to be continuous when its parts adhere closely together, so that the ending of one part coincides exactly with the beginning of the next; to this species belong the magnitudes of extent or space, *e. g.* a surface. A magnitude is interrupted or discrete when its parts are separated one from another, as the individual stones in a pile. Pure Mathematics is therefore divided into two principal sections, Geometry and Arithmetic (in its wider sense). Arithmetic, which includes Arithmetic proper, Algebra, and the analysis of finite and infinite quantities, offers from its very nature hardly any material for pictorial representations: we must therefore confine ourselves to geometry alone

Geometry (earth measuring) derives its name from a single application of the science, which will be treated of hereafter; and, as before remarked, has reference to continuously extended magnitudes, or magnitudes of space. As there are three different directions or dimensions of extension, so are there also three different kinds of magnitude or space,—lines, surfaces, and solids; of which lines extend only in one direction (length), surfaces in two directions (length and breadth), and solids in three (length, breadth, and height or thickness). Lines are bounded by points, surfaces by lines, and solids by surfaces. Lines are divided into straight and curved; in like manner, surfaces into plane and curved. Geometry itself, nowever, is not divided according to the three kinds of magnitudes, but only into two principal sections,—Planimetry (Plane Geometry), and Stereometry (Geometry of Solids). The former treats of such magnitudes of space or combinations of magnitudes as are found in a single plane, or in which only two dimensions occur (length and breadth); the latter, of those in which all these three are found (length, breadth, and height or thickness), and hence it refers to

solids. Geometry is again divided into the lower and higher, of which the former treats of rectilineal figures and the circle, of bodies bounded by planes, and finally of the cylinder, cone, and sphere; the latter of curved lines, of surfaces inclosed by them, and of the solids and curved surfaces which they generate.

I. PLANIMETRY OR PLANE GEOMETRY.

1. GENERAL IDEAS.

Lines, as already mentioned, are divided into straight (*pl.* 1, *fig.* 1) and curved (*fig.* 2). A broken line (*fig.* 3) cannot be said to be a particular species of line, but only a combination of several straight lines. A mixed line (*fig.* 4) is the union of straight and curved. The idea of horizontal and vertical lines (*figs.* 5, 6) is essentially foreign to pure Geometry. In applied or practical Geometry we call a line horizontal, when it runs in the same direction with the plane of the horizon, or the surface of still water, sometimes termed level; and vertical or perpendicular, when it corresponds to the direction of the plumb-line, or that of a string to which hangs a freely suspended weight; every other straight line is called slanting or oblique (*fig.* 7). Two straight lines in the same plane are said to be *parallel* (*fig.* 8) when they never meet however far they may be produced, or when they are everywhere equally distant from each other. Curved lines also, if they possess the latter property, are sometimes called parallel, although this extension of the idea is hardly allowable.

Two straight lines, when they meet in a point (*fig.* 9), form an angle with each other: this is the name which is given to their inclination or separation. The lines are called the sides, and the point where they meet the vertex. Any angle, even if it has both sides curved (*fig.* 10), or one side curved and the other straight (*fig.* 11), may still be reduced to a rectilineal angle. In elementary Geometry, all angles are rectilineal. If one of the sides of an angle is extended beyond the vertex, a second angle is formed, which is called the adjacent angle of the first. If the two adjacent angles are equal (*fig.* 7), each is called a *right* angle (also *fig.* 12). All right angles are equal; and on that account they are used as a standard by which to measure other angles. Every angle smaller than a right angle is called acute (*fig.* 13), and every angle that is larger is called obtuse (*fig.* 14). Two or more angles that have a common vertex, and lie on the same side of a straight line in such a manner that this line constitutes one side of the first angle and one of the last, are altogether equal to two right angles. The angles about a point are together equal to four right angles.

A flat space bounded by lines is called a *figure*. A figure is called *rectilineal* if it is bounded by straight lines; if by curved lines, *curvilineal;* and it is a *mixed figure* when it is inclosed by both straight and curved lines. Plane Geometry deals only with plane figures (figures that lie in a plane

surface). If a rectilineal figure is bounded by three lines, it is called a *triangle;* if by four, a *quadrilateral;* if by more than four, a *polygon.*

Triangles are divided, *first,* according to their sides, into *equilateral* (*pl.* 1, *fig.* 15), in which all the sides are equal; *isosceles* (*fig.* 16), which have only two sides equal; and *scalene* (*fig.* 17), in which all the sides are unequal. *Secondly,* they are divided with reference to their angles, into *right angled triangles* (*fig.* 18), when they have one angle right and two acute; *obtuse angled triangles* (*fig.* 20), when they have one angle obtuse and two acute; and *acute angled triangles* (*fig.* 19), when all the angles are acute. That side of a right angled triangle which is opposite to the right angle is called the hypothenuse, the two others are called the legs.

Among quadrangular figures, the parallelograms form a remarkable class. They are quadrilaterals in which the two opposite sides are equal and parallel. Every parallelogram is divided by a diagonal—a straight line joining the vertices of two opposite angles—into two equal triangles (*fig.* 23). There are four different kinds of parallelograms: the *square* (*fig.* 21), in which all the sides are equal, and all the angles right angles; the *rectangle* (*fig.* 22), which has also its angles right angles, and only its parallel sides equal; the *rhombus* or *lozenge* (*fig.* 24), in which all the sides are equal, but only its opposite angles equal; and the *rhomboid* (*fig.* 25), which has only its opposite sides and angles equal. A quadrilateral in which only two sides are parallel is called a *trapezoid* (*fig.* 27). It is called a right angled trapezoid when it has two right angles (*fig.* 26), and equilateral when the two sides that are not parallel are equal (*fig.* 28). A trapezoid may have three sides equal, but the parallel sides must always be unequal. A quadrilateral in which none of the sides are parallel is called a *trapezium* (*fig.* 29).

A polygon is called regular when all its sides and angles are equal, and irregular when this is not the case. *Figs.* 30–37 represent regular polygons, viz. *fig.* 30, a 5-sided figure, or *pentagon; fig.* 31, a 6-sided, or *hexagon; fig.* 32, a 7-sided figure, or *heptagon; fig.* 33, an 8-sided, or *octagon; fig.* 34, a 9-sided figure, or *nonagon; fig.* 35, a 10-sided, or *decagon; fig.* 36, one of 11 sides, or *undecagon; fig.* 37, one of 12, or *dodecagon;* all of which are accompanied by circles, either circumscribed or inscribed.

The only curved line which occurs in elementary geometry, is the circular. The extremities of this line meet, and every point in it is equally distant from a point in the space inclosed, called the centre. The surface inclosed by the circular line is called the circle (*fig.* 38). In its relation to this, the circular line is called the circumference or periphery. A portion of the circular line is called an *arc,* e. g. *abc* (*fig.* 39). The size of an arc with reference to the whole circumference is measured by degrees. Every circle is divided into 360 equal parts, which are called degrees; each degree contains 60 minutes, and each minute 60 seconds. A straight line, drawn from the centre of a circle to its circumference, is called a *semi-diameter* or *radius,* e. g. *cd* (*fig.* 40); a straight line uniting two points of the circumference, a *chord,* e. g. *ef* (*fig.* 40, also *fig.* 43); and a *diameter* when, passing through the centre, it unites two opposite points of the

circumference, as *ab* (*pl.* 1, *fig.* 40). Every chord cuts from a circle a *segment*, as *bac* (*fig.* 41). The part of a circle included between two radii and an arc is called a *sector* (*fig.* 42). The angle formed by two radii is called a centre angle, as *bac* (*fig.* 42). An angle formed by two chords meeting in the line of the circumference, is called an inscribed angle (*fig.* 44). When a straight line, produced at pleasure, touches a circumference only at one point, it is called a tangent, e. g. *bc* (*fig.* 46); any such straight line, however, which either immediately or when produced cuts the circumference in two points, is called a secant, *ab* (*fig.* 46). A rectilineal figure is said to be inscribed in a circle when all its sides are chords (*fig.* 45); it is said to be circumscribed about a circle when all its sides are tangents (*fig.* 54). Two or more circles are said to be *concentric* when they have a common centre (*fig.* 47); circles of different centres are excentric. Two excentric circles touch one another when their circumferences have only one point in common: this point may be either on the exterior (*fig.* 53) or on the interior of the circumference. In the first case the sum, in the second the difference of their radii, will be the distance between their centres; in both cases the centres and the point of tangency will be in the same straight line. Two excentric circles cut each other (*fig.* 48) when their circumferences have two points in common. Each point of intersection forms a triangle with the two centres.

2. OF THE POSITION OF STRAIGHT LINES IN THE SAME PLANE.

Only one straight line can be drawn between two given points, so that the position and direction of a line is completely determined by these points. On the other hand, innumerable curved lines are possible between two points. A straight line is the shortest distance between two given points. Hence it follows that in a triangle each of the sides is less than the sum, but greater than the difference of the two others. If two triangles have the same base, so that the one lies entirely within the other, the outer has a greater perimeter than the inner (*pl.* 3, *fig.* 82). Two straight lines on one plane may intersect each other, either directly or when produced; they can, however, have but one point in common, or they may never meet even when produced. In the first case, they converge and form an angle; in the second, they are parallel. If two lines, whether parallel or not, as *kl, mn*, or *op, qr* (*fig.* 1), are intersected by a third straight line, *st*, then there will be eight angles formed,—four internal and four external: of these, the two internal or two external angles which lie on opposite sides of the secant line, and are not adjacent to each other, are called *alternate* angles, as *a* and *h*, *c* and *f*, *i* and *u*, *m* and *n*. Then again, two angles, one internal and the other external, lying on the same side of the secant without being adjoining, are called *opposite* angles; e. g. *a* and *i*, *c* and *g*, *k* and *o*, *m* and *n*. When parallel lines are intersected by a third straight line, each two of the alternate angles, as well as of the opposite, are equal to each other. In every triangle, the sum of all the angles is equal to two right angles. The angles

of a quadrilateral are together equal to four right angles, as will become evident if we divide the quadrilateral by a diagonal into two triangles (*pl.* 1, *fig.* 23). The sum of the angles in a pentagon is equal to six right angles, since two diagonals divide it into three triangles (*pl.* 3, *fig.* 4). And in general, the sum of the angles of a rectilineal figure is always equal to twice as many right angles, less 4, as the figure has sides. This proposition will become clearer if we draw, from a given point within the figure, lines to all its corners (*fig.* 5), and remember that the sum of all angles that have their vertex in one common point, is equal to four right angles. This proposition also holds good if the figure have a re-entrant angle (*fig.* 6); but in order to prove it in that case, it will be better to divide the figure by diagonals that must not intersect one another, into triangles, of which there will always be two less than the figure has sides.

3. OF THE EQUALITY OF FIGURES.

Two figures are said to be equal when they can be so applied to each other as to coincide throughout. The sides and angles of a figure are in such intimate and dependent relation, that from the equality of some of them we may infer the equality of the rest. For example, if of two triangles we know that three parts are mutually equal—either the three sides, or two sides and the included angle, or two sides and the angle which is opposite the greater of the two, or two angles and the included side—then may we conclude from this that the rest of the parts are also equal each to each, and that the triangles themselves are equal (*pl.* 3, *figs.* 11, 12). But of the three parts ascertained, one must always be a side, since two triangles of unequal sides may have equal angles, as in *fig.* 7. If, in this triangle ade, we add to de the parallel bc, then it is plain that the triangles abc and ade have their angles equal, since $q = q$, $m = n$, $o = p$; but the triangles themselves are by no means equal, since the one is only a part of the other. From these cases of equality it follows also what parts are necessary to construct a triangle. This is most easily done by having the three sides, a, b, c, given (*fig.* 8); but we may also employ two sides, a, b, and the included angle m (*fig.* 9), as well as two angles, m, n, and the included side a (*fig.* 10); or finally, two sides, a, b, and the angle m (*fig.* 18) lying opposite to one of them. It is to be observed, however, that when this angle lies opposite to the smaller of the two sides, two different triangles may be constructed, both of which will answer the conditions of the proposition, so that in this case the triangle is not completely defined. By means of the equality of triangles the following, among other properties, may also be proved: 1. In an isosceles triangle, the angles opposite the equal sides are also equal (*fig.* 13), for let $ab = ac$, and from a draw a line which bisects bc, then there will be two equal triangles in which $\angle m = \angle n$, from which it follows that $o = p$, and $q = r$, which shows that the line is perpendicular to bc, and bisects the angle at a. In an equilateral triangle, all the three angles are equal. 2. The greater angle of a triangle lies opposite to

the greater side, and vice versâ. If, in the triangle *abc*, (*fig.* 14), *ab* is greater than *ac*, then is also ∠ *acb* greater than ∠ *abc*, &c. 3. If two triangles have two sides of the one equal to two sides of the other, each to each, and the included angle unequal, the third sides will be unequal, and the greater side will belong to the greater triangle, which has the greater included angle; or if an angle of a triangle is increased, while its including sides remain the same, then will the side opposite the angle be also increased (*figs*. 15–17).

A quadrilateral is a parallelogram, when it can be proved,—1, that two of its opposite sides are equal and parallel; or 2, that all the opposite sides are equal; or 3, that all the opposite sides are parallel. The opposite angles of a parallelogram are also equal: two adjacent angles form together two right angles (*fig*. 22).

Hence all the angles in a parallelogram are either right angles or two are acute and two obtuse (rhombus and rhomboid). The two diagonals of a parallelogram mutually bisect each other (*fig*. 19).

To construct a parallelogram it is generally necessary to have two sides and an angle given. For a rectangle, however, two sides are sufficient; for a rhombus, one side and an angle; and for a square, one side. A trapezoid can be constructed by means of its four sides (*pl*. 3, *fig*. 21), by constructing first a triangle out of the two sides *ad*, *ae*, which are not parallel, and the difference of the parallel sides *de*, then producing *de* to *c*, so that *ce* may become equal to the lesser parallel, and with *aec*, the parallelogram *abce* will be completed. A trapezium may be constructed,—1, with four sides and an angle; 2, with three sides and the two included angles; 3, with three sides and the angles lying about the unknown side; 4, with two adjacent or two opposite sides and the three angles (*figs*. 23, 24).

Every regular polygon has a central point which is at an equal distance from all its sides and from all its angles. This point is found by bisecting two angles, for the bisecting lines always meet in that common centre (*fig*. 25).

The preceding propositions concerning the equality of triangles, enable us to solve and prove the solution of a number of problems of easier construction. Among these are: 1. to construct a triangle equal to a given triangle (*fig*. 26). 2. To describe a given angle *m*, on a given straight line *ab*, at a given point *a* (*fig*. 27). 3. Through a given point, to draw a line parallel to a given line. 4. To bisect a given angle (*pl*. 2, *fig*. 77). 5. To bisect a given line (*fig*. 75). 6. To draw a perpendicular to a line at a given point. 7. From a given point out of a line, to let fall a perpendicular on that line (*pl*. 3, *fig*. 28, and *pl*. 2, *fig*. 75). 8. To draw a perpendicular at the extremity of a given line (*pl*. 3, *fig*. 29, and *pl*. 2, *fig*. 76). 9. To draw a given line between the sides of a given angle *bac*, so that it may form a given angle *m* with one of the two sides (*pl*. 3, *fig*. 30). 10. To construct a triangle with a given line *a*, and two given angles *m* and *n* [the sum of which must be less than two right angles], (*fig*. 31). 11. To construct an equilateral triangle upon a given base (*pl*. 2, *fig*. 78). 12. To construct a square upon a given base *ab* (*pl*. 2, *figs*. 79, 80). In order

to obtain a regular octagon from a square, we must proceed as follows:—Draw (*pl.* 3, *fig.* 32) the two diagonals of the quadrilateral intersecting at *e*; bisect the angle *ced;* make the bisecting line *ef* = *ce* or *de*, and draw *cf* and *df;* finally, describe upon the remaining three sides of the quadrilateral, equilateral triangles, which are equal to the triangle *cdf*.

4. OF THE SIMILARITY OF FIGURES.

Two figures are called *similar* when they agree in their form, or more definitely, when the angles of the one are equal to the angles of the other each to each in the same order, and the sides of the one are in the same proportion as those of the other. We arrive at the latter definition as respects triangles, by examining two lines not parallel to each other, which are intersected by several parallel lines. If we divide one of the two lines *aq*, *ar* (*pl.* 3, *fig.* 33), into the equal parts, *ab* = *bc* = *cd* = *de*, commencing at the point of intersection, and then draw from the points of division, *b*, *c*, *d*, *e*, parallel lines, then the divisions of the other lines thus formed will also be equal to each other. If, again, we take in one of the two lines *aq*, *ar* (*fig.* 34), two or more unequal parts, beginning at the point of intersection *a*, and then draw parallel lines from the points of division *b*, *c*, then will the resulting sections, *ad*, *de*, of the other line, be in the same proportion to one another as the sections, *ab*, *bc*. In this case the triangles *abd*, *ace*, are similar; we readily perceive that their angles are equal, and that two sides of the one triangle are always in the same proportion as the corresponding and similarly placed sides of the other. To be certain that two given triangles, *abc*, *def* (*fig.* 35), are similar, it is only necessary to know,—1, that two angles, or 2, that one angle and the ratios of the including sides, or 3, that the ratios of two of the sides and the angles opposite to the greater of them, or 4, that two ratios of sides, are equal.

The following propositions, among many others, may be proved by means of the similarity of triangles. If in a triangle *abc* (*pl.* 3, *fig.* 36), lines be drawn from two angles, *a*, *b*, to the middle of the opposite sides, each of these lines cuts the other into two parts, of which the one lying towards the bisected side is half of the other. From this it readily follows that all the three lines drawn from the angles of a triangle to the middle of the opposite sides, pass through one and the same point. If we bisect the angle *a* of the triangle *abc* (*pl.* 3, *fig.* 37), it may be readily shown that the segments into which the opposite side *bc* is divided by the bisecting line, are in the same proportion as the two other undivided sides, thus, $bd : ed :: ab : ac$. Hence we deduce the proposition, that lines bisecting the three angles of a triangle cut each other in one and the same point, which is of importance, as being the centre of the circle inscribed in the triangle. To cut off from a given triangle a smaller one similar to it, we may either, 1, draw a line parallel to one side of the triangle, or 2, from one angle of the triangle, not the least, cut off by a line another angle equal to a smaller one of the same triangle. E. g., if in *fig.* 38 the angle $n = m$, then

will the triangle *acd* be similar to *abc*. And if the second triangle, *aba* thus formed, is also to be similar to the original triangle, then must $q = o$, also $n + q$ or $bac = m + o$, which is only possible when *bac* is a right angle. In this case the bisecting line *ad* is perpendicular to *bc*. If, therefore, in a right angled triangle, we let fall a perpendicular from the vertex of the right angle upon the hypothenuse, the perpendicular thus let fall will divide the triangle into two smaller ones, similar to each other and to the original triangle (*fig*. 39–41). From this may be easily deduced, 1, that the perpendicular let fall from the vertex of the right angle, is a mean proportional between two segments of the hypothenuse ; 2, that either side about the right angle is a mean proportional between the whole hypothenuse and adjacent segments. From the latter proposition follows another: that when the sides of a right angled triangle are expressed in numbers, the square of the hypothenuse will be equal to the sum of the squares of the other two sides.

With respect to the similarity of such rectilineal figures as have more than three sides, we will confine ourselves here to the following proposition: two figures are similar, when they can be divided by similarly situated diagonals into triangles which are similar each to each (*pl*. 3, *fig*. 42).

Similarity of figures may also be applied to the solution of numerous problems of construction, of which we will here mention only one ;—to find a fourth proportional to the three given lines, *a*, *b*, *c* (*fig*. 43). This is a problem of the same importance in Geometry as the Rule of Three is in Arithmetic.

5. OF THE EQUIVALENCE OF AREAS IN FIGURES.

Figures are said to be equivalent when they occupy equal areas. In equality we combine similarity with equivalence. We must here premise that in triangles and parallelograms, some one side is assumed as the ground line or basis upon which the figure is supposed to rest, and that then the height or altitude is the perpendicular distance from this basis to the opposite side or angle.

Two parallelograms are equivalent, when their bases and altitudes are equal (*pl*. 3, *fig*. 45–47). Here we may always consider them as erected upon the same base, and the opposite sides will then be in one and the same parallel; in which case, apart from the condition of equality or complete coincidence, three conditions, as represented in *figs*. 45, 46, 47, are possible. A triangle is always the half of a parallelogram of the same base and altitude, therefore equal to a parallelogram of the same altitude and half the base, or to one of an equal base and half the altitude; whence it follows that triangles of equal bases and altitudes are equivalent (*fig*. 48). If we assume in succession two different sides of the same triangle as bases, they will be inversely proportional to their corresponding altitudes, viz. $ab : ac :: bc : cd$ (*fig*. 49). A trapezoid may be divided by a diagonal into two triangles, which will have the parallel sides of the trapezoid for their bases, and the perpendicular distance between these sides for their common alti-

tude; it is, therefore, equal to a parallelogram whose base is equal to half the sum of the parallel sides, and whose altitude is equal to their perpendicular distance from each other (*fig.* 50). A rhombus, whose diagonals are perpendicular to each other, will be four times as large as a right angled triangle, which has for its two legs half the diagonals of the rhombus (*fig.* 51). The areas of two parallelograms as well as of two triangles of the same base, are to each other as their altitudes; of the same altitude, as their bases; and generally, parallelograms are to each other as the products of their bases by their altitudes. The areas of two squares are to each other as the squares of their sides. The areas of two similar triangles are to one another as the squares of their homologous or similarly situated sides (*fig.* 53); the same is true generally with regard to the areas of two similar figures. If on the three sides of a right angled triangle, three similar figures, triangles or any others, be constructed, the figure on the hypothenuse will be equivalent to the sum of those on the two legs (*pl.* 3, *fig.* 54). A particular case of this proposition is known as the Pythagorean: the square described upon the hypothenuse is equivalent to the sum of the squares described on the other two sides.

As the unit of measure for the determination of the superficial relations of figures, we use a square whose side is equal to the unit of length, which, therefore, according to the length of the side, is called a square foot, a square inch, &c. To ascertain how many times one square is contained in another, it is necessary to find out how many times the side of the one is contained in that of the other, and the number thus obtained multiplied by itself; hence a square foot contains not 10 or 12 square inches, but 100 or 144, according to the number of inches, 10 or 12, into which the foot is divided, &c. The area of a square may thus be found, by measuring one of its sides and then multiplying the number expressing its length by itself. Hence we are accustomed to call the product of a number by itself, or the second power, its square. The area of a parallelogram is found by multiplying the base by the altitude (expressed in the same unit of measure); that of a triangle by multiplying the base by half the height, or the height by half the base; that of a trapezoid by multiplying half the sum of the parallel sides by their perpendicular distance; that of a regular polygon by multiplying its circumference or perimeter by half the perpendicular let fall from the centre on one of the sides; that of an irregular polygon by dividing it by diagonals into triangles, whose areas must be separately ascertained and added together.

By the assistance of the preceding propositions, many problems relative to the changing and dividing of figures may be solved. A few of these problems are the following:—1. To change a triangle into a parallelogram of equal area, or the contrary (*fig.* 55). 2. To change the triangle *abc* into another of equal area, and with a given side *be* (*fig.* 56). 3. To change a parallelogram into a rhombus of a given side *cf* (*fig.* 57), or of a given angle *m* (*fig.* 58). 4. To change a given triangle *abc* into an equilateral triangle (*fig.* 59). 5. To change a quadrilateral *abcd* into a triangle (*fig.* 60). 6. To change a given figure into another of a prescribed

shape, e. g. the triangle *abc* (*fig.* 61) into a quadrilateral similar to the given quadrilateral *defg*. 7. To divide a triangle into a certain number of equal parts by lines proceeding from one angle. 8. To cut off from a triangle a certain portion, as for instance a third, by means of a line which is to proceed from a given point, *d*, in one of its sides (*fig.* 62). 9. To cut off from a triangle, *abc*, a certain part, as a third, by lines proceeding from a given point, *d*, within the triangle (*fig.* 63). 10. From a triangle, *abc*, to cut off a certain portion, by a line parallel to one of the sides (*fig.* 64). 11. To divide a parallelogram into a given number of equal parts, by lines parallel to one of its sides. 12. From an acute angled parallelogram to cut off a given part by a line perpendicular to two of the sides (*pl.* 3, *fig.* 65). 13. To divide a parallelogram, *abcd*, into a certain number of equal parts, by lines proceeding from a given point in one of the sides (*fig.* 66). 14. From a trapezoid, *abcd*, to cut off a given part, for instance the half, by a line parallel to its parallel sides (*fig.* 67). 15. To cut off from any quadrilateral, *abcd*, a given part, by a line proceeding from a corner, *a*, or from a given point, *e*, in one of its sides (*figs.* 68, 69).

6. OF THE CIRCLE AND ITS MEASUREMENT.

A circular line cannot have more than two points in common with a straight line (*fig.* 70). A straight line intersects or touches the circle, according as it has two points in common with the circumference, or only one; in either case we must consider the line as indefinitely produced in either direction. We obtain a tangent, when we draw a perpendicular to the extremity of a radius or diameter (*fig.* 71). On the other hand, a radius drawn to the point of tangency of a tangent, will be perpendicular to it; whence it follows, that to any point of a circumference only one tangent can be drawn. Lines drawn from the same point, tangent to a circumference, are equal to each other, e. g. $su = sv$ in *fig.* 72.

Equal angles at the centre of the same circle, or of equal circles, have equal chords and areas, and the reverse. An angle at the centre is measured by the number of degrees contained by its arc. An inscribed angle is half the angle at the centre of the same arc, and is therefore measured by the half of its arc. An angle formed by a tangent and a chord is measured by half the arc included between the tangent and the chord (*fig.* 73). Inscribed angles resting upon the same or upon similar arcs are equal (*fig.* 75). When two chords intersect each other, either within the circle, or when produced, without it, the angle thus formed is measured in the first case by half the sum, and in the second by half the difference of the two arcs included between the chords (*fig.* 74). Every angle inscribed in a semicircle is a right angle (*fig.* 77). If at any given point of a diameter a perpendicular be drawn to the circumference, it will be a mean proportional to the two segments of the diameter (*fig.* 78).

From the preceding propositions may be obtained the solution of the following problems: 1. To find the centre of a circle or of a circular arc

(*pl.* 2, *fig.* 84). 2. To bisect a circular arc. 3. To araw a tangent to a given point in a circumference (*pl.* 2, *fig.* 85). 4. From a given point out of a circle, to draw a tangent to the circle (*pl.* 3, *fig.* 80). 5. Upon a given base, *ab*, to construct a triangle in which the angle opposite the base is equal to the given angle *m* (*fig.* 76). This problem is indefinite, since every point of an arc may be taken as the vertex of the triangle; but it becomes definite if the height of the triangle is also given. A particular case is exhibited in the problem: upon a given line, as hypothenuse, to construct a right angled triangle. 6. To construct a mean proportional to two given lines. 7. To divide a given triangle, *abc*, by lines running parallel to a given side, into a certain number of of parts, five for instance, that shall be either equal, or in a definite proportion (*fig.* 79).

The construction of regular polygons in and about the circle, is of importance in understanding its theory. A regular polygon is said to be inscribed in the circle, when all its sides are chords; and circumscribed about the circle, when all its sides are tangents. A regular polygon is inscribed in a circle, by dividing the circumference of the latter into as many equal parts as the polygon is to have sides, and connecting these points by chords. The difficulty here lies only in dividing the circumference into a given number of equal parts. The division into four or six parts is most easily made; the former, by drawing two diameters perpendicular to each other; the latter, by using as chords, lines equal to the radius (*fig.* 81). To divide the circumference into ten equal parts, we draw two radii perpendicular to each other, bisect the one, and connect the point of bisection with the extremity of the other, and then cut off from this connecting line a section equal to the half of the radius; the remainder will be the length of a chord whose arc is the tenth part of the circumference, or the side of a regular inscribed decagon. *Pl.* 2, *fig.* 81, shows the construction of a regular pentagon in a circle. *AB* is here a diameter, *CD* a radius perpendicular to it; from *F* the middle point of *BC*, with a radius equal to *FD*, we describe an arc intersecting *AC* in *G;* draw *DG;* this will be the side of the regular pentagon (*CG* will be the side of the regular decagon). We may obtain a pentagon by connecting the alternate angles of a decagon. From the division into four equal parts, we may readily obtain that into 8, 16, 32, &c., and the division into 10, 20, 40, 80, &c. The fifteenth part of the circumference is found by subtracting the 10th part from the 6th, for $\frac{1}{6} - \frac{1}{10} = \frac{4}{60} = \frac{1}{15}$.

A regular polygon is circumscribed about a circle, by dividing the circumference into as many equal parts as the polygon is to have sides, and then drawing tangents to all the points of division. From a polygon of any given number of sides inscribed in the circle, we may obtain a regular polygon of double the number of sides, by bisecting the arcs, whose chords form the sides of the former, and drawing chords to the half-arcs. The circumference (as well as the area) of a circle is always greater than the perimeter (or area) of an inscribed polygon, but is less than the perimeter (or area) of one circumscribed about it (*pl.* 3, *fig.* 83).

The circumference of a circle cannot be directly measured, since it is not a straight line; but if two polygons of a great number of sides be

described in and around it, and their circumferences determined, that of the circle will be intermediate. In this way Archimedes determined the ratio of the diameter to the circumference as $7:22$, and Ludolph, as $1:3\ 14159$. The latter number is employed and indicated by π, as the ratio to a diameter of 1 (or unity). Accordingly, since circles are to each other as their diameters, the circumference of any circle may be found by multiplying its diameter by π ($= 3.1415926$).

Every circle may be regarded as a regular polygon of an infinite number of sides; hence, also, as a triangle whose base is equal to the circumference of the circle, and whose altitude is the radius. We consequently obtain the area of a circle by multiplying the circumference by half the radius, or according to the preceding proposition, by multiplying the second power of radius by π. A sector is equal to a triangle whose base is the length of the arc, and whose altitude is equal to the radius (*pl.* 3, *fig.* 84).

Allied to the circle are the symmetrically curved lines, the oval and the ovate: each one consisting of four elongated quadrants. In the former the quadrants are all equal; in the latter only the two lying on the same side of the short axis. The following are some constructions of ovals. In *pl.* 2, *fig.* 87, an isosceles triangle, *CDE*, is constructed upon the base *CD*, and under it another and equal arc, *CDF*. From *C* and *D*, with any radius, $CA = DB$, describe arcs intersecting the equal sides produced of each triangle in *G* and *I*, *H* and *K*: and finally, connect these points by arcs described with the radii $FH = EK$, from *F* and *E* as centres. *Fig.* 88 agrees with the preceding construction, except in that the two equal triangles employed, are equilateral. In *fig.* 88, the length of the oval, or of the major axis, *AB*, is given. Divide it at *C* and *D*, into three equal parts. From the points *C* and *D*, with radii equal to $\frac{1}{3}$ *AB*, describe circles intersecting in *E* and *F*. From these points draw two diameters in each circle, *GF*, *EI*, *FH*, *EK*, and with one of these diameters as radius, from the points *E* and *F*, describe the arcs *IK* and *GH*, completing the outline. In this construction the breadth of the oval is a little more than $\frac{3}{4}$ of the length. An oval of less breadth, with the same length, *AB*, may be thus obtained (*fig.* 89). Divide *AB* into four equal parts, and from the points of division, *C*, *D*, *E*, with a radius equal to $\frac{1}{4}$ of *AB*, describe three circles, intersecting each other in *F*, *G*, *H*, and *I*. Through these points draw in the first and third circles the diameters *MH*, *NI*, *FK*, and *GL*, prolonging them until they intersect in *O* and *P*. From *O* and *P*, with radii $OK = PM$, describe the arcs *KL* and *MN*. In this construction the breadth of the oval is not quite $\frac{2}{3}$ the length. In *fig.* 91, a half-oval of given length, *AB*, is constructed in the following manner. From the points *A* and *B*, in the line *AB*, any part, $AH = BK$, is taken, and with this distance as radius, arcs are described cutting each other in *I* and *L*; with the distance between *I* and *L* as radius, describe, from these points as centres, arcs cutting each other beneath *AB*; finally, from *D* as centre, complete the circle by the arc *IL*.

II. STEREOMETRY, OR THE GEOMETRY OF SOLIDS.

1. OF THE POSITION OF LINES AND PLANES IN SPACE.

Through one or two points, as well as through one straight line, innumerable planes may pass. Only one, however, can pass through three points not in the same straight line, through two parallel or intersecting straight lines, or through a straight line and a point external to it. Two planes meeting each other without coinciding, form a straight line by their intersection. A straight line not in a plane, can have only one point in common with it. It is perpendicular to the plane when it is perpendicular to all straight lines drawn through its foot in the plane; this is likewise the case when it forms a right angle with two lines lying in the plane (*pl.* 3, *figs.* 86, 87). The angle of inclination of a line to a plane not perpendicular to it, is found by letting fall a perpendicular from any point of the line upon the plane, and connecting the extremities of the two lines by a line situated in the plane. This is the least angle which the straight line can make with lines drawn through its foot on the plane. Two straight lines perpendicular to the same plane, are parallel to each other (*fig.* 90). If, of two parallels, one stands perpendicular to a plane, the other must also. A straight line is parallel to a plane, as well as a plane parallel to a straight line, when they will not meet if produced. If a straight line be parallel to a plane, and we pass through the line, planes cutting the first plane, the lines of intersection will be parallel to each other and to the plane (*fig.* 88). Two planes perpendicular to the same straight line are parallel to each other (*fig.* 90). Two parallel planes, intersected by a third, will have the lines of intersection parallel (*fig.* 91). If two straight lines in space be intersected by three parallel planes, the segments of the one will be proportional to those of the other (*fig.* 93). Parallels or perpendiculars between two parallel planes are equal; hence the distance between two parallel planes is measured by a perpendicular let fall from one upon the other. Two angles which have three sides parallel are equal; if they lie in different planes, these latter are parallel (*fig.* 92). The inclination or separation of two planes not parallel, is measured by the angle formed by lines in each plane, drawn perpendicular to a point in the line of intersection of the planes. Planes, like lines, may form adjacent and opposite or vertical angles, which, with respect to their magnitudes, have the same properties as those of lines (*fig.* 89). Two planes are perpendicular when their angle is a right angle. If a plane be perpendicular to two intersecting planes, it will also be perpendicular to their line of intersection. If a straight line be perpendicular to a plane, every plane passing through the former will be perpendicular to the latter. When three or more planes meet in one point, they form a corner or solid angle (*pl.* 3, *fig.* 94). The edges or lines of intersection of planes meeting in this manner, form as many plane angles as there are planes. If the solid angle be formed by

three planes, then is the sum of any two of the plane angles greater than the third. In any case, however, the sum of any number of plane angles, forming a solid angle, is less than four right angles.

2. OF ANGULAR SOLIDS.

A solid may be inclosed either by plane surfaces alone, in which case it is called a polyhedron, or by curved surfaces alone, or by both plane and curved at the same time. Bodies of the first and third kind have a base, that is, a plane surface upon which the solid is supposed to rest. If such a body should have another plane bounding surface parallel to this base (in which case this plane may be considered another base), or a vertex opposite to the base, the distance between the two surfaces, or between the vertex and the base (in both cases measured by a perpendicular let fall), is called the altitude of the solid. The planes bounding a polyhedron are called its faces; their intersections, its edges. No polyhedron can have less than four faces, four solid angles, or six edges. Furthermore, no polyhedron can be inclosed by figures of six or more sides, or have equal solid angles formed by six or more plane angles.

Two solids are said to be equivalent when the spaces inclosed between their bounding surfaces are equivalent; they are equal when they agree exactly in shape and size, so that the one may be taken for the other.

A polyhedron is called regular when it is inclosed by perfectly regular and equal figures, and has all its angles equal. There are only five regular solids; 1, *tetrahedrons*, bounded by four triangles (*pl.* 2, *fig.* 56); 2, *octohedrons*, by eight (*fig.* 58); 3, *icosahedrons*, by twenty (*fig.* 60); 4, *hexahedrons*, bounded by six squares (*fig.* 57); 5, *dodecahedrons*, by twelve pentagons (*fig.* 59). The expansion of some of these solids, or the representation of their surfaces as spread out in a plane, may be found in *pl.* 4, where *fig.* 49 is the expansion of the tetrahedron, *fig.* 50 that of the hexahedron, *fig.* 51 of the dodecahedron. A solid, bounded by regular figures of two kinds, and which has, at the same time, all the solid angles equal, is called an Archimedean solid. If we limit ourselves to polyhedrons having triangles and squares for faces, such a solid may be contained, 1, by two triangles and three squares (a special case of the three-sided prism); 2, by eight triangles and six squares; 3, by eight triangles and eighteen squares (*pl.* 2, *fig.* 73^b); and, 4, by thirty-two triangles and six squares.

The most important angular solids are the prisms and pyramids. A prism is a solid bounded by two equal and parallel rectilineal figures (forming the bases) and as many parallelograms as each base has sides. It is called three, four, five-sided, as the bases are triangles, quadrilaterals, pentagons, &c. (*pl.* 2, *figs.* 61, 62, 63). The prism is called a right prism if the lateral faces are perpendicular to the bases, otherwise it is oblique. A four-sided prism whose bases are parallelograms, is called a parallelopipedon; when all the faces are squares, it is a cube or hexahedron. If a prism be intersected by a plane parallel to the base, the section formed will be equal

to the base (*pl.* 3, *fig.* 96). The sections of two planes parallel to each other, but not to the base, are equal. *Plate* 2, *fig.* 64, represents a parallelopipedon intersected by a plane, not parallel to the base. Prisms of equivalent bases and equal altitudes are equal to each other (*pl.* 3, *figs.* 97–99). Prisms of equal bases, but of unequal altitudes, are to each other as their altitudes; those of equal altitudes, as their bases; those of unequal bases and altitudes, as the product of the two. A cube whose edge is the unit of length, serves as the unit of measure for determining the volume of a solid; it is called cubic foot, cubic inch, &c., as the edge is a foot, inch, &c. The volume of a cube is obtained by raising the number expressing the length of its edge, to the third power; that of any prism in general by multiplying the area of the base by the altitude, the same unit of measure being used in both.

A pyramid is a solid, bounded by any rectilineal figure as base, and as many triangular planes meeting at the vertex as the base has sides. It is called three, four, or five sided, &c., as the base has three, four, five, or more sides (*pl.* 2, *figs.* 65, 66, 67, 70). If a plane be passed through a pyramid, parallel to the base, the section thus formed will be similar to the base, and will bear to it the same proportion as the square of the perpendicular let fall from the vertex on the section, to the square of the altitude of the pyramid (*pl.* 3, *fig.* 95). A three-sided prism may be divided into three equivalent pyramids, of which two have the same base and altitude as the prism. Hence it follows that every pyramid is $\frac{1}{3}$ the prism of equivalent base and altitude. Consequently, the solid content of a pyramid is obtained by taking $\frac{1}{3}$ of the product of the base by the altitude. If from a pyramid we cut off a smaller pyramid, by a plane parallel to the base, the part that is left is called a truncated pyramid or a frustum. Such a solid is equivalent to three perfect pyramids of the same altitude with it, and having for bases the upper base of the frustum, the lower base, and a mean proportional between the two bases (*fig.* 101). If a three-sided prism be intersected by a plane not parallel to the base, the part remaining is equivalent to the sum of three pyramids of the same base as the prism, but which have for vertices the corners of the triangle in which the prism is intersected by the plane (*pl.* 3, *fig.* 100).

3. OF THE ROUND BODIES.

Among those solids inclosed by both plane and curved surfaces, the cylinder and cone are the best known and most important, as is the sphere among those the whole of whose surfaces are curved; these together are known as the round bodies. The common or typical cylinder is bounded by two equal and parallel circles (forming the bases), and a curved lateral surface uniting their circumferences. The latter is a simple curved surface, and may be generated by the revolution of one straight line around the circumference of a circle, but not in its plane, and constantly parallel to a fixed line which then forms the axis. The cylinder is *right* (*pl.* 2, *fig.*

68), or *oblique* (*fig.* 71), as this axis is perpendicular or oblique to the base. A right cylinder will manifestly be generated by the revolution of a rectangle about one of its sides. On the convex surface of the cylinder, innumerable straight lines may be drawn, parallel to each other and the axis. If a cylinder be intersected by a plane passing through the axis, the section will be a parallelogram (a rectangle in the right cylinder); if the plane be parallel to the base, the section will be a circle equal to the base; if it have any other position, an ellipse will be formed. Every cylinder may be considered as a prism of an infinite number of sides; its volume, as in the prism, will evidently be obtained by multiplying the area of the base by the altitude. The convex surface of the right cylinder is equal to the area of a rectangle whose base is equal to the circumference of the base of the cylinder, and whose altitude is the altitude of the cylinder. The determination of the convex surface of an oblique cylinder is very difficult.

A cone (*fig.* 69) is bounded by a circle as base, and a convex surface running to a point. The latter is a simple curved surface, and is generated by the revolution of a line around the circumference of a circle, and fixed to a point not in the plane. A straight line from the vertex to the middle of the base, is called the axis of the cone, which is termed right or oblique, as this axis is perpendicular or oblique to the base. The ordinary right cone is produced by the revolution of a right angled triangle about one of the short sides. On the convex surface of the cone, from the vertex to the circumference of the base, innumerable straight lines may be drawn, which in the right cone are all equal to each other. Every cone may be considered as a pyramid of an infinite number of sides. Since, then, the pyramid is the third part of a prism of the same base and altitude, the cone will be the third part of a cylinder of the same base and altitude. When a cone is intersected by a plane we obtain, 1, a triangle, when the plane of intersection is parallel to the axis (isosceles, in right cones); 2, a circle, when the plane is parallel to the base; in any other position, one of the three curves, known as the conic sections, which are next in importance to the circle (ellipse, parabola, and hyperbola). When a cone has its upper part or vertex cut off by a plane parallel to the base, it is said to be truncated: this is equivalent to the sum of three cones, whose altitude is that of the truncated cone (or frustum), and which have for bases, the upper base of the frustum, the lower base, and a mean proportional between the two bases. The area of the convex surface of the right cone, is equal to that of a sector of a circle whose radius is the length of the side of the cone, and whose arc is equal to the circumference of the base The area of the convex surface of a truncated cone is equivalent to that of a rectangle whose altitude is the length of the side of the truncated cone, and whose base is equal to half the sum of the circumference of the two bases.

A sphere is inclosed by a single curved surface, all of whose points are equally distant from a point within, called the centre. A straight line drawn from this centre to any point of the surface is called a radius; all radii of a sphere are equal. A diameter is a straight line passing through

the centre, connecting two points of the surface. The section of a sphere by a plane is a circle, which is smaller as the distance of the plane of intersection from the centre is greater (*pl.* 3, *fig.* 103). If the plane pass through the centre, the circle thus formed whose diameter is that of the sphere, is called a great circle. All others are small circles. A line connecting the centre of a sphere with that of a circle of intersection, is perpendicular to the plane of the latter. If two or more circles therefore are parallel to each other, their centres will all be in a diameter of the sphere, perpendicular to their planes; this is called their axis, and its extremities their poles. Every great circle bisects the sphere; two great circles mutually bisect each other, and divide the surface into four parts. If one great circle pass through the poles of another, their planes will be perpendicular. The angle between two great circles is measured by the arc of a circle they intercept, whose plane is perpendicular to that of the two circles (*pl.* 3, *figs.* 103, 110). Two parallel circles include a part of the sphere called a spherical segment, and a part of the surface called a zone. If one of the circles be tangent to the sphere, the zone has only one base. The altitude of a zone or spherical segment is the perpendicular distance between the planes of the bases. The area of a zone is obtained by multiplying its altitude by the circumference of a great circle (*fig.* 102).

The surface of a sphere is equal to the area of four great circles. The solidity of a sphere is obtained by multiplying the third power of the diameter by π (3·1415926) and dividing by 6. If we take a cone, hemisphere, and cylinder, of the same base and altitude (the altitude equal to a radius of the hemisphere), the solidities of these three bodies will be to each other as 1, 2, 3, that is, the cone will be one half the hemisphere, and this, two thirds of the cylinder; a cone, sphere, and cylinder will be in the same proportion, if the first and last have for bases, a great circle of the sphere, and for altitudes, a diameter (*pl.* 3, *fig.* 104).

III. TRIGONOMETRY, OR THE MEASUREMENT OF TRIANGLES.

1. PLANE TRIGONOMETRY.

Plane Trigonometry teaches how to obtain all the parts of a plane triangle, three numerically expressed parts being given, one of which must always be a side. Since every rectilineal figure may be divided into triangles, trigonometry serves for the determination of all rectilineal figures. Geometry gives directly but a single example, viz. the determination of the third side of a right angled triangle, knowing the other two. To obtain this result we square the numbers expressing the lengths of the known sides, add them together, if the hypothenuse is desired, or subtract the less from the greater, for one of the legs. The square root of the result will be the length of the third side.

Instead of the angles, certain quantities are employed whose value depends on that of the angle, and which are called the trigonometrical functions. The most important of these are the sine, cosine, tangent, and cotangent. The explanation of these may be best made in a right angled triangle (*fig.* 105). Here the side opposite an acute angle, as *abc* or *m*, divided by the hypothenuse, as $\frac{ac}{ab}$, is called the sine of that angle, likewise the cosine of the other acute angle, *bac* or *n*; 2, the side opposite an acute angle, *abc*, divided by the other short side, as $\frac{ac}{bc}$, is the tangent of that angle, and likewise the cotangent of the other acute angle, *bac*. Consequently—

$$\text{Sin. } m = \text{cos. } n = \frac{ac}{ab}$$

$$\text{Sin. } n = \text{cos. } m = \frac{bc}{ab}$$

$$\text{Tang. } m = \text{cot. } n = \frac{ac}{ab}$$

$$\text{Tang. } n = \text{cot. } m = \frac{bc}{ac}$$

Consequently, in similar right angled triangles of different size, the sines, cosines, &c., of the homologous or corresponding angles will be equal. If the hypothenuse of the right angled triangle be taken as unity, then the side opposite an acute angle may be taken as the sine of that angle and the cosine of the other. How far the sine, cosine, &c., of an angle varies with its size, may be seen in *fig.* 106. Here *abc* is a quadrant whose radius is taken as unity. Consequently, $de =$ sin. *dbc*; $fg =$ sin. *fbc*; $be =$ cos. *dbc*; $bg =$ cos. *fbg*; whence it follows that the sine of an (acute) angle is greater, and the cosine less, as the angle is greater. Consequently, the tangents likewise increase, and the cotangents diminish as the (acute) angle increases. The sines and cosines of (acute) angles are evidently always fractions, while the tangent and cotangent of $45° = 1$; tangents of more than 45° are greater than unity, and as the angle approaches 90°, they become very great, tang. $90° =$ infinity; the same is the case with the cotangents of angles less than 45° and approaching 0.

The sines, cosines, tangents, and cotangents of all acute angles, have been calculated and arranged in tables called trigonometrical tables, which are indispensable in all trigonometrical calculations. The ordinary tables, however, do not contain the sines, cosines, &c., themselves, but their logarithms, as these are more readily employed in calculations.

From the preceding explanations may be readily derived rules for solving all possible cases of right angled triangles. For acute angled triangles, the following two propositions are of the greatest importance:—1, any two sides of a triangle are to each other as the sines of their opposite angles (*pl.* 3, *figs.* 107, 108). In *fig.* 107, the triangle *abc* is divided into two right angled triangles, *abd* and *acd*, by the perpendicular let fall from *a* on *bc*.

From the first we have sin. $m=\frac{ad}{ab}$; from the second, sin. $n=\frac{ad}{ac}$; whence sin. m : sin. n :: $\frac{1}{ab}$: $\frac{1}{ac}$ :: $ac : ab$. In *fig.* 108, where the triangle abc is obtuse angled, and the perpendicular let fall from c meets only the prolongation of ab, we have sin. $o=\frac{cd}{ac}$, and sin. $n=\frac{cd}{bc}$, whence sin o : sin. n :: $bc : ac$; so that the preceding proposition holds good also for obtuse angled triangles, if, instead of the sine of the obtuse angle, we take that of the angle which must be added to the obtuse angle, to make two right angles. 2. The sum of two sides of a triangle is to the difference of these sides, as the tangent of half the sum of the angles lying opposite to them, is to the tangent of half their difference. In the triangle abc (*fig.* 109), we accordingly have $ab+ac : ab-ac$ :: tang. $\frac{1}{2}$ $(acb+abc)$: tang. $\frac{1}{2}$ $(acb-abc)$. In the figure, with the lesser of the two sides, ab and ac, namely ac, a semicircle is described cutting ab and its prolongation in d and e, the chords cd and ce drawn, as also df parallel to ce. Then cdf and dce being right angles, we have $be : bd$, that is $ab+ac : ab-ac :: ce : df$. But $ce=cd$ tang. x, and $df=cd$ tang. y; moreover, $x=\frac{1}{2}\,cae=\frac{1}{2}\,(acb+abc)$; and $y=x-n=\frac{1}{2}\,(acb-abc)$, whence the preceding proposition immediately follows.

If we distinguish the angles of a triangle by A, B, C, and the sides opposite to each by a, b, c, we have the following formula for the solution of triangles.

I.—*For right angled triangles, when* A *is the right angle.*

1. Given the hypothenuse a, and a side b; then sin. $B=\frac{b}{a}$; $c=a$. cos. B.
2. Given the hypothenuse a, and an acute angle B; then $b=a$. sin. B; $c=a$. cos. B.
3. Given the two sides b and c; then tang. $B=\frac{b}{c}$; $a=\frac{b}{\text{sin. B}}=\frac{c}{\text{cos. B}}$.
4. Given the side b, and an acute angle B or C; then $a=\frac{b}{\text{sin. B}}=\frac{b}{\text{cos. C}}$; $c=b$, cot. $B=b$, tang.C.

II.—*For acute angled triangles.*

1. Given a side, a, and two angles; then $b=\frac{a\,\text{sin. B}}{\text{sin. A}}$; $c=\frac{a\,\text{sin. C}}{\text{sin. A}}$.
2. Given two sides, a, b, and an opposite angle, A; then sin. $B=\frac{b\,\text{sin. A}}{a}$; $c=\frac{a\,\text{sin. C}}{\text{sin. A}}=\frac{b\,\text{sin. C}}{\text{sin. B}}$.

Obs. If the side a, opposite the given angle, A, be less than the side b, there will be two solutions possible, since for B, we may take the acute angle answering to sin. B in the tables, and likewise its obtuse supplemental angle, whence there will also be two values for C and c.

3. Given two sides, a, b, and the included angle C ; then tang. $\frac{1}{2}$ A—B $=\frac{(a-b) \text{ tang. } \frac{1}{2}(A+B)}{a+b}$; $A=\frac{1}{2}(A+B)+\frac{1}{2}(A-B)$; $B=\frac{1}{2}(A+B)-\frac{1}{2}(A-B)$; c remains as before.

4. Given the three sides, a, b, c. Then indicating by s, the half sum of the sides, $\frac{a+b+c}{2}=s$; we have tang. $\frac{1}{2}$ A $=\sqrt{\left(\frac{(s-b)\ (s-c)}{(s-a)\ s}\right)}$; tang. $\frac{1}{2}$ B $=\sqrt{\left(\frac{(s-a)(s-c)}{(s-b)\ s}\right)}$; tang. $\frac{1}{2}$ C $=\sqrt{\left(\frac{(s-a)\ (s-b)}{(s-c)\ s}\right)}$.

2. SPHERICAL TRIGONOMETRY.

Spherical Trigonometry teaches the calculation of spherical triangles; that is, of such triangles as are formed on the surface of a sphere, by arcs of great circles. In such a triangle there are also six parts, of which three must be given to determine the rest.

Every spherical triangle answers to a three-sided solid angle, from whose vertex, with any radius, circles are described. Consequently the three sides of the spherical triangle on the surface of the sphere, measure the plane angles at the centre forming the solid angle, and its angles, the inclination of their planes. Hence spherical trigonometry serves for calculating solid angles, and may thus be called solid trigonometry.

On account of what is to follow, some of the most important properties of spherical triangles may here be introduced, although they belong properly to Stereometry. Every two sides of a spherical triangle are together greater than a third (*pl.* 3, *fig.* 111). If through the centre of the sphere, and the sides of the spherical triangle *abc*, we pass three planes, these latter will form a solid angle, whose three plane angles are measured by the arcs, *ab*, *ac*, *bc*. Since any one of these three plane angles is less than the sum of the other two, the same must be true with respect to the three arcs or sides of the spherical triangle.

The sum of the three angles, *aob*, *aoc*, *boc*, is less than four right angles; likewise the sum of the three sides is less than the entire circumference or 360°.

The area of a spherical triangle is proportional to the excess of the sum of its angles over two right angles (called the spherical excess). A spherical triangle, *def*, is called the polar or supplemental triangle of another, *abc* (*pl.* 3, *fig.* 112), where the vertices of the angles of this second triangle are respectively poles of the sides of the first. If *def* be the polar triangle of *abc*, the latter will be, on the other hand, the polar triangle of the former. Every angle of the polar triangle is measured by a semi-circumference minus the side lying opposite to it in the other triangle, whence the name

(supplemental triangle). Hence it follows that the sum of the angles of a spherical triangle must be greater than two right angles, and less than six. A spherical triangle is called right angled, when at least one of its sides is a right angle. If the triangle *abc* (*fig.* 113) be right angled at *c*, and we produce the sides *ab* and *cb* to *d* and *e*, so that $ad = ce = 90°$, and unite *d* and *e* by the arc of a great circle, then *bde* is called the complemental triangle of *abc*, and $de +$ the angle $bac = 90°$; as also $bed +$ the side $ac = 90°$.

The sines of the sides of a spherical triangle are to each other as the sines of the opposite angles. Let *abc* (*fig.* 114) be a spherical triangle, whose sphere has its centre in *o*, and unity for radius. If now from *c*, on the plane *aob*, we let fall the perpendicular *cd*; from *d* on *ae*, *bo*, the perpendiculars *de*, *df*, and draw *ce*, *cf*; it would be easy to show that the triangles *ceo*, *cfo* are right angles, and consequently that $ce =$ sin. *cod*, $=$ sin. arc *ca*, $cf =$ sin. $cob =$ sin. arc *cb*.

One of the most important formulæ in spherical trigonometry is that which expresses the cosine of an angle of a triangle, in terms of the three sides. To obtain this formula we may employ *fig.* 115, where *abc* is a spherical triangle, *o* the centre of the sphere, *cd* and *ce* tangents to the sides *ca* and *cb*, meeting the radii *oa* and *ob* in *d* and *e*. Drawing *de*, then according to a proposition of plane trigonometry, $\overline{de}^2 = \overline{cd}^2 + \overline{ce}^2 - 2cd.\ ce.\ \cos.\ dce$; and also $= \overline{od}^2 + \overline{oe}^2 - 2od.\ oe.\ \cos.\ doe$. But (indicating the radius by *r*) $cd = r.\ \text{tang.}\ ac$; $ce = r,\ \text{tang.}\ bc$; angle $dce =$ angle $acb = c$; $od = \frac{r}{\cos.\ ac}$; $oe = \frac{r}{\cos.\ bc}$; $doe = ab$. Substituting these values, we have $\cos.\ acb = \frac{\cos.\ ab - \cos.\ ac.\ \cos.\ bc}{\sin.\ ac.\ \sin.\ bc}$. If we indicate, as is customary, the angles by the capital letters A, B, C, and the sides corresponding to these letters by *a*, *b*, *c*, respectively, the preceding formula becomes $\cos.\ C = \frac{\cos.\ c - \cos.\ a,\ \cos.\ b}{\sin.\ a - \sin.\ b}$. If, however, we indicate the sides and angles by small letters, so that the side a' answers to the angle *a*, &c., then $\cos.\ c = \frac{\cos.\ c' - \cos.\ a',\ \cos.\ b'}{\sin.\ a'\ \sin.\ b'}$. These formulæ are not suited to calculations of angles by means of logarithms.

Two simple rules may be adduced, of universal application in calculating right angled spherical triangles. If, for instance, we write down the sides and angles of one of these in their natural order of sequence, omitting the right angle altogether, and taking for each side about the right angle, 90— that side, we shall have, 1, the cosine of any part = the product of the cotangents of the including parts, and 2, the cosine of any part = the product of the sines of the second and third parts following. Thus, if *c* be the right angle, and we take b' for $90 - b$, and a' for $90 - a$, we shall have as the order of succession, a', B, *c*, A, b', a', B, *c*; then, for example, $\cos.\ a' = \cot.\ B,\ \cot.\ b'$; and $\cos.\ A = \sin.\ B,\ \sin.\ a'$, &c. The solutions thus obtained may be ambiguous when a part is given by its sine, since any

two angles or arcs, which, when added together, make 180°, have equal sines. Thus, if in the triangle ABC, A and a are given, we have sin. $B = \frac{\cos. A}{\sin. a'}$ whence there may be two values for B—one above, the other under, 90°. In fact, *pl.* 3, *fig.* 116, shows that the two triangles, bac and $ba''c$, have a side, bc, common, and the angles opposite to A equal (since the angles bac and $ba''c$ are equal), while all the remaining parts of the one triangle are supplements (180 -- the part) of the corresponding parts in the other.

In the solution of acute angled spherical triangles, two cases occur in which the results of trigonometrical calculations are ambiguous: 1, when two sides and the angle opposite the smaller of these are given; 2, when two angles and the side opposite the smaller one are given. *Fig.* 117 illustrates the latter case. If, in the triangle abc, we have given the angles abc and acb, and the side, ac, opposite the smaller angle, then a second and entirely different triangle, acb'', may be constructed, of very different parts, provided that ab' is so taken that its prolongation $ad = ab$, and consequently $abc = adc = ab''c$.

In astronomy, it is frequently desirable to ascertain what effect a very slight alteration of one part (a side or angle) of a triangle produces on another part, all the rest remaining unchanged. These effects may be often determined by geometrical considerations, as, for instance, when the change sought is that which alteration of an angle of a spherical triangle produces on the opposite side. In *fig.* 118, convert the triangle acb into acb'' by a slight alteration of the angle acb, and indicate the change of the angle c by δc; that of the opposite side, c' by $\delta c'$. If we let fall from b on ab'', the perpendicular, bx, we may take $ax = ab$, and $b''x = \delta c'$, and we will have $\delta c' = \sin. abc, \sin. a', \delta c$.

The application of trigonometry, both plane and spherical, to geodesy, is of great importance. The piece of land to be surveyed is divided into triangles whose corners are indicated by signals; of the sides of these triangles only one need be measured, as a basis from which, with the help of the observed angles, to calculate the remaining sides. In this respect, some special formulæ are still necessary, of which we here give but one example: —given the angular interval of two signals of moderate height above the horizon, to deduce the horizontal angle of the two points of the horizontal plane on which the signals are erected. In *fig.* 119, let a, b, be the signals observed from o; and let the angle aob be measured. If we suppose a sphere constructed with o as the centre, and from z, the vertical point or zenith of o, the great circles zac, zbd, described, cod being the horizontal plane, cod or czd will be the horizontal angle sought. If we make the angle $aob = m$, cod or $czd = m + x$; $ac = h$, $bd = h'$, then the correction of the measured angle m is $x = \frac{1}{4} ([h + h']^2 \text{ tang. } \frac{1}{2} m - [h - h']^2 \text{ cot. } \frac{1}{2} m)$.

For the solution of triangles which, supposing the earth to be a perfect sphere, may be taken for spherical, three methods are principally used: they may be either considered as spherical triangles, in which case the central angle corresponding to each side is deduced from the known radius of the earth; or from the angles of the spherical triangle, the angles of their

chords are obtained, and the triangle of these solved as a plane triangle; or finally, the spherical triangle is treated as plane, in which case a correction is applied to the angles, each one being diminished by about the third part of the spherical excess. This latter rarely reaches five seconds.

Knowing the angles and sides of the triangle, as also the relative positions of the signals, we have still to determine the angle which one of the lines makes with the meridian. To this *fig.* 120, *pl.* 3, has reference, where *z* is the zenith, *p* the pole, *s* the pole star, *zs* a great circle. Hence the following problem is to be solved by means of the formulæ of spherical trigonometry: From the sides *ap*, *ab* (*fig.* 121), and the angle *pab* of a spherical triangle *abp*, to determine the side *pb*, and the angles *p* and *b*, where *pa* and *pb* are the complements of the breadths of the positions A and B, and the angle *p*, the difference of their lengths.

IV. HIGHER GEOMETRY, OR GEOMETRY OF CURVES.

The higher Geometry treats, as above mentioned, of curved lines, curved surfaces, and the solids bounded by these. In applying Algebra and Analysis to Geometry, and establishing its principles by calculation, a marked difference is observed between it and the lower Geometry. This application of Analysis to Geometry is known as Analytical Geometry, which is by no means limited to the cases of the higher Geometry, since straight lines, the circle, and planes may be treated of analytically. The position of a point in a plane is indicated in Analytical Geometry by its co-ordinates (so called). By this is generally understood the distance of a point from two straight lines whose position is known, generally at right angles to each other, and called the axes (of ordinates and abscissas). The distances are parallel to the axes, and are known as the abscissa or ordinate of the point, accordingly as they are parallel to the axes of abscissas or of ordinates. The two together are called co-ordinates. The point of intersection of the two axes is called the origin of co-ordinates; since the two co-ordinates of a point form a parallelogram with the portions of the axes cut off by them, these latter may also be considered as co-ordinates; hence the ordinate only is generally drawn parallel to the corresponding axis, and the portion of the axis of abscissas cut off by it, called the abscissa. Thus if in *pl.* 3, *fig.* 106, *bc* represent the axis of abscissas, and *b* the origin of co-ordinates, supposed to be rectangular; then the perpendicular *fg* let fall from *f* on *bc*, will be the ordinate, and *bg* the abscissa of the point *f*.

Polar co-ordinates are different from the co-ordinates first explained. Here we assume only one fixed straight line, and a point in it (called the pole) as known, and determine the position of every other point by its distance from the pole, or the length of the connecting line (Radius vector) between point and pole, and the angle inclosed between it and the fixed straight line; a point in space is known by its distance from these known planes, cutting each other in the origin of co-ordinates, and generally

perpendicular to each other. If, however, a point in space is to be determined by its polar co-ordinates, a line and two angles are required.

Every line, straight or curved, is in analytical geometry expressed by an equation from which all the peculiarities of the line may be derived by calculation. If we suppose all co-ordinates to be expressed in numbers, and indicate the abscissa by x, and the ordinate by y, then for every line the dependence between abscissa and ordinate of one and the same point of the line may be expressed by an equation, which holds good for every point of one and the same line. Thus for the equation of the straight line we have $y = ax + b$, or $ax + by + c = o$.

Curved lines, or curves, are divided into curved lines of simple curvature which lie in one and the same plane, and into curved lines of double curvature which lie in different planes. The former, to which we here limit ourselves, are again subdivided into algebraic, which may be expressed by an algebraic equality; and transcendental, whose equations are transcendental, that is, consist of an infinitely great number of terms.

Algebraic curves are divided according to the degree of their equations, into lines of the first, second, third, &c., order. Since, however, the straight line alone is expressed by an equation of the first degree, and is consequently the only line of the first order, we term lines of the second order, also, curves or curved lines of the first class; lines of the third order, curves of the second class, &c.

Every curved line may have a touching line or tangent, as well as the circle. By this is understood a straight line which has one point in common with the curve, and indicates the position of the cürve with respect to that point. Thus in *pl.* 3, *fig.* 134, a tangent is drawn through the point m. The part of the axis of abscissas between the ordinate and the tangent of a point, is called the sub-tangent. If we erect a perpendicular to a tangent at the point of tangency, and prolong it to the axis of abscissas, the part of the perpendicular (mn in the figure) contained between the latter and the point of tangency, is called the *normal;* that part of the axis of abscissas (np in the figure) between normal and ordinate, the *sub-normal.*

The most important curves, as well as those of most frequent occurrence, belong to the first class. These are the ellipse, parabola, and hyperbola. They are also called the conic sections, because they are produced by intersecting a cone by a plane in various directions. If the plane of intersection be parallel neither to the axis nor side of the cone, the outline of intersection is called an ellipse (*pl.* 1, *fig.* 55). This is a closed curve line, having the peculiarity that in one of its axes there are two points. termed the foci, so situated that the sum of the distances of any point of the curve from the foci, will be the same. The more the direction of the generating plane approaches a perpendicular to the axis of the cone, the more do the foci approach each other; and when the perpendicular is attained, the foci meet in the centre, and the ellipse becomes a circle. Every line passing through the centre of an ellipse, is called a diameter; the longest diameter (called *major axis*) is that which passes through the

foci; the shortest (called *minor axis*) is perpendicular to the former and bisects it.

The distance from a focus to the centre is called the eccentricity (in the circle $= 0$); the equation of the ellipse is $y^2 = \frac{b^2}{a^2}(a^2 - x^2)$, where a and b are the semi-major and minor axes. In the circle $a = b$, therefore, $y^2 = a^2 - x^2$ is the equation of the circle of radius, a.

A hyperbola is produced when the intersecting plane is parallel to the axis of the cone. As this intersection always meets the base of the cone, the hyperbola is an open curve. It also has two foci, the difference of whose distance to any point in the circumference will always be the same. It is composed of two equal parts, each of two branches, which, stretching into infinity, approach continually without ever meeting two straight lines (the asymptotes) which intersect each other in the centre of the major axis. The equation of the hyperbola is $y^2 = \frac{b^2}{a^2}(x^2 - a^2)$. When $a = b$, it becomes $y^2 = x^2 - a^2$; such a hyperbola is called equivalent. The asymptotes of this form a right angle with each other.

The parabola is produced when the plane of intersection is parallel to the side of the cone; it also is an open curved line, but has only one focus. Every point of the curve is equally distant from the focus and a fixed straight line called the *directrix*. It also consists of two symmetrical, infinitely extending branches, which unite in a point half way between the focus and directrix, called the vertex. A straight line drawn through the vertex and the focus is called the axis. The equation of the parabola is $y^2 = px$.

The following algebraic curves may be mentioned in addition:

1. *Parabolas* of higher orders. These are curves in which a power of the ordinate is proportional to some other power of the abscissa: their general equation is $y^m = ax^n$. If $n = 1$ and $m = 2$, the equation becomes a quadratic (thus, $y^2 = ax$ is the same with the common or Apollonian parabola); a cubic when $m = 3$, &c. The parabola of Neil (*pl.* 3, *fig.* 124), whose equation is $y^3 = ax^2$, is particularly remarkable. It is that curve in which a heavy moving body falls equally in equal time.

2. The *cissoid* (*fig.* 122), a curve of the second class, discovered by the Greek geometrician, Diocles. It consists of two infinite branches, uniting in a point, a, and continually approaching a tangent of the circle (the asymptote) without ever meeting it. Its equation is $x^3 = (a - x)\,y^2$.

3. The *conchoid* (*pl.* 3, *fig.* 123), a curve of the third class, discovered by Nicomedes, whose equation is $\frac{x^2y^2}{(b+y)^2} + y^2 = a^2$. Its construction is very simple: draw a straight line, and out of this line take any point, a; from this point draw a straight line cutting the firstin q; from q take off $qm = qn$ in this second line equal to a given or fixed length: m and n will be points of the two infinite branches of the conchoid, which also has qq for its

asymptote. Müller of Gröningen has proposed to apply the conchoid to the measurement of barrels.

4. The *cardioid* (*fig.* 125), a curve of the third class, properly an epicycloid of two equal generating circles. Its equation is $y^4 - (a^2 + 2ax - 2x^2) y^2 - 2ax^9 + x^4 = 0$.

5. The *lemniscata* (*fig.* 130), a curve of the third class, discovered by Jacob Bernouilli, and investigated by Euler and Fagnano, whose equation is $(x^2 + y^2)^2 = 2\,a^2\,(x^2 - y^2)$.

6. The *ophiuride*, discovered by Uhlhorn, for the trisection of angles, constructed as follows (*fig.* 131): Construct a right angle, *abc*, with determined sides, *ab*, *bc*; draw from *c* to any point in the line *ab*, or its prolongation, a straight line, *cd*; erect at *d* a perpendicular, *dn*, to *cd*, and upon this, from the end of the other side of the angle, let fall a second perpendicular, *am*, then *m* will be a point of the curve. Taking $ab = a$, $bc = b$, the equation of the ophiuride will be $x^3 + (y^2 - ay)\,x - by^2 = 0$.

7. The *scyphoid*, according to Uhlhorn, is formed in the following manner (*fig.* 132): If, from any point, *o*, out of an unlimited straight line, *yy'*, a perpendicular, *ob*, and any oblique line, *oc*, be drawn to the line, and through *c* a line, *nz*, perpendicular to *oc*, and on *nz* the distances $cm = cm' = bc$, then will *m* and *m'* be points of the scyphoid. Taking $ob = a$, then, with *o* as origin of co-ordinates, *ob* as axis of abscissas, and *yy'* as ordinates, the equation of the scyphoid will be $y^4 - 4a\,(a - x)\,y^2 - (a - x)^4 = 0$.

Examples of curves whose equations are most readily expressed by polar co-ordinates, are afforded by the spiral lines (*pl.* 1, *fig.* 51), which wind continually around a fixed point, either continually approaching to, or receding from it, according to a given law. The simplest of these is the Archimedian or equable spiral (*pl.* 3, *fig.* 133), which is generated when a point moves uniformly along the radius of a circle, this radius describing an uniform rotation around its extremity, so that the distance of the moving point from the centre is always proportional to the angles described by the radius. It is generally provided, in addition, that the moving point shall meet the circumference of the circle by the time that the radius has described its first entire revolution.

Spirals may also be described on the surface of a cylinder, a sphere, or a cone: the well known screw line (*pl.* 1, *fig.* 52) belongs to the cylindrical spirals.

The cycloid or trochoid belongs to the transcendental curves. This is described by a point in the circumference of a circle which rolls along a straight line until it has completed a revolution; the circle, curve, and line, being supposed to continue in the same plane (*pl.* 3, *fig.* 135). If the revolution be started when the point lies in the straight line (at *a*), and is consequently the point of tangency between the circle and line, and continues until it again meets the straight line (at A), then the line A*a*, called the base of the cycloid, will be equal to the circumference of the generating circle. The cycloid cuts the base at A and *a'*, therefore the

point i', lying half way between the two, called the vertex, is the furthest distance from the base, this distance being equal to a diameter of the generating circle. When the generating point lies without the circle, the cycloid produced is called the *curtate* or *contracted* cycloid (*fig.* 136). If it be within, it becomes *prolate*, or *elongated* (*fig.* 135^a).

If the circle with its generating point revolve, not on a straight line, but upon the circumference of another circle, fixed, and in the same plane, then the curve produced will be an *epicycloid*, if the revolution be on the outer side of the fixed circle, and a *hypocycloid* when on the inner. We have here, as in the cycloid, the same distinctions into *ordinary* or *common* epicycloid (*figs.* 137, 140); *prolate* or *elongated* (*figs.* 138, 141); and *curtate* or *contracted* (*figs.* 139, 142).

Another transcendental curve, or rather genus of curves, is the *quadratrix*, a curve line, described on a common axis with any other given curve, and indicating by its ordinates the area of the latter curve, since their ordinates are as the areas answering to the corresponding abscissas, with the given line as axis of ordinates. The oldest quadratrix is that of Dinostratus (*fig.* 126): let *ab* be a diameter of a circle, and the triangle *acb* so constructed that *the height cn* : *the base ab* : : *angle cab* : *a right angle*, then *c* will be a point of the quadratrix, whose equation is $x\ g\ \frac{\pi(a-x)}{2a} - y^0$.

Another construction of the quadratrix is given by Tschirnhausen (*fig.* 127). Let *adb* be a semicircle, *o* the centre, and *m* a point in the circumference, furthermore *n*, a point of the diameter which lies in such a manner that *quadrant ad* : *arc am* : : *ao* : *an*, and draw through *m* and *n* to *ao* and *do* parallels meeting in *p*, then *p* will be a point of this quadratrix whose equation is $y = \sin.\ a\ \frac{\pi x}{2a}$.

We have still to explain the meaning of the terms *evolutes* and *involutes*. Suppose that on the elevated side of a curved line, a perfectly flexible thread be laid. If, now, this thread be kept continually stretched, and unlapped by degrees from the curved line, its end will describe a new curve, which is called the *involute* of the old curve, this latter being the *evolute* of the former. Thus the parabola of Neil is the evolute of the common parabola. In *pl.* 3 (*fig.* 128), the involute of the circle is represented, which is constructed as follows: Through any points, *b*, *c*, *d*, of a circle, tangents are drawn, and on these the points, b', c', d', so taken that the tangents, bb', cc', dd', shall equal the length of arcs of circles contained between the points of tangency and a fixed point, *a*. The points *b* , c', d', will then be points of the involute of the circle, which is a transcendental curve.

Among the solids produced by the higher curves, the spheroid is the most important, resembling the sphere, and like it having a centre in which every diameter is bisected, but differing in these diameters being of unequal length (*pl.* 2, *fig.* 73). Among all the diameters of a spheroid, three, perpendicular to each other, called its axes, are best worthy of mention.

A plane passed through one of the three axes forms an ellipse by its intersection with the surface. When two axes are equal, the spheroid becomes *elliptical*, being generated by the revolution of an ellipse around one of its two axes (forming an ellipsoid). A paraboloid is generated by the revolution of a parabola about its axis, and a hyperboloid by that of a hyperbola.

V. APPLIED GEOMETRY.

1. GEODESY, OR SURVEYING.

Practical geometry, which is itself only a part of applied mathematics, embraces, in a restricted sense, 1, the greater and lesser arts of surveying, or geodesy; 2, descriptive geometry, or theory of proportion. In a restricted sense, we understand by practical geometry only the first of these divisions, which proposes to itself the problem, accurately to determine the size, shape, and position of a larger or smaller part of the earth's surface, and to represent it pictorially on a reduced scale.

We distinguish, as above mentioned, a lower geodesy or field surveying, which has to deal only with small parts of the earth's surface, as a field, or estate, and a higher geodesy, having reference to whole countries.

Under geodesy are also reckoned, generally, *levelling* and *surveying* of mines.

The first problem in field surveying is to mark off a straight line. This is done by means of straight cylindrical staves of wood, from 6 to 8 feet high, and 1 to $1\frac{1}{2}$ inches thick, with iron points at the lower end for more convenient insertion into the ground; together with a number of stakes, called also arrows, pickets, &c. Of these staves, A and B are placed perpendicularly in the ground about 100 feet apart, and a third, C, still further forward in the same straight line. In order to place these staves in the same straight line, we may have them so adjusted, that, standing behind A, the others, B and C, shall both be covered by it; or A and B may be covered by C in the line of sight. This latter method is, perhaps, preferable. We must proceed in the same way to extend the line of these staves.

The second problem is to measure a line which has already been staked off. This is done either by means of the measuring chain, which is most generally employed, or by measuring tapes or threads, which are commendable for their cheapness and convenience, but do not afford accurate results; or, finally, by measuring staves, which give by far the most correct measurements.

With stakes and a chain, or some other means of measuring a given line, quite a number of the more difficult problems may be solved, without any other apparatus. We can, in the first place, survey any irregularly curved line on the surface of the ground, as, for instance, the outline of a field or plain (*pl.* 4, *fig.* 1). To this end, a straight line, AB, is staked off, and on this as many successive distances, A*a*, *ab*, *bc*, &c., as possible, measured.

The distances, aa', bb', cc', &c., from a, b, c, &c., are next to be measured at right angles to the base (which may be done by the eye, or more accurately with a string divided as the numbers 3, 4, 5, of a right angle). The measured distances of both kinds, the abscissas, Aa, Ab, Ac, &c., as also the ordinates, aa', bb', &c., are traced on paper on a reduced scale, and the points, a, b, c, &c., united. The accuracy of the outline will be evidently in proportion to the number of abscissas and ordinates measured. The outline may sometimes be such as to render it advisable to stake off two lines, as in *fig.* 1, whose relation to each other must be known.

In the second place, the distance between two points may often be determined even when no direct measurement is possible. Three principal cases are here to be distinguished: 1. When the distance between two points cannot be measured directly, but only that from a third point to each of these two (*fig.* 2). In this case, we measure the distances, CA, CB; continue the prolongations of these lines beyond C, towards D and E; take CE = CA, and CD = CB, or the reverse; the measured distance from D to E will be the same as that from A to B. It is much more convenient, when the prolongations of the lines, AC and BC, cannot be made equal to them, to take a certain part of the distance, as one fourth $Cd = \frac{1}{4}Cb$, $Ce = \frac{1}{4}Ca$; then de will be the same fraction of AB, or $de = \frac{1}{4}$AB, or AB $= 4de$. 2. When we can reach only one of the two points whose distance from each other is desired, as in *pl.* 4, *fig.* 3. Here we assume any point, C, at pleasure, from which B may be reached in a straight line, measure CB, and continue the prolongation of this line to D, so that CD = CB, and then in the direction DE, making the angle CDE = CBA. (To effect this take on BA and BC, any portions, Ba, Bb—five feet, for instance—measure the distance ab, make Dd = Bb, and with a beam compass, from d as centre, with ab as radius, describe an arc, intersecting another arc from D as centre, with aB as radius. Stake off the line DE through e, and we shall then have the direction of the required angle.) We then continue in the direction DE or De until we reach a point, E, which lies in the same straight line with C and A, as ascertained by two staves. The distance DE will then equal AB. *Fig.* 4 represents another method of attaining the same result: Take on AB any point, C, between A and B, and then a point, D, whose distance from B and C may be directly measured; continue the lines CD and BD beyond D, making DF = CD, DE = DB. Finally, draw EF, and continue it to a point, G, in the same straight line with D and A; EG will be the distance required. We may here also, instead of the whole line, BD, CD, take a fractional part of these prolongations; thus, if we make D$e = \frac{1}{4}$DB, and D$f = \frac{1}{4}$CD, then, if g lie in a straight line with AD, as well as with ef, eg will $= \frac{1}{4}$AB. 3. When we can reach neither of the points, A, B (*fig.* 5). In this case, many methods may be employed; the one represented is perhaps the simplest: lay off the line CD approximately parallel to AB, and on it take cD equal to an aliquot part, as $\frac{1}{4}$ of CD; make Dca = DCA, and Dcb = DCB, taking the distance ca so that a may be in the line AD, and Db so that b may be in the line BD, then ab will, in our figure, be $\frac{1}{4}$ of AB (the line cb is not represented).

To determine with staves alone, the height of an object whose foot cannot be reached, we employ two of unequal lengths, DE and FG (*fig.* 8). Erect the two staves so that the eye, placed at the ground at J, shall see their summits, E and F, in the same line with that of the object, B. The staves being of known height, measure the distances, JD and DG (together equal to JC): with the same staves repeat the operation at another point of the line, JA, as at C′ and D′, obtaining the values J′D′ and D′C′ (together $=$ J′C′). As the triangles JDE, JGF, and JAB, are similar, and also J′D′E′, J′C′F′, and J′AB, as well as GF $=$ C′F′ and ED $=$ E′D′, we will have

$$\begin{array}{l} JD : DE :: JG : GF \\ JD : DG :: JA : AB \\ \hline JG : GF :: JA : AB :: JG : JA :: GF : AB. \end{array}$$

$$\begin{array}{l} J'D' : DE :: J'C' : GF \\ J'D' : DE :: J'A : AB \\ \hline J'C' : J'A :: GF : AB. \quad \text{But} \\ JG : JA :: GF : AB \\ \hline JG - J'C' : JA - J'A :: GF : AB. \end{array}$$

JG and J′C′ are, however, known; $JA - J'A = JJ'$ is also known, consequently $AB = \frac{GF \times JJ'}{JG - JC'}$.

The shadow of an object when the sun shines may be used for measuring its height, although this method has no claim to great accuracy. Erect a perpendicular post or staff, of known length, and measure as nearly as possible at the same time, the length of its shadow and that of the object; then the length of the staff will be to that of the object as the lengths of their shadows. If in the line of the shadow we erect a post so that the end of its shadow coincides with that of the object's shadow, then the same proportions will hold good, and the method is at the same time more convenient (*pl.* 4, *fig.* 6).

If the foot of the object to be measured cannot be reached, we may apply the preceding method on two different days, when the sun has a decidedly different height, best of all at the time of true noon, when the shadow falls exactly in the true meridian. If we indicate the length of the object's shadow at the two different times, by C and C′, those of the post's shadow by c, c', and the length of the post by a, then the height of the object will be $\frac{a(C - C')}{c - c'}$.

Instead of the shadow we may use a horizontal reflecting surface (of oil or mercury). Erect at any point, D (*fig.* 7), a staff, DE, of known length, not to exceed a few feet; find a place, C, between the staff and the object, where the mirror shall reflect the top of the object to the eye placed at E. In this case, the triangles, CDE, ABC, are similar, and if AC can be mea-

sured, the height desired will $= \frac{AC.DE}{CD}$, or (substituting c for AC, a for DE, and d for CD) $= \frac{ca}{d}$. If the foot of the object cannot be reached, the same process may be repeated with the same staff, at another place, D′, and the height will $= \frac{c'a}{d'}$, if $C'D' = d'$, $AC' = c'$. If now, $CC' = b$, and consequently, $c = c' + b$, then $\frac{c'a}{d'} = \frac{(c'+b)a}{d}$, whence $c' = \frac{bd'}{d-d'}$, and the height required $= \frac{ab}{d-d'}$.

The describing of a part of the earth's surface, *i. e.* making a perfect representation of it on a reduced scale, is to be considered as one of the principal problems in surveying. Three methods may be employed when only a small part, easily overlooked, is in question: in these the plane table is the most convenient instrument to be employed. 1. By *sighting forwards and measuring*. Sight from a given station A (*pl.* 4, *fig.* 9), situated in tne inside or in the circumference of the figure, towards all its corners, which are to be indicated by signals or other marks; measure also the distance of this point from all the corners, determine the sight lines on the plane table by means of a diopter ruler, and mark off, according to a scale, the proportional lengths of the distances above mentioned. By connecting the extremities of the lines thus obtained, we shall have a figure similar to that of the field. 2. By *going round the figure*, or *sighting backwards*. All the sides of the figure (except two) must be measured, and sights taken from one corner to the others. This is also called surveying the figure from the circumference. The method is inconvenient, but oftentimes the only one practicable (*fig.* 11). 3. *Surveying from two stations.* Measure a base line, AB (*fig.* 9), and sight from its ends to all the corners of the figure; transfer this base, reduced, to the paper, and draw from its extremities the lines of sight: the intersections of these two sets of lines will determine the corners of the figure. This method, when it can be employed, is always preferable to the other two. It is more fully illustrated in *pl.* 5 (*fig.* 57). Here *ab* is the base, which may be 100 or 1000 feet long. After it has been measured, the plane table is set up at A, and from *a*, sights taken to the other extremity of the line, as also to all the principal points visible from A, as C, D, E, F, &c. The corresponding sight lines are then to be drawn on the table by means of the sight or diopter ruler. The length of the base line is then to be marked off on the sight line running towards *b*, on a reduced scale, as $\frac{1}{1000}$ or $\frac{1}{5000}$; and the table removed to B. It is here to be set up in such a manner that the point *b* lies directly over that part of the base answering to *b*. It is furthermore erected so that the sight line to *a* corresponds with the line *ab* drawn on the table. The points C, D, E, F, &c., already sighted from A, are to be sighted from B, and the corresponding sight lines drawn upon the plane table. Their intersec-

tions with those drawn from the other extremity of the line, determine the position of these points on the reduced plane of the table.

If a great surface is to be measured—an entire country, for instance—a trigonometrical net-work must be constructed, as already mentioned under the head of Trigonometry. This consists in dividing the part of the earth's surface in question, into a great number of connected triangles, whose corners form stations visible one from the other. In these triangles, only one side, rarely over $1\frac{1}{2}$ mile long, needs to be measured; in addition to which, the angles must be measured with a theodolite. Care must be taken that the angles of these triangles be neither too acute nor too obtuse; those most nearly equilateral are most convenient. The net must be so arranged that each sheet of the plane table contains at least two of the corners of the trigonometrical net. In *pl.* 4, *fig.* 10, AB represents the base; from this the points D and C are determined, by measuring the angles BAC, DAB, and ABC, DBA, and two triangles thus obtained, whose sides, AC, BC, AD, BD, may be calculated trigonometrically. AD may now be taken as base, and the point E determined; as also K, from the base DE, &c. In this manner the network, ABCDEKH, is produced. It will add greatly to the accuracy of the work to determine each point from several stations if possible. This serves to control the various measurements. Suppose the point K to be determined from DH, and likewise from DE, if it should fall towards L, some error must have occurred, which must be detected either by repeated measurements or by special calculations to which we have not time here to refer.

A very important problem, and one of frequent occurrence, is to determine the point on the plane table corresponding to the one where it was originally set up; this, knowing the positions, α, β, γ, of the three points of the field, A, B, C (*fig.* 64–68). If the triangles, α, β, γ, can be brought into a position perfectly parallel to the field triangle ABC, then the point required will be determined by applying the sight ruler at α, β, γ, and sighting towards A, B, C; the intersections of these three lines will determine the point. It is very difficult, however, to attain this parallel position. If the two triangles are not parallel, the three sight lines will form a triangle, by means of which the desired point may be attained. We cannot here go into the minute details of the operation.

To determine the area of a rectilineal figure, all that is necessary is to divide it by diagonals into triangles, whose individual areas are to be computed from their ascertained bases and altitudes, and added together. The figure may also be divided into trapezia and triangles, which method is sometimes preferable. When the figure to be calculated is curvilineal, the latter method may sometimes be employed (as in *fig.* 13), if the parallel lines are drawn so closely to each other that the included parts of the circumference may, without material error, be considered as rectilineal. In the case represented in *fig.* 12, the two triangles, ABC, BCD, are first calculated, then the mixed lined parts by which the triangles exceed the curvilineal figure. This latter is effected by dividing them into trapezia and triangles, by perpendiculars erected, and subtracting their sum from that of the

triangles. In *fig.* 14, the computations are to be made in the same way, and the sum of the mixed lined portions added to that of the triangles. In *fig.* 15, the parts BCL, DEK, &c., may be determined in the same manner.

Levelling forms a particularly important branch of Geodesy. This consists in ascertaining the difference in height of two points of the earth's surface, by direct measurement, and not by projection or calculation. The object of levelling may be thus expressed generally: to ascertain how much further one point of the earth's surface lies from its centre, than another point. As a general rule, it is not great elevations that are here in question, but simply the gradual rise and fall of the ground. The instruments necessary for this purpose will be described hereafter; the operation itself is explained by *pl.* 4, *figs.* 16–19. In general, two methods for determining the difference of height of two points, may be distinguished—either to set up at one of the two points (*fig.* 16), or between the two (*fig.* 17): the latter is, perhaps, preferable. The distance between the two points whose difference in height is to be ascertained, must not be very great (from 1—2000 feet). At a greater distance, intermediate stations must be assumed, which the nature of the surface sometimes renders necessary for slight distances. Thus, in *fig.* 18, the difference of elevation between A and J is to be ascertained by means of the intermediate stations, C, E, G, assumed in the lines A, J; four levellings are here required. In *fig.* 19, this difference between A and D is determined by means of the two intermediate stations, B and C.

It will be necessary to add, in conclusion, a few words with respect to *topographical drawing.* This consists in representing portions of the earth upon paper, in their natural appearance. A topographical drawing differs from a chart in its much larger scale, which admits of the insertion of more details. While, for ordinary maps or charts, the scale rarely exceeds $\frac{1}{20000}$ of the natural size, in special plans for economical or military purposes, it may amount to $\frac{1}{2500}$, so that one line, or $\frac{1}{100}$ of a foot in the drawing, would represent 25 feet of ground.

A topographical drawing represents not only streams, roads, houses, forests, &c., but also mountains and valleys; and this in such a manner that from the drawing the steepness of the declivities may be ascertained. This is done, according to the almost universally adopted method of the engineer Lehman, by means of rectangular pen strokes, made side by side, in such a manner that the amount of black is to the amount of white, as the given angle of inclination to 45° minus the same angle; consequently, a horizontal surface appears entirely white; that inclined at an angle of 45°, entirely black; at 5, 10, 15, 20° of slope, the breadth of each black space is respectively $\frac{1}{8}$, $\frac{2}{7}$, $\frac{1}{2}$, $\frac{4}{5}$, of the white interval succeeding; while at 40, 35, 30, 25 degrees in succession, the reverse order takes place. This method is not calculated for slopes of from 45° to 90°, for the simple reason that they seldom occur, are always much broken in their declivities, and are entirely impracticable for military purposes, which the inventor had chiefly in view. *Figs.* 58–60, on *pl.* 5, are intended to elucidate the preceding remarks.

Fig. 60 represents the environs of the town of Greitz, and *fig.* 59, the plan and profile of a mountain top, drawn according to the declivities of the surface.

2. DESCRIPTIVE GEOMETRY.

A. Projection.

a. Projection in vertical and horizontal planes.

By the theory of projection is understood in general, a combination of all those propositions by whose application we are enabled to represent an object as it appears to us in a certain direction, and from a certain distance. If we suppose lines to be drawn from our eyes to all points of the object, representing lines of sight, a pyramid of rays will be formed, whose base is the surface of the object, whose sides are the rays of sight, whose apex is the eye, and whose altitude is the perpendicular distance of the object from the eye. If we suppose a plane to be passed through this pyramid, parallel to its base, according to the principles of similar figures and the laws of Stereometry, we will have an image in the plane of intersection which is similar to the body in question, and which is smaller as the distance of the plane from the eye is less. If we suppose the object to be at an infinite distance, the pyramid produced will be of great altitude, and the angle made by the sight rays with the base of the pyramid will be obtuse; if the section be made tolerably near the base, we may assume the portions of the sight rays thus cut off as parallel to each other, and perpendicular to the base of the pyramid. The intersecting plane is called the plane of projection, and upon it the image of the object is supposed to be represented.

According to the preceding principles we can find the projection of a point, by drawing a perpendicular from it to the plane of projection; the intersection of the line with the plane will be the projection of the point. Nevertheless, as the distance from the plane at which the point is situated is not determined, we cannot ascertain its actual position from this projection. This will be possible, however, if we employ a second plane, upon which we may suppose the distance of the projection from the point itself to be described. The first plane is called the *plane of elevation,* or the *vertical plane;* the second, the *ground,* or *horizontal* plane. Both planes may be considered as perpendicular to each other, and the position of a point in space may be accurately determined by the intersection of the two perpendiculars erected from the projection of the point on the two planes. In *pl.* 4, *fig.* 20, AB is the vertical plane, and BC the horizontal plane; $a'b'$ is the vertical, and $a''b''$ the horizontal projection of a line. If we suppose lines to be drawn from the four points, perpendicular to their respective planes, a' and a'' will intersect each other in ab' and b'' in b, and the position of the points, a, b, in space will thus be determined. Now whenever

two points in any straight line are known, the line itself will be determined.

We can imagine the horizontal plane to be revolved about its axis in such a manner as to form an angle (= 2 R) with the other plane, leaving only one plane upon which objects are to be projected. If a line out of this plane is to be projected upon it, perpendiculars must be let fall from the two extremities upon the horizontal plane; the straight line connecting the feet of these perpendiculars is called the projection of the line.

As a straight line is determined by two of its points, and as curved lines require several, every curved line may be considered as consisting of infinitely small straight lines. The projection of a curved line, then, is the same as that of a straight line, only requiring more points in the line. Thus in *pl.* 4, *fig.* 23, let 1 2 3 4——9 indicate the position of a line in horizontal projection—we here suppose the horizontal plane to be revolved—and 1′ 2′ 3′——9′ be the position of the line with respect to the vertical plane *xy*; the projection of the line is now to be found. First of all a number of points is to be assumed in the line 1——9, the same determined in the line 1′——9′, and perpendiculars drawn to the corresponding plane, which determine the feet. Prolong these perpendiculars beyond their feet and they will intersect each other, the points of intersection of the corresponding perpendiculars forming the corresponding points of the projection. Thus, from the intersection of the perpendiculars 3 and 3′, the point 3^2 lying in the projection is ascertained. When all the points are found, the projection will be obtained by joining 1^2, 2^2, 3^2——9^2, this will be the line.

As surfaces are bounded by lines, we can obtain the projection of surfaces by finding the lines inclosing them. In *fig.* 21 let *abcd* be the position of a surface in plan, a^3 c^3 is the position of *ac* in elevation. To determine the two corners b^3 and d^3 on this line, project *b* and *d* upon *ac*, in b^2 and d^2, and take off these points on a^3 c^3. Drawing perpendiculars from the four extremities of the two horizontal figures, we shall have the points a', b', c', d' as the corners of the projection, which is itself obtained by connecting the corners by straight lines.

If the figure be bounded by curved lines, a mode of proceeding similar to that employed in the case of straight lines will be necessary. In *fig.* 22 let *ab* be the view of a circular plane, in ground plan, f^5 f^4, the same in elevation. It is well known that the end view of a circle perpendicular to a plane appears as a straight line, this in the ground plan being the horizontal, and in the elevation the vertical diameter. We must thus, first of all, find the points in the curved line which are to be projected. For this purpose describe the two semicircles, divide them into an equal number of equal parts, for instance, in c', d', e', &c., and in a^4, c^4, d^4, &c., and project these upon the diameter; we shall thus obtain the points *a*, *c*, *d*, &c., and a^5, c^5, d^5, &c. By drawing the lines of projection from the like named points, we shall obtain the projected points of half the curved line. Thus, for instance, from the lines of projection from *d* and d^5, we get the point d^3, and as *d* and *h* lie at an equal distance from the centre, we obtain by means of the lines from *h* and a^5 the point h^3 symmetrical with d^3. After the

points of projection of the upper semicircle have been found, we describe the semi-curve a^2c^3——b^2, and corresponding to it, the symmetrical half, lying beneath.

Solids are bounded by surfaces, as these are by lines; the problem of finding the projection of a solid resolves itself, then, into finding the projection of points. In illustration of this, we will explain the method of projecting a regular six-sided pyramid, in its various positions. *Pl.* 4, *fig*. 24, exhibits this pyramid in its regular position, in a horizontal and vertical plane: *acdbef* is the polygon forming the base of the pyramid, and which we have placed in the same position in the ground plan, with respect to the bases *xy*, that the pyramid is to have in elevation. If we suppose the pyramid to be completed above this base, we shall have a *view from above* of the former. For this purpose, if the pyramid be right, we find the centre of the polygon: this will be the projection of the apex *g* of the pyramid. Lines drawn from the point *g* to the corners of the base, form the projections of the edges of the pyramid. To the eye, however, the projection thus obtained will suit for any height of the pyramid, as the point *g* is not determined with respect to its distance from the base; we must therefore have a side view of the pyramid, since, as already mentioned, two projections, at least, are necessary to determine the position of a point. As the lines which stand perpendicularly to the ground plane are projected as points, under the same conditions surfaces will be projected as lines. The case is the same with respect to the plane of elevation: *g* is the projection of the altitude of the pyramid in the plane, which appears as a line in elevation; *acdbef* is the projection of the base in plan, which appears in the plane of elevation as a series of lines, whose position and individual extremities are determined by drawing the perpendiculars aa', cc', &c. The position of the point g', in the perpendicular gg', is determined by the method already explained. By connecting the point g' with the points a',b'——f', we shall obtain the vertical projection of the pyramid.

Suppose that an oblique section, $h'n'$, be made through the pyramid, perpendicular to the plane of elevation, and its projection in the ground plane required; the first step will be to indicate the plane $h'n'$ by a straight line. The lines $a'g'$ and ag, $c'g'$ and cg, &c., are corresponding projections. If, then, from the points where the plane $h'n'$ cuts the different edges of the pyramid in the elevation, perpendiculars be let fall upon the corresponding edges of the plan, the points of intersection will determine the corners of the plane of intersection, *h, i, k, l, m, n*.

If we suppose the pyramid to rest with one corner, b^3, upon the basis *xy*, as in *fig*. 25, its axis, however, still parallel to the plane of projection, the projection on the horizontal plane must be changed, as the altitude of the pyramid is no longer perpendicular to this plane. To describe this projection, place the elevation obtained in *fig*. 24 upon the corner b^3, at the required angle, and then draw from the point *g* a perpendicular to the ground plan. From the point *g*, of *fig*. 24, draw a line parallel to the basis *xy*, until it cuts the perpendicular in g^2; then g^2 is the apex of the pyramid for the new projection. It is evident that the line gg^2 must be parallel to the basis

xy, as, according to the assumption, the axis of the pyramid remains parallel to the plane of elevation: the line gg^2 is the projection of the circular arc described by the apex g, during the supposed change of position about the corner b^3. The same condition holds good for all the other points of the pyramid; and by drawing perpendiculars from points a^3, b^3——f^3, their intersections with lines parallel to xy, drawn from a, b, c——f, will give the points a^2, b^2——f^2, determining the projection of the obliquely situated base: connecting these points with each other, and with the apex g^2, we obtain the horizontal projection of the oblique pyramid. In a similar manner we obtain the projection of the plane of intersection, h^3l^3, exhibited in the elevation as a straight line: this projection is h^2, l^2——n^2.

Let us now suppose the pyramid to be rotated upon the corner b^3, still at the same inclination to the base xy; the axis of the pyramid will no longer be parallel to the plane of elevation. It is evident that all points of the pyramid must describe horizontal arcs during this rotation, whose centres will lie in a perpendicular, supposed to be erected from the point b^3. Their perpendicular distance from the base must, consequently, remain the same as before. As, however, the inclination to the base remains the same, the projection in the ground plane needs to be changed only with respect of the direction of the edge g^2b^2 to the base xy. *Pl.* 4, *fig.* 26, represents the upper view seen in *fig.* 25, at the same angle with the basis xy. The preceding explanations have taught us that we can draw horizontal lines from all points of the elevation, in which the new projections of these points, for the new position, must lie. The points are absolutely defined, by drawing perpendiculars from the corresponding portions of the plan to these horizontal lines. Thus, to obtain the position of the point g^4 in the new projection, we draw the horizontal lines g^3g^4, and the perpendicular, g^2g^4. In like manner we obtain the projection of the base a^4, b^4——f^4, and consequently the projection of the entire pyramid, by uniting g^4 with these corners. As this pyramid is no longer parallel to the base, the plane of intersection, h^3l^3 (*fig.* 25), can no longer appear as a straight line in this last position of the pyramid. Its projection, h^4, l^4, k^4——n^4, is obtained by the preceding methods.

As an additional illustration, we give the projection of the three principal conic sections. If we imagine a plane to be passed through a cone, which is parallel neither to the axis nor to one of the sides, we shall obtain a regular, symmetrical, curved line, termed an *ellipse;* if the plane be passed parallel to one side of the cone, a *parabola* will be produced; and when parallel to the axis, a *hyperbola.* The development and properties of these three curves are cases of the higher Geometry and Analysis (see pages 24, 25). In this place we have to do only with their projections.

Pl. 4, *fig.* 27, is the projection of a right cone in the horizontal and vertical planes. The circle, A′B′, and the straight line, AB, are the projections on the vertical plane of the base, C is the apex, and DE the intersecting plane, appearing in elevation as a straight line, and whose intersection is to form the ellipse, whose shape in horizontal projection is to be obtained. The question reduces itself to finding the breadth of the ellipse for the different points of the circumference. These points lie symmetrically upon the

surface of the cone, only in different planes above the base; and it is necessary to find the projections of these planes in both views of the cone. In the vertical projection, these planes appear as straight lines; in the horizontal plane, as circles. When we pass planes through D and E, in the vertical projection, the points D and E, of the curve, will be situated in them; if, then, the length DE be divided into any number of equal parts, and horizontal planes be passed through the points of division, there will be two points of the ellipse in each plane, which will be situated in that part where the line DE cuts these planes in succession. To obtain the form of the ellipse in the horizontal projection, we draw in it the diameter, A′B′, and let fall upon it perpendiculars from the points 1. 2. . . . DE: the points 1′, 2′, &c., will answer to the horizontal projection of those points, and as the horizontal projections of the surfaces projected as straight lines in elevation must be circles, these can readily be determined, knowing their radii, C′, I′, &c., and their common centre, C. These circles are cut successively by the ellipse. By drawing the perpendiculars, DD′ and EE′, we obtain the projection of the extremities, since the axis of the ellipse lies parallel to the plane of projection. Drawing a perpendicular from the point where the ellipse cuts the plane marked 6, until it cuts the circle 6′ in the horizontal projection, we shall obtain one point of the horizontal projection of the ellipse, or two as the ellipse is symmetrical. By a repetition of the process, a number of points in the horizontal projection will be obtained, through which the ellipse itself may be passed. The figure standing near *fig.* 27, exhibits the actual view, or the orthographic projection of the ellipse. It is obtained by taking off the axis, DE, of the ellipse from the vertical projection, with its planes of intersection, which would here be represented as straight lines. From the horizontal projection, we obtain the true breadth, and if these be described one after the other upon the corresponding planes on each side of the axis, we shall obtain the points through which the ellipse is to pass.

Pl. 4, *fig.* 28, exhibits the vertical and horizontal projection of a right cone, with a parabolic intersection. DE is the projection of the parabola, which, for the vertical plane, is a straight line. The horizontal projection is obtained precisely as in the case of the ellipse. Thus, 1, 2, 3, — — — horizontal planes are passed through that part of the front view of the cone traversed by the parabola, at equal distances from each other; these appear as straight lines: they are circles in the horizontal view of the cone. In *fig.* 28, semicircles only are drawn. From the points, 1′, 2′, &c., where the planes passed through the elevation cut the parabola, draw perpendiculars to the horizontal projections of these planes; the perpendicular, DD′D″, will form the foot of the parabola, EE′ its vertex, and perpendiculars from 1′, 2′, — — — let fall upon the cirles 1^2, 2^2, 3^2, — — — will form intersections, all lying in one arm of the parabola, the other being easily constructed, as shown in *fig.* 28. The orthographic projection of the parabola is shown in the figure near *fig.* 28. It is obtained by erecting a perpendicular from the middle of DD′, and marking successively upon this the height D1′, D2′, D3′, &c., and drawing throngh these points, parallels to

DD′. Marking off on these horizontal lines, the breadths taken at the corresponding parts of the horizontal projection of the parabola, we shall obtain the points, 1′, 2′, 3′, 4′, and 5′ E, which determine one half of the parabola. The other half is to be drawn symmetrically with this.

A simple method of describing a parabola, when its breadth below and its altitude to the vertex are given, is shown in *fig.* 31. At the middle of the line AB, erect the perpendicular, CD=twice the height of the parabola, and determine the vertex, C; through this xy is passed, parallel to AB. At A erect the perpendicular, Ax, and divide it in f, g, h, &c., into equal parts, which are then marked off from D, at a, b, c, &c. From C to k, and from A to a, draw straight lines; their intersection will give one point of the parabola. Another point will be obtained by the lines Ci and Ab, &c. The second limb of the parabola, being symmetrical with the first, is easily constructed.

The projection of the third conic section, the *hyperbola*, is explained by means of *fig.* 30. As this is perpendicular to the base, it can appear only as a straight line in horizontal projection, this line being D′F′. The vertex is projected at E′. If the line E′F′ were also perpendicular to the base line, the hyperbola would be projected on the vertical plane in another straight line: this is the case in *fig.* 55, with respect to the line CA. As this view can give no satisfactory representation, we have in *fig.* 30 made the plane of intersection parallel to the vertical plane. To determine the vertex of the hyperbola in the vertical view, we first draw the axis, CC′, take off from C, and parallel to the base, the distance C′E′, and let fall from the point thus obtained, a perpendicular to the side of the cone. From the point where this meets the side, draw to the axis a parallel to the base; this determines the point E on the latter. Place any number of planes of intersection, in the horizontal projection, parallel to the base; these will form circles with the common centre C′, and whose vertical projections as straight lines may be readily ascertained by methods already explained. From the points 1, 2, 3, &c., of the ground plane, where the projections of the intersecting surfaces cut those of the hyperbola, draw perpendiculars to the corresponding vertical projections of the aforesaid planes of intersection; we thus obtain the intersections 1′, 2′, &c., as points of the hyperbola, which may then be joined by a line continuous with the vertex E. Another method of describing the hyperbola is presented in *pl.* 4, *fig.* 29.

If one body penetrate another, a surface of intersection or penetration will be formed. It is one of the problems of projection to determine the outlines of such surfaces of intersection, and their projections under different circumstances. The number of possible cases is infinite, and we can here only adduce a few as examples. In *fig.* 32, two cylinders are shown, of unequal diameters, and penetrating each other at right angles. The base line, or the one in which the surfaces of horizontal and vertical projection intersect each other, may be represented by xy. The one cylinder is represented in vertical projection as circle E, and in horizontal projection as rectangle AB; the other, being parallel to both planes, is in both cases

exhibited as a rectangle, until it touches the second cylinder; the problem, therefore, is reduced to finding the part which intersects the surface of the first cylinder. In vertical projection this is easy, as here the line of intersection coincides with the convexity of the cylinder; nothing more is necessary, then, than to prolong the corners *a* and *g* to the cylinder E. The case is different, however, in the horizontal projection, the line of intersection here being a curve. If the axes of the two cylinders lie in the same plane, the curve will be symmetrical; if, as in our example, this is not the case, the superior half will be different from the inferior, and it becomes necessary in all cases to seek similarly situated points of the intersection surface, in the horizontal and vertical projection. For this purpose, under *ag* and *a'g'*, describe the semicircles whose projections are the lines *ag* and *a'g'*; divide their circumferences into any number of equal parts, and draw lines parallel to the edges of the small cylinder. These must reach to the circumference of the larger cylinder, in the vertical projection, and may be of any length in the horizontal. From the points where these lines intersect the circumference of the larger cylinder in the vertical projection, let fall perpendiculars upon the corresponding parallels in the horizontal projection. We shall thus obtain the points *h, i, k, l, m, n,* and *o*, which are common to the convexities of both cylinders, and must consequently lie in the contour of the surface of intersection; this latter may then be easily described. We have represented the lower half of the curve; the upper is obtained in a similar manner. As the cylinders approach towards equality in thickness, the curve becomes abrupt; when both are equal, the intersection appears in the projection as two straight lines, which meet above the axis of the cylinder.

Pl. 4, *fig.* 33, represents the intersections of two cylinders, of different diameters, when their axes lie in the same plane. The mode of constructing the cylinders and their bases follows from what has already been said. We must remark, however, that the two upper ellipses in the vertical view have arisen from a misapprehension of the engraver; the upper bases should have been projected as straight lines. The construction of the intersection follows from what was said in explanation of *fig.* 32. With respect to the horizontal projection, the views of the bases are readily found, these being ellipses, whose perpendicular axes are the respective diameters of the cylinders, the horizontal being determined from the vertical view, by means of the perpendiculars *gg', hh', ee', ff', aa', bb', cc',* & *dd'*. To project the line of intersection, the points of division of the projection *e'f'* are projected upon the ellipse *ef*, at 1', 2', &c., parallels to the surface, drawn through the points of the ellipse thus obtained, and at the corresponding points the line of intersection in the vertical view, cut by perpendiculars. The points of intersection will be common to both cylinders, or be points in the line of intersection. The lower line of intersection, dotted only in the figure, is obtained in a similar manner.

Fig. 34 exhibits the intersection of a cylinder and a sphere, where the cylinder has the smaller diameter of the two, and is not parallel to the surface of projection. The development of both projections presents no difficulty in itself, if what has already been said on the subject be kept in mind;

it is only the circumferences of the surfaces of intersection that require to be attended to here. These surfaces of intersection must, in all cases, be curves: they are obtained by dividing the circumference of the cylinder into any number of equal parts, and, through these, drawing parallels to the sides of the cylinder. The figure represents only a few of these parallels. Where these lines intersect the perpendicular diameter of the sphere, in the vertical view, planes of intersection are to be passed through the sphere. These can be very readily transferred to the horizontal view, where they appear as circles. The points where the parallels to the circumference of the cylinder intersect the corresponding circular sections, are points of the surface of intersection, which may then be readily described. On account of the small scale of our figure, only a few points have been determined; the rest are readily formed in the same manner. A concluding example of the intersection of bodies is presented in *fig.* 35. Here, an oblique cone penetrates an oblique cylinder, in such a manner that part of the cone passes through the cylinder. To develope the intersection, the method employed in reference to *fig.* 33 must be again brought into play, with this difference only, that the lines drawn from the points of division of the base to the cone must not be parallel to the lateral edges, but converging to the apex of the cone.

b. The Reticulations of Bodies, and the Unfolding or Development of Surfaces.

By the reticulation of a body is meant the continuous description of its inclosing surfaces in one plane. This is easiest in bodies which are inclosed entirely by plane surfaces, as is the case in the so-called regular bodies. It is only in this case that the reticulation of a body can exhibit a perfectly true picture of its surface. *Plate* 4, *fig.* 49, is the reticulation of a tetrahedron, formed by four equal equilateral triangles; *fig.* 50, that of a cube, or hexahedron, formed by six equal squares; *fig.* 51, that of a dodecahedron, formed by twelve equal regular polygons; *fig.* 52, that of an icosahedron. The figure is not quite complete, as in addition to the fourteen equal equilateral triangles, six more must be added, viz. one in the upper row, next to 11, three in the middle row, next to 7, and two in the lower row, next to 14.

In conclusion, we will present one or two examples, in which not the entire reticulation, but merely the convex surface, will be referred to. The bottoms, so to speak, are very easily constructed.

Fig. 53 represents, at A, the horizontal intersection of a cylinder by the plane CD, this latter being itself visible in *fig.* 54. Let portions be cut off obliquely from the lower part of the cylinder, by the lines BC and BE, and an oblique portion from the upper part by the lines FG. Suppose, now, that half the convex surface of this remnant of the cylinder is to be ascertained. If the cylinder had not been mutilated in this manner, its development would be a rectangle, the altitude being equal to the height, and the base equal to the circumferenee of the cylinder, or, as in our illustration, to half

this circumference. This rectangle must, in fact, be constructed upon the elements first mentioned. The semi-circumference, in *fig.* 53, is to be divided into 8 equal parts, as shown at *a, b, c, d,* &c.: these must be so small that, without material error, the arcs may be considered as straight lines. These eight parts are to be transferred to the rectangle at a^2, b^2, &c., and the perpendiculars a^2a^3, b^2b^3,——— i^2i^3, drawn, which will all lie in the convex surface of the cylinder. From the points *a, b, c,* ——— *i,* in the horizontal projection of the cylinder, lines are to be drawn until they intersect the oblique section of the cylinder at a', b',—— i'. From the points a', b',——— i', draw parallels to the base, xy, until they intersect their corresponding lines; the points a^3, b^3, c^3,——— i^3, will thus be obtained, which may be connected by a curve. This will be the development of half the ellipse of which the line FG represents the vertical projection. The perpendiculars aa', bb',——— ii', also intersect the projections CB and BE of the semi-ellipses of the lower cylinder sections; accordingly, here, as in the upper ellipse, the corresponding points may be connected by parallels to the base, xy, and points of the curve obtained on the lines a^2a^3, b^2b^3,——— i^2i^3. *Fig.* 54 represents rather more than half the development of the cylinder.

Fig. 55 exhibits the horizontal and vertical projection of a right cone, intersected in the three conic sections, and projected according to the rules given for *figs.* 27, 28, 29. The convex surface of this cone is now to be found, and upon it the developments of the three conic sections, described.

The convex surface of a right cone is a circular sector, whose radius equals the slant height of the cone, and whose arc equals the circumference of the base. If, then, from any point, with a radius equal to the slant height of the cone, an arc, x^2y^2 (*fig.* 56), be described, this sector, with its two radii, will determine the convex surface of the cone, provided that the proper length of the arc has been obtained. This may be done by dividing the circle whose projection is xy (*fig.* 55), just as was done in the case of *fig.* 53, and transferring the arcs of division. The sum of these arcs, which will be few or many as the result is to be less or more accurate, will determine the extent of the circumference of the base. Lines drawn from the individual points of division to the centre, in *fig.* 56, will represent so many lines of the convex surface of the cone. Their projections in elevation (*fig.* 55) will be obtained by transferring the parts from the plan, $x'y'$, of the base, to its vertical projection, xy, by means of perpendiculars. Lines must then be drawn from the points of intersection thus obtained, to the apex. These will intersect the vertical projections of the conic sections.

From the vertex, in *fig.* 56, lay off on the middle line the distance from the apex of the cone (*fig.* 55), to the point G′; G′ will then be a point in the development of the ellipse. The distance from the apex of the cone (*fig.* 55) to the first intersection of the ellipse by the projection of the sides of the cone in *fig.* 55, laid off on both sides of the point G^2 in *fig.* 56, gives two new points in the development of the ellipse; and the same is to be done with respect to the remaining lateral lines of *fig.* 55. Connecting these points in *fig.* 56, will give the development of the ellipse. In like manner the curve $B^2A^2C^2$ is found, as the projection of the hyperbola The

parabola, in its development, appears divided into two symmetrical parts, owing to its falling on the line in which the convex surface of the cone is supposed to be divided. By these various constructions, we obtain the symmetrical figure, $H^2E^2F^2B^2C^2A^2D^2E^2H^2G^2$, which forms that part of the convex surface of the cone bounded by the three conic sections.

B. PROJECTION OF SHADOWS.

By projection of shadows is to be understood the method of representing bodies as they appear to an observer under illumination from a certain direction. It is evident that both the direction and the nature of the illumination (whether from a point or a surface) must greatly influence this mode of representing objects. If the illumination be supposed to proceed from a single point, it involves a department of the subject which will not be treated of in this place, as it more properly belongs to another part of our work. We here treat only of that description of shadows produced by an infinitely great luminous surface, considered as the source of light. In the former case, the rays of light form a cone, and diverge the more the nearer the source of light (the apex of the cone) lies to the object illuminated. In the latter case, and the one to be now treated of, the rays are all parallel to each other. In what follows, we suppose the plane of illumination to be so situated with respect to the surface of representation, as that all the rays come in the direction of the diagonal of a cube, i. e. incident at an angle of 45° on both the horizontal and vertical plane. The rays of light are supposed to come over the left shoulder, and to fall upon the paper and the object to be represented.

The general head of shadows embraces two subdivisions, viz. *shadows* proper and *shades.* The *shade* of a body is that part of its own surface from which light has been intercepted by some other part of the body itself. The *shadow of* a body is that part of indefinite space from which light is excluded by the body. The *shadow on* a body is that portion of its surface from which light has been intercepted by some other body, placed between it and the source of light.

With respect to the shades of bodies, it is evident that the rays of light can exert their greatest powers of illumination only when they fall at right angles upon a surface, and that the illumination will be less, the more oblique the angle of incidence. The deepest shade must be produced where the rays are only tangent to the body, as they there no longer illuminate the body. Taking for illustration a half cylinder, as exhibited in horizontal projection in *pl.* 4, *fig.* 38, the line bb' will be perpendicular to the surface, and the illumination of the cylinder will consequently be greatest in this part. This is the point of *highest light.* The ray of light dd' will only be tangent to the circumference of the cylinder: here then will be the *darkest shade.* Between b' and d the rays of light will fall more and more obliquely, and the illumination become less and less. The same must be the case from b' towards a. Beyond d the body would be entirely dark, were it not for

the reflection of light from other bodies about or beyond this. The body will therefore become again somewhat lighter, after its darkest shade. Similar conditions must be presented in respect to the illumination of all other surfaces, and it will not be difficult to determine the tone of light of each surface, knowing the direction of the incident rays. It is evident that by taking this direction at other angles, any other illumination may be constructed. It must not be forgotten, however, that the angles of incidence upon the two planes of projection must always be complements of each other, so that light, incident at an angle of 40° upon the plane of elevation, will strike the horizontal plane at an angle of 50°, &c.

Shadows, as already mentioned, are produced when one part of a body projects beyond another, or one body is interposed between the source of light and another body. These shadows, like the bodies themselves, may be constructed when the measurements of all the parts are known, *i. e.* the body itself, with all its accompaniments, may be constructed in plan and elevation. The rules for the reception of light must always be the same as in the case of shades, and the same angle of incidence of the light must be employed for both the shades and the shadows.

When a shadow is to be determined, it is, first of all, necessary to determine the lines which cast the shadow; and after these have been found, to seek the projections of these lines of shadow. After this, those parts of the entire surface from which light is intercepted may be readily determined.

a. Shades and Shadows upon Plane Surfaces and Curved Surfaces of Elevation.

Let us suppose *figs.* 36——44, *pl.* 4, to represent the vertical, and beneath this the horizontal projection of a plane wall, to whose anterior face six bodies, of different forms, are attached, and covered above with partly circular, partly rectilineal plates: let now the problem be, to determine the shadows cast by the plates upon the solids, and by both plates and solids upon the wall.

Fig. 36 is a four-sided prism, A, covered with a somewhat projecting plate, B, likewise four-sided. To find the various shadows, it becomes necessary first to find the line of shadow. The direction of the rays of light is here, and in the following cases, assumed at 45°, for the horizontal and vertical projection. Draw lines in the horizontal projection, in the direction of the rays, to the bodies A' and B'; then the first rays passing by the body will go through the points c and d. These are the projections of the two right edges of the bodies A and B, in vertical projection; the latter will therefore be two lines of shadow. If, moreover, rays be drawn at an angle of 45° in the vertical projection, one will pass by at a' and others at c^2, d^2, d^3: the line $a'd^2$ will consequently cast a shadow. As c^2 is the projection of the upper right hand edge of the prism, d^2 that of the lower right hand edge of the plate, and d^3 of the upper right hand edge of the same, the four lines just mentioned will cast shadows. Of all these edges, the line $a'd'$

alone casts a shadow upon A; all the rest, and even a part of $a'd^2$, cast shadows upon the wall only.

As the direction of every line is determined by several points lying in it, to determine the boundary of shadow in both projections, we need two points for a straight line, and a greater number for curves.

The shadow of the line $a'd^4$ falls upon the body A, and it becomes necessary to obtain the point b^2, from which the shadow must run parallel to the shadow-casting edge, the two surfaces of the body and the plate being parallel. In any case, the point which casts its shadow upon b^2 must lie in the line $a'd^2$, whose horizontal projection is ad, and this, in a direction of 45°. If, therefore, in the horizontal projection, a line be drawn from the left corner of A′ to ad, at an angle of 45°, it will determine b as a shadow-casting point. This point is then transferred to the vertical projection, by means of the perpendicular bb', and b' is then exhibited as the shadow-casting point in the latter. Drawing a line from b', at an angle of 45°, its intersection with the left edge, b^2, of the prism determines the shadow of the point b', and the direction of the line of shadow. As the line $a'd^2$ determines the boundary of the surface which prevents the incidence of light upon the body, A, that part of A above the line passing through b^2, lies in the shadowed portion.

With respect to the shadows cast by the prism and plate upon the wall, the edge a casts a shadow, which necessarily begins in the point where this edge touches the wall. Drawing a line from a', at an angle of 45°, this line, $a'a^2$, will be the line of shadow. The side of the plate behind the line d^2d^3 will likewise cast a shadow, whose direction is determined by the two lines drawn at an angle of 45°, in the vertical projection. The length of this shadow is obtained by considering that the point d, the horizontal projection of the line d^2d^3, and consequently the edge which is projected through it, casts its shadow to d'. The edge $a'd^2$ also casts a shadow upon the wall, which must run parallel to the edge, the edge and the wall being themselves parallel. The point d^4 is the point of shadow for d^2; if then, through d^4, a parallel to $a'd^2$ be drawn, this will be the line of shadow of the plate on the wall behind it. Finally, the side of the prism lying behind c casts also a shadow upon the wall, whose limits will be the shadow of the edge which is projected through c. If then the tangential ray cc' be drawn, and the point c' be projected on the vertical plane, the perpendicular through this point will determine the line of shadow, whose upper point still remains to be determined. The point c^2 in the vertical projection answers to c in the horizontal; then, if we draw a line, at an angle of 45°, through c^2, it will intersect the above mentioned perpendicular in e, and this point, e, will be the shadow of c, or, what is the same, of c^2, and will limit the shadow of the edge of the prism.

Pl. 4, *fig.* 37, exhibits the half of a hexagonal prism, covered by a four-sided plate, under the same conditions as in the preceding case. The shade of the body is found according to the principles already laid down. The surface receives the strongest light to the left, the rays here falling perpendicularly; the light, however, fades somewhat towards the extreme left.

The anterior surface receives the light more obliquely, and thus appears somewhat darker and uniformly illuminated, the surface being parallel to the plane of projection. The light passes entirely by the right surface, which consequently appears entirely dark, brightening a little, however, towards the wall, where it receives a certain amount of reflected light. The shadows on the wall are constructed as in *fig.* 36; those cast by the plate on the body, are obtained as follows: that for the anterior surface is obtained, as in *fig.* 36, by the lines bb', b^2b^3, as this surface is parallel to the surface of representation, and to the anterior surface of the incumbent plate. The shadow cast by the corner a of the plate, will be determined by a ray of light passed through the point. It will fall upon the left lateral surface at a^2, which will therefore be the projection of this shadow. To obtain this point in vertical projection, draw the perpendicular a^2a^3, and intersect this by a ray through a, a^3 will be the point desired. If this be connected with the extremity of the shadow or the anterior surface, the broken line through a^3b^3, gives the shadow of the anterior face of the plate. Since a is the projection of the horizontal lateral edge of the plate, it follows that every ray through that edge must be parallel to aa^3; the direction of the shadow, therefore, from a^3 towards the left, will be regulated by that of the ray.

Fig. 38 represents a half cylinder, covered by a four-cornered plate. The shade of the body is obtained by drawing a line from the left-hand side to the centre, at an angle of 45°. Where this line cuts the convex surface of the cylinder, the light will be greatest, the rays falling here in a plane perpendicular to the surface. A second ray, tangent to the surface, determines the line of deepest shade, up to which the light decreases more and more, and beyond which reflected light comes into play. The shadows of the body and the plate on the wall are obtained as before. The shadow cast by the plate on the body must be a curve, the body itself being curved. To obtain this shadow, find first the shadow of the point b, the corner of the plate, which is done by means of the lines bb', b^2b^3. As it is a curve that we are seeking, it will be necessary to obtain a number of points in it, so as to determine its direction. The points casting shadows lie, however, in the line be. One of the points of shadow is given by the line cc', c^2c^3, and as many more can be obtained as is necessary for the required degree of accuracy. The shadow naturally ceases where the ray, dd', is tangent to the cylinder: the shadow then passes into the shade of the body. To find the direction of the shadow from b^3 to the left, seek first the shadow of the point a. This falls at a', and the vertical projection of this point must lie on the line a^3. But b^2 is the projection of the left side of the plate; the point a^3 lies, therefore, behind b^2, and if a ray be passed through b^2 (actually through d^2), it will determine the point a^2. This lies in the line b^2b^3, and all the other points of the shadows cast by the lateral edge will fall in the direction b^2b^3.

Pl. 4, *fig.* 39, represents the half of a hexagonal prism, covered by a semicircular plate. The shade of the body has been already constructed in *fig.* 37, as well as the shadow of the body. It now remains to determine the shadow of the plate on the wall. As the plate is circular, its shadow must

be a curve. The method of finding this curve is easily determinable from the preceding considerations. Take a certain number of points in the horizontal projection, obtain their shadows in this projection, and transfer them to the vertical projection. Thus, the point c will cast its shadow to c'. and this must lie on the perpendicular line, $c'c^3$, of the vertical projection; transfer the point c to c^2, and from this point draw the ray c^2c^3; we thus obtain the projection of the point of shadow; and after a sufficient number of points has been obtained, an indication of the curve of shadow produced by the lower edge of the plate. As the upper edge must cast a similar shadow this is to be obtained in the same way. The point d^3 determines the extreme point of this curve, which is closed by a perpendicular representing the shadow of the vertical edge or line of shade of the plate. The shadow of the plate upon the prism must also be curved, as it falls from a curved upon a plane surface. First of all, it is necessary to find the point of shadow upon the edge. For this purpose, draw the line aa' in horizontal projection; we shall thus obtain the shadow-casting point, a^2, in the vertical projection, and a^3 as the point of shadow. According to the method described in *fig.* 38, the points b^3, c^3, &c., are then obtained, and consequently the curve of shadow upon that side of the prism which lies in the light.

Fig. 40 exhibits the half of a truncated cone, covered by a four-sided plate. The construction of the shade of the body, and the shadow of the cone and plate upon the wall, differs very little from what has been described in *fig.* 38; it is different, however, with respect to the shadow on the cone. This shadow is analogous to that of the cylinder; as, however, the surface of the cone is not perpendicular, but deviates every moment from the perpendicular, its shadow must fall somewhat differently. To obtain this shadow, suppose several horizontal planes to be passed through the vertical projection of the cone, appearing in it as straight lines, and in the horizontal projection as semicircles; they are indicated by the figures 1, 2, 3, &c. Suppose the rays necessary for producing the shadow to be drawn in the horizontal projection, they will intersect the cone, and as the sections are parallel to the axis of the cone, these sections will appear in the vertical projection as hyperbolas, or at least parts of such, and may be constructed according to *fig.* 30. These hyperbolas serve instead of the perpendiculars employed in *fig.* 38, and by means of them, and of the projections of rays as bb', b^2b^3, the curve of the shadow may be very readily determined.

Fig. 41 represents a half cylinder, covered by a semicircular plate. Here one plane casts a shadow upon another parallel to it; the shadow will therefore be parallel to the shadow-casting line, and to determine this shadow nothing more is necessary than to pass a ray through the corner where the shadow begins. From the point where this ray intersects the edge of the cylinder, draw a line parallel to the plate: this will be the line of shadow.

Innumerable cases might be adduced, but the general principles involved in all are nearly such as have been explained and illustrated in the preceding instances.

b. Shades and Shadows upon hollow, straight, and curved Surfaces.

Fig. 42 exhibits a four-cornered niche, closed above. The point a is the horizontal projection of the shadow-casting line, and must itself be the shadow-casting point. Passing a ray, aa', through this point, it will determine the situation of the point of shadow upon the back wall, whose projection in elevation may be determined by the lines $a'a^3$, and a^3a^2 at a^3. As, however, a is the projection of the entire shadow-casting edge, the limit of shadow for this edge must lie in the perpendicular $a'a^3$; a parallel, therefore, to the corner of the niche, drawn through a^3, will determine the shadow of the cover.

Fig. 43 represents a niche, forming the half of a hexagon, and covered rectilineally above. The ray of light, aa', determines the extremity of the shadow in horizontal projection. The intersection of the perpendicular $a'a^3$, with the ray through a^2, determines its position, a^3, in elevation. The part of the perpendicular below this point, a^3, will be the line of shadow cast by the vertical edge of the niche. As the right side of the niche is oblique to the surface of representation, the shadow must run obliquely from d^3; now, as the cover coincides with the edge of the niche in c, the shadow must run in this direction; d^3c will therefore be the line of shadow on this oblique side.

Fig. 44 exhibits the half of a hollow cylinder, open above, and the problem is, to find the shadow cast by the edge of the cylinder upon its inner surface. Its boundary in vertical projection is obtained, in the first place, by passing a ray through the point a, the horizontal projection of the edge of the cylinder. The vertical projection of the point a', where it meets the inner surface of the cylinder, is obtained at a^3, by the intersections of the lines $a'a^3$ and a^2a^3. A part of the upper edge of the cylinder also casts a shadow. This begins in the point c^2, which is the vertical projection of the point c', where a ray is tangent to the surface of the cylinder. The point c' is obtained by drawing a line from c to the surface of the cylinder, at an angle of 45°. The shadow runs from c^2 to a^3 in the curve. This curve is found by obtaining individual points, as before explained.

Pl. 4, *fig.* 45, exhibits the shadow of a straight cover to a semi-cylindrical niche. The straight part of the shadow is obtained as in the preceding case; the figure itself shows the method of finding the shadow of the covering. Thus for instance, the shadow of the point b is obtained by means of the lines bb', b^3, and b^3, and the shadow ends in the point c', where the anterior edge, a^2c', of the covering, meets the wall of the cylinder.

Fig. 46 explains the method of finding the shadow cast upon its inner surface by the edge of a niche, dome-shaped above. This shadow has a very peculiar outline, and can only be determined for the dome by a very exact projection of the rays, and the accompanying subsidiary lines. The limit of the straight part of the shadow is easily found to lie at a, according to *fig.* 45; the extremity of the compound shadow must necessarily lie at e, where a ray of light would be tangent to the dome. To obtain the curve between a and e the following method is to be employed, which is quite

analogous to those already mentioned, and peculiar only in the construction of the perpendiculars, by means of which the vertical projection of those points is found, whose horizontal projection has already been ascertained. These perpendiculars, which in all the other constructions have appeared as straight lines, here present themselves, in the space of the dome, as curves. To determine this curvature, horizontal sections are passed through the vertical projection, at 1, 2, 3, &c., projected as straight lines here, but as semicircles in the horizontal view. Suppose in the ground-plan a ray to be passed from any point in the anterior edge of the dome, against the inner wall; this will intersect the projections of all the above mentioned horizontal projections. It will then be easy to determine, from the preceding observations, the vertical projections of these intersection lines, which will be straight to the commencement of the dome, and will then curve according to the curvature of the dome. If these lines be intersected by a line, at an angle of 45°, from the vertical projection of the point from which the ray has been drawn in the horizontal projection, we shall obtain one point in the curve of shadow; the other points necessary fully to determine this curve, may be obtained in the same manner.

Fig. 47 teaches the method of finding the shadow cast upon the inner wall of a dome-shaped, closed, half-round niche, where a part of the dome is cut away above, and the niche continued in a semi-cylinder. This is a combination of the cases treated in *figs.* 44 and 46, so that it would merely be necessary to construct two shadows, one after another, but for the fact that a part of the dome (that lying between b' and c'), has been cut out. Here it is not the contour of the dome that casts the shadow, but the boundary of the section. In this manner a part of the shadow seen in *fig.* 46 is cut away, as shown in *fig.* 47.

It is necessary, before concluding these remarks on shades and shadows, to advert to the cases where the body is not attached to a wall, but stands at some distance from it. Although these cases present now no difficulty whatever, it may be advisable to give an example, as in *fig.* 48. Let a six-sided prism stand in front of a wall, as shown by the horizontal projection of *fig.* 48. The problem is, to ascertain the shadow cast by the pyramid, in horizontal projection upon the floor, and in vertical projection upon the wall behind. First, to find the point where the apex g casts its shadow. Draw rays from g and g', in horizontal and vertical projection, and combining them in the usual way, find their intersection at g^3, the vertical plane. The line $g'g^3$ is the projection of the shadow of the axis of the prism, and g' is a point in this shadow. We obtain the shadow-casting point of the axis by drawing a line to that axis, from g', at an angle of 45°. If a horizontal plane, d^2b^2, be passed through g^4, this plane will be projected in plan as a small hexagon. Pass tangent rays to this hexagon; they will determine the points d' and b' as the projections of the shadow-casting points of the pyramid, in horizontal projection. If then from d and b the lines dd' and bb' be drawn, these will determine the outline of the tapering shadow of the pyramid. The lines $d'g^3$ and $b'g^3$ will determine the outline of the

shadow in the vertical view, as is clearly shown by drawing the rays d^2d and b^2b'.

C. Linear Perspective.

We shall here confine ourselves to the mathematical principles of perspective, as occasion will be had to speak of perspective in general and its application to the arts, in another part of the work.

We have already remarked, in the introduction to projection, that in perspective the visual rays are all supposed to proceed from one point—the point of sight; while in projection they are supposed to be parallel to each other. We may here again employ the illustration of the plate of glass interposed between the spectator and the object, and upon which the perspective image of the latter is represented.

In *fig.* 57, *pl.* 4, let XY be the ground plan upon which a square, *abcd*, is supposed to be drawn, and whose perspective representation is to be obtained on the vertical glass plate RS. Let F be the station of the observer, and A the point from which he sees the square *abcd*. This point is called the *point of sight.* The distance of the observer from the glass plate—the plane of projection or the plane of the picture—is determined by the line AA′; the point A′ is called the *point of distance.* If visual rays be drawn from the point of sight to all the corners of the square *abcd*, these must necessarily intersect the glass plate or the plane of the picture. It is then necessary to find the points of intersection, so that by joining them by straight lines the perspective of the square may be obtained. For this purpose, in the horizontal plane draw perpendiculars through the points *a*, *b*, *c*, *d*, extending to the foot of the glass plate. In the position of the square here assumed, two such lines are all that is necessary, as two of the corners lie in one line. Drawing lines from *f* and *g* to A′, these must lie in the plane of the picture, and the same must be the case with their intersections with the visual rays. The points a', b', c', d' will then be the perspective representation of *a*, *b*, *c*, *d*, the corners of the square. By properly connecting the points a', b', c', d', by straight lines, the quadrilateral thus obtained will be the perspective of the square.

The line DD′ supposed to be drawn at a height equal to that of the point of sight A, is called the horizon of the picture: it is the principal line in a picture, as by its height all the parts of the picture are regulated. All visual rays tend to the point of sight, and lines perpendicular to the plane of the picture will have their perspectives all tending to the point of sight. The point of sight is also called the *vanishing point,* because in it the lines appear to vanish. Other parallels not perpendicular to the plane of the picture meet in the horizon, but not in the point of sight. There may, therefore, be several vanishing points in the same picture, but only one point of sight, since an object for one and the same picture can only be observed from a single point.

The preceding construction, however satisfactorily it illustrates the principles of perspective, is yet too complicated in practice; simpler constructions therefore become necessary.

Fig. 58 exhibits a simplified construction for representing the square just mentioned. Let xy again be the basis of the plane of the picture, lying then above this line; the space below xy is the ground plane upon which the square *abcd* is described, and which rests immediately against the basis. We assume for the first case that the point of sight A is opposite to the middle of the square, and that DD′ is the horizontal line. The distance of the point A from the plane of the picture or the perspective plane must be given in numbers or otherwise, and laid off right and left from the visual point on the horizon; D and D are then the two points of distance. From all points of the square draw perpendiculars to the base, meeting it in *a* and *b*, from which points, rays, as *a*A, *b*A, are to be drawn to the visual point. The lines *a*D and *b*D are also to be drawn from the points *a* and *b* to the points of distance D and D opposite to them; they will intersect those first drawn in *e* and *f*. By connecting the points of intersection thus obtained by straight lines, we shall obtain the figure *abef* as the perspective of the square *abcd*, for the situation of the visual point at A. If the visual point be not in the middle, but at A′ for instance, the points of distance will lie at D′ and D′: visual rays from *a* and *b* to A′ willi ntersect the lines *a*D′ and *b*D′, and thus determine *abgh* as the perspective of the square. It must be remarked that two points of distance are not always necessary, one being sufficient in most cases, as will be shown in the next example.

If the square *abcd* does not lie immediately against the basis, as in *fig.* 60, the process is somewhat different, as the distance from the basis is to be taken into account. In this case let D be the point of sight, and A the point of distance, and draw perpendiculars to the basis from the four corners. These lie here in two lines, as the square is parallel to the basis. From the points where these perpendiculars meet the basis, draw lines to the point of sight D. Take off the distance of the corners from the basis, in the direction opposite to the point of distance D; for the point *a* we obtain a'; for b, b'; for c, b'; and for d, d'. Drawing lines from a', b', and d' to the point of distance A, they will intersect those drawn to the point of sight in a^2, b^2, c^2, d^2; connecting the four corners a^2, b^2, c^2, d^2 by straight lines, we shall have the perspective picture of the square at its proper distance from the basis.

From this figure we perceive that all lines in the object which run parallel to the basis, must be parallel to it in the perspective representation.

Pl. 4, *fig.* 59, exhibits a complicated rectilineal figure, with the construction of its perspective representation. The mode of operation is precisely the same as in the instance just explained.

Fig. 61 shows how a curve is to be represented in perspective. The curve is here a circle, *ab*; A is the point of sight, and D the point of distance. Here it is necessary to determine the perspective of several points, through

which the curve is then to be passed. Divide the circle into any number of equal parts, the number being greater with the size of the circle and the degree of accuracy required. From the points of division draw perpendiculars to the basis. As two of these points lie exactly behind two others, we shall have only five points on the basis, from which lines are to be drawn to the visual point, A. Each one of the three middle lines determines two points; the two external lines, only one each; these last points being the extremities of a diameter parallel to the basis. The distances of the five points are now to be laid off on the basis, in a direction opposite to the point of distance, and through the points thus determined, lines drawn to the point of distance: these, by their intersections with the visual rays, will determine five points in the curve. The remark made with reference to *fig.* 60, that all natural lines parallel to the basis, are parallel to it and to each other in perspective, enables us to obtain the remaining three points. Through the three points of division to the left of *ab*, parallels to the basis are supposed to be drawn, which must then meet the division points of the circle, opposite to them. Drawing, in the perspective view, lines parallel to the basis, from the three perspective points obtained for those points, these will intersect the visual rays corresponding to the opposite division points. The three deficient points of the curve will then be obtained, with which the perspective view of the circle can be readily completed, as shown in *fig.* 61.

We have hitherto spoken of figures in a plane, that is, of surfaces: to deal with solids, we must determine the height, in addition to the length and breadth. It is to be observed that while the depth of the figure, speaking with reference to the plane of the picture, has its vanishing point in the point of distance, and the breadth in the point of sight, the height must likewise have its vanishing point in the horizontal line. If, then, a certain height be applied to the base, and lines be drawn from its top and bottom to a point in the horizontal line—generally the point of sight—the top and bottom of all bodies which have the height supposed, will lie in these lines.

Fig. 62 exhibits the method of determining the perspective height. Let a four-sided prism be here drawn, whose plane is given, and whose height is yz. A is the point of sight, D the point of distance, and xy the basis of the plane of the picture. After the base, *abcd*, of the object has been perspectively represented at a^3d^3, by the methods already given, apply the height, yz, to the basis, and draw the lines yA and zA; these will contain the top and bottom of the prism for the different planes in the plane of the picture. To ascertain, for instance, the perspective height of the prism at the point d^3, draw a line parallel to the basis, meeting the line yA in 2. A perpendicular line, 2,2′, cutting the line zA, determines at 2′ the perspective shortening of the height yz, for the plane through d^3, and so on for any other point. Let perpendiculars be next erected at the four corners of the perspective ground plane, and parallels be drawn to the basis, intersecting the line Ay, and perpendiculars again from these points intersecting the line Az; finally, draw

from these last points, parallels to the basis: their intersections with the perpendiculars first erected, will determine the four corners of the perspective representation, a^4d^4, of the upper base of the prism.

Pl. 4, *fig.* 63, exhibits the perspective representation of a compound body, viz. a pedestal surmounted by a cross. All the measurements necessary, are obtained from the plan *abcd*, and the geometrical elevation shown on the right hand side. By keeping in view the principles already explained, and observing the correspondence of lettering, the perspective construction will not be difficult. The line of height drawn on the right hand side serves to determine all the heights, as any geometrical height may be marked off upon it, and innumerable lines drawn from the relative top and bottom points: from the different intersections, the shortening in height for any plane in the picture may be determined.

VI. OF THE MOST IMPORTANT MATHEMATICAL AND SURVEYING INSTRUMENTS.

For the better elucidation of the preceding observations, the most important instruments used in geometrical drawing, and the various geodetical operations, have been represented on *pl.* 5, *figs.* 1–56.

The simpler drawing instruments, as the simple compasses, the drawing compasses with movable lead tube and pen point, the drawing pen, the ruler, the scale, and the square, need not be mentioned here, as they occur in every box of mathematical instruments, and consequently are too well known to require description. The first of the rarer instruments to be mentioned, is the *hair compasses* (*figs.* 1, 2), serving for very minute measurements. One foot, *b*, of a common compass, *ab*, is cut off, and a spring prolongation riveted at *d*, so that when it is attached to the head piece, it forms a leg, as in *ab*. The screw *c* can turn at *e* in the rivet, and has its nut in the upper leg. Turning this screw will cause a very slight motion of the spring joint to or from the other leg, independently of any motion in the joint. Very minute differences of measurements or lines may be thus appreciated.

The repeated efforts necessary to divide lines into a certain number of equal parts by the ordinary method, have caused the invention of the proportional compasses. These depend upon the geometrical principle, that in similar isosceles triangles the bases are as the sides. If, then, the legs of two such triangles are to each other as 1 : 2, the bases must be so likewise, and the smaller base will be one half the size of the larger. If we suppose two compasses, so united by their heads as to have a common joint, and the legs in the above mentioned ratio, all the conditions will be fulfilled, and the *bisecting compasses* will be the result. In this, the space between the points of the small legs will be just half of that between the points of the larger ones.

As, however, other divisions besides bisections are required, the head of the compasses has been made movable, thus forming the *proportional*

compasses (*pl.* 5, *figs.* 3, 4). Here the legs *ad* and *bc* are applied to each other, and have two equal slits, in which the plates *f* and *g* pass, convertible by the screw *e* into an ordinary compass joint. This screw forms, then, the point of rotation of the two legs, thus divided into four. Upon the leg is placed a graduation, determining the proportions $\frac{1}{2}$, $\frac{1}{3}$ $\frac{1}{4}$, &c., between the two legs. Loosening the screw *e*, and moving the head until the index mark on *f* coincides with a certain part of the division, $\frac{1}{4}$ for instance, and then tightening the joint, we shall have a relation of $1 : \frac{1}{4}$ between the two sets of legs. The space included between the points of one set of legs, will then be four times that between the points of the other set, whatever be the opening of the compasses. As there is a definite relation between the radius and the side of a regular polygon, another graduation is placed upon the compass to indicate this relation. This graduation is so arranged that when the index, *f*, stands at 1, where all the legs are equal, and consequently $ab = cd$, *ed* will be the side of a hexagon, inscribed in a circle of radius, *ab*. (The side of the inscribed hexagon = radius.) If the index *f* stands at 15, as in *fig.* 3, then $ab =$ radius, and $cd =$ the side of the pentadecagon. *Fig.* 4 exhibits the compasses edgewise, showing the nut of the screw. For the sake of great accuracy, a micrometer screw is sometimes attached to this instrument, as in *fig.* 5. In addition to the arrangements already described, there is a movable nut at *h*, through which passes the slide *k*. To move the head, *e*, by a very slight amount, the slide *k* is attached to the screw *b*, and rendered fast. By loosening or tightening the screw *l*, the slide *k* will be moved backwards or forwards, and with it the head, *e*, to which it is attached. When this is done, *e* is screwed fast, and *k* brought to *i*, where the micrometer arrangement is again brought to play in moving the points *a*, *b*, and consequently, *c*, *d*. We thus obtain *hair proportional compasses*. The lengths *ad* and *bc* must be perfectly equal, or else all indications will be erroneous. The great utility of the proportional compasses consists in their enabling us to enlarge or reduce all parts of a drawing to a certain scale, all that is necessary being to adjust (once for all) the two sets of legs in the same proportion as the required reduction or enlargement.

The common compasses do not answer for long lines. In this case the *beam compasses* are to be used, *pl.* 5, *figs.* 6, 7. These consist of a prismatic beam of wood or hollow prismatic brass rod, *ab*, upon which the boxes, *c*, *d*, slide, capable of being fastened by clamp screws. To each box is attached a point *h*, and *i*, one of them being replaced, when necessary, by a lead tube or pen point. One of the boxes is provided with a micrometer arrangement. For this purpose a head is attached at *f*, through which passes the smooth end of the screw, *e*, without moving back or forwards. The nut of the screw is at *g*; so that when the screw *e* is turned, the spindle being fixed, the nut *g* moves forwards and backwards, and with it the box and point attached.

The *triangular compasses* (*fig.* 8) serve to take off all three corners of a triangle at once, rendering the operation of transferring triangles one of great ease. This instrument has three legs, united in one head in such a manner, that two of the legs form ordinary compasses, in whose head, *a*, a

spindle *f* is inserted by means of the spring, *c*. The end of this spindle is round at one end, where it receives the eye of the third leg, *c*, which may be fastened by means of the plate, *g*, and the head screw, *h*. In this manner the leg *c* may be made to assume any given position with respect to *b* and *d*, and thus any given triangle be taken up with the compasses.

A convenient variety of the triangular compasses is the *plate compasses*, *fig*. 55, which consists of a three-limbed plate, AA, provided with a button, B. At the extremities of the limbs A, A, A are attached the secondary limbs, C, lying in the same plane. Their attached points, *a*, *a*, *a*, may be placed at any required position in the plane of the paper.

It has been seen under the head of perspective and projection, that when a circle is viewed from any direction other than one perpendicular to its centre, it will appear as an ellipse. As such foreshortened circles very frequently occur, and ellipses are troublesome to describe, considerable attention has been directed to the invention of instruments by means of which these curves may be readily drawn. These are called *elliptographs*, of which various forms have been devised. Two of these are represented in *figs*. 9–13. *Fig*. 9 is an upper and *fig*. 10 a perspective view of the oldest and most imperfect of these instruments. This consists of a cross, *abcd*, which has two grooves on its upper surface, crossing each other at right angles. Under these grooves are four points, which when in use are placed in the two axes of the ellipse, so that the middle of the cross stands exactly above the centre of the circle or ellipse to be described. The movable slides *g* and *h* work in the grooves in such a manner that *h* always moves in the groove *ad*, and *g* in the groove *bc*. The slides have boxes above, through which the ruler *ef* is passed, to which the boxes may be fastened by head screws. One end of the ruler *f* carries the drawing point *i*. To use the instrument, place the point *i* upon the extremity of the long axis, and fix *h* at the middle of the cross. Then bring *i* to the extremity of the short axis, and fix *g* at the middle of the cross. If now the ruler be moved, the box *h* will slide in *ad*, and *g* in *bc*, by means of which the drawing point will describe the ellipse represented in the figure.

The defect in this instrument consists in the impossibility of describing ellipses whose axes are shorter than or much different in proportion from those of the cross.

Farey's elliptograph, *pl*. 5, *figs*. 11—13, is perhaps the best yet invented. As far as its size will admit, the entire series of ellipses from a straight line to a circle may be drawn with great accuracy. It consists of two circles, A and B, lying one above the other in such a manner that, by means of the pinion K and the rack-work *d*, any required eccentricity may be given to them. They are fixed at any position by the screws *cc*. The two bridges, *aa*, and the curved arms, *bb*, serve to give the necessary strength to the circles, and to make them sufficiently open to see underneath them. The two circles may be shoved backwards and forwards between the four beams, D, E, F, and Q, which together form a frame-work. These beams lie in two planes, as shown by *fig*. 12, so that F and Q determine the path of the upper, D and E that of the lower circle. In this way the two circles may be moved

so that the centre of each moves parallel to its frame, the paths described being perpendicular to each other, thus constituting the axis of the ellipse. The ellipse itself is described by one point of a small pair of compasses, M, during the working of the two circles, the other point being placed in the foot H. *f* and *f* are buttons by means of which the circles are moved along, while the frame lies upon the paper. The frame is held down with the left hand by means of the buttons N and O. Any required eccentricity can be given to the foot H, as it slides with its frame, *g*, in a groove, and is united to the rack-work, *h*, which is moved by the pinion L. *Fig.* 13 presents a clearer view of the whole arrangement; it is a section perpendicular to the two beams, *a, a*, of the circles A and B. *Fig.* 12 is a side view of the instrument.

If both the circles are placed concentric with each other, and the point M placed in the centre, the latter will describe a point when both circles are moved in the frame. If L be turned, M becomes more and more eccentric, and will describe a circle in the movement of the circles, or an ellipse with equal axes. By turning K, both circles become eccentric. If M stand in the centre, it describes a straight line in the movement of the circles; in other terms, an ellipse with one of its axes = O. The different ellipses are obtained by the eccentricity of M. The two buttons, N and O, pass through the ruler, P, which has slits, as well as the prolongations of F and Q, so that the whole instrument can be moved without displacing N and O, which is necessary to the adjustment.

Another instrument for describing various curves, principally epicycloidal, is the eccentric compasses, represented in *pl.* 5, *fig.* 14. This was invented by Suardi, who described 1273 different curves which could be drawn with it. The three legs, A, B, C (the head only of C is visible), form a stand, at whose point of union, C, is placed the principal axis, *a*. Upon this the tube, *b*, provided with a milled head at *r*, may be turned. The strip *d* is fastened to the tube so as to turn with it. The wheels, *e* and *f*, are attached at *d* in such a manner that the axis of *e* may be moved in a slit made in *d*, and that of *f* may be moved on *d* by means of a slide; *g* is a toothed wheel, fixed upon the main axis, *a*. All three wheels may be interchanged or replaced by others, according to the character of the curve. They must all, however, be kept in close contact during their use. The pierced head, *n*, is attached beneath the prolonged axis, *f*, in which may be moved the small strips, *h*, carrying the drawing point, *k*.

The *Pantograph*, in its original form, is an instrument invented by Father Scheiner, intended to reduce or enlarge drawings to any required scale: in other words, to draw similar figures. As the similarity of figures can be attained in various ways, it follows that there must be various forms of pantographs. The two principal systems have been represented in *figs.* 15, 16. Passing by the earlier and more imperfect forms of the pantograph, we represent, in *fig.* 15, the first of these systems. Two rulers, AB and BC, are connected by a joint at B; two others, DF and FE, hinged together at F, are combined with the first two at D and E, so as to form a parallelogram. However much the angular positions of these four rulers may change, they

must always form a parallelogram. Small rollers are placed beneath the four points, D, E, F, and B, to allow the instruments to move freely over the paper. A, G, and C, are three points on the rulers where pipes are attached, and which must always lie in a straight line. C is the fixed point; the tracing point to be moved over the drawing, R, to be copied, is at A, and the drawing point with which the copy, S, is made, is at G. It is evident that in all movements of the instrument, the line CGA will remain straight; the figures described must, therefore, be similar, and the amount of reduction will be greater as G is nearer to A.

The pantograph represented in *fig.* 16, or *eidograph,* as it was called by its inventor, Professor Wallace of Edinburgh, has a very different construction. The bar D moves in the socket A, in which it may be fixed at any point by a screw. To the socket is attached a heavy foot, about which it can rotate. The arms, EH and FG, may also be moved, with reference to the axis of the bar D, the amount of motion being determined by the amount of reduction required in the copy. For this purpose, scales are attached to the bar and arms; a tightly stretched endless string, *aa,* passes round the two pulleys B and C, attached to which are the sockets, in which the two arms, EH and FG, slide, so that these must always make corresponding motions. If the tracing point be at H, and the drawing point at G, and if the three points, H, G, and A, lie in a straight line, then S will be the reduction of R. It is evident that if the tracing and drawing points change places, the copy will be larger than the original, in all pantographs, instead of being smaller.

By a slight change, the pantograph can be so arranged as to copy objects on the same scale as the original; and if the drawing pencil acts at its upper end, and the tracing point at its lower, then the copy will be inverted. This is of great importance to engravers, who are obliged to invert the designs on their plates, so as to have the engravings direct. For this purpose, the plate is placed above the drawing-point, which here consists of a fine etching needle. The pantograph has also been employed for engraving writing.

Two other drawing instruments of great utility are the *spring compasses* (*pl.* 5, *fig.* 17), and the *parallel ruler* (*figs.* 18, 19). The spring compasses, instead of a head, have a strong, curved, steel spring, *b,* to which the feet, *f* and *g*, are soldered. A curved screw, *d,* is fastened in the foot *f,* and passes through *g*. At *e* is the nut of the screw, against which *g* is constantly pressed, by the elasticity of the spring. By the action of this nut the points are separated or approximated. For convenience of handling, a button is attached at *a.* The spring compasses supply the place of the hair compasses, and have the advantage of being firmer, and less liable to derangement when once fixed, for which reason they are employed in describing circles on wood and metal.

The parallel ruler is used to draw parallel lines. Its earliest construction consisted of two parallel rulers, connected by a cross-piece at each end: the more modern instrument, represented from above in *fig.* 18, and laterally in *fig.* 19, consists of a broad ruler, A, upon which the axle B turns, in two small boxes, *aa.* At each end of this axle are attached small and perfectly

equal milled rollers, CC. These rollers pass through apertures made in the ruler, projecting on the other side sufficiently to rest upon the paper. The ruler, carried on this axle with its rollers, will admit of any number of parallel lines being drawn. The greatest possible care must be taken to have all parts of these two rulers perfectly accurate. If the two rulers of the first form be not constantly parallel, or the rollers of the second be not perfectly equal in diameter, the results will be erroneous. The common square, or T square, with movable head, supplies the place of the parallel ruler, and is more certain in practice.

Here also must be introduced the *protractor* (*pl.* 2, *fig.* 74), used in measuring and describing angles. It consists of an arc of brass, generally a semicircle, although sometimes a full circle is employed. In the semicircular protractor, the diameter falls along a ruling edge, and a small notch is made in the middle of this, answering to the centre of the circle. The circumference of the semicircle is divided into 180°, and, if of sufficient size (8—10 inches in diameter), into five minute subdivisions. A rule is sometimes attached to the protractor, turning about its centre, for the purpose of measuring or describing angles more accurately. By attaching a vernier to this rule, very small subdivisions, even to a minute, may be read off.

Directing our attention now to the instruments used in geodetical operations, the first one to be mentioned is the *measuring staff*. This is nothing but a staff of dry, oiled wood, tipped at each end with brass or iron, and divided into feet and inches, for the purpose of measuring straight lines in the field. When in use, it is either laid flat on the ground, or upon posts whose tops lie in a horizontal plane. Two or three of these are always required, in order that when one is laid down, another may be placed in contact with it,—the two extremities touching, and both lying in the same straight line. The first one is then to be taken up and placed at the other extremity of the second, and the operation thus continued. The base line for determining the length of a degree of the meridian in France, was measured by this instrument. Glass rods are sometimes employed, as was the case in the great English trigonometrical survey.

The *measuring chain* (*pl.* 5, *fig.* 27) affords results that are sufficiently accurate for ordinary purposes: it is therefore the one generally employed in common surveying. This consists of links of strong brass or iron wire, connected together by rings, and so arranged that the interval from the centre of one ring to that of the next amounts to just one foot. Ten of such links, or in some cases 12, form one rod, and this amount, and sometimes half and quarter rods, is indicated by rings, D, of peculiar shape. In America *Gunter's Chain* is universally used. Here the entire chain is 66 feet in length (= to four rods or poles), and is divided into 100 links, every tenth link being indicated by a piece of brass of peculiar shape. Each link, then, including the connecting rings, is 7.92 of an inch. At the ringed extremities, A and B, of the chain when in use, chain staves are driven into the ground over which these extremities pass. Each staff (*fig.* 28) has a pointed iron foot, *c*, and a cross piece, *b*, upon which the ring rests, and which is of use to enable the staff to be driven into the ground, by the

application of the operator's foot. In surveying, two chain carriers carry the staves with the chain; the forward one has in addition a number of *arrows* or *pickets* (*fig.* 29). The chain staves are sighted in the line to be measured and driven in, the chain stretched tight between them. The chain is then carried forwards, the forward carrier taking with him the chain staff, inserting an arrow in the hole made by the latter. When the hind carrier comes to the arrow, he takes it up, and inserts his staff, over which the ring of the chain is slipped; the forward carrier then stretches the chain and inserts his staff, which he again replaces by an arrow, and the operation is continued until the line is measured, or the forward carrier has exhausted all his arrows, which are generally ten in number. In this latter case a transfer of the arrows to the forward carrier is made, and the measuring proceeds as before. Careful count must of course be kept of the number of such transfers. This operation is sometimes carried on by means of the arrows and chain alone, the chain staves being omitted.

Of the more complicated surveying or measuring instruments we shall first of all mention the *plane table.* This was invented by Prætorius, hence called mensula prætoriana, and is used to obtain a reduced plan of a tract of land. Of the various improvements made in this instrument since its invention, the two principal are represented in *pl.* 5, *figs.* 20 and 21. *Fig.* 20 exhibits the instrument as modified by Major Lehman of Saxony. It consists of a stand with three feet, *a, b, c,* shod beneath with iron, and so attached to the stand by joints and winged screws as to allow a horizontal position of the table even on uneven ground. The support, *l,* upon which the board, *m,* rests, may be rotated on its axis; the board itself can be rendered perfectly horizontal by the three adjusting screws, *g, h,* and *i,* passing through the plate K. At a later period Lehman did away with these screws, producing the adjustments entirely by the feet of the stand. He also changed the mode in which *l* was turned, by causing it to run out into a disk below, turning on the plate K. This disk had an endless screw turned upon its edge; a spindle attached to *k* caught in this, so as to produce a very slow rotation of the board. The spindle was capable of being removed for coarse adjustments.

Fig. 21 represents a section of the upper part of *Mayer's plane table.* It has also a three-footed stand, upon which the socket, *a,* fits. In the upper part of this socket is an excavation in which the ball or nut, *b,* turns, capable of being fixed by the screw *c.* This nut carries above, the spike upon which the plate AB rests, and to which the metal ring, *lk,* is screwed. *df* is a portion of a middle piece with three curves, through whose extremities, *f,* pass three screws; one of them represented at *h.* These catch in the stirrups, *gi, mn,* upon which rests the ring *lk,* with its incumbent plate, AB. By these adjusting screws, the horizontal position of the table is secured. Instead of the female screw at *g,* a knuckle may be attached, as shown at *o.* The screw *e* serves to fix the apparatus to the spindle. This form of the plane table does not give the fine adjustment of the first, as the screws allow a certain amount of spring.

As part of the apparatus of the plane table, we will mention, first, the *fork* (*fig.* 22), which consists of two strips, A and C, fastened by the head, B, in such a manner that they may be slid upon the drawing board. The upper strip, A, runs out to a point which lies exactly above the spot, C, of the lower strip, to which the line for the plummet D is attached. A second plummet is often attached above B. These forks serve to bring a given point on the table directly above a certain point on the ground. The point A is laid upon the point in question, and the table moved until the plummet hangs directly over the spot of earth.

The *erecting compass* is another piece of supplementary apparatus, serving to set up the table in a direction parallel to the original one. This is nothing more than a small magnetic compass with a well divided dial, which can be screwed on the edge of the table.

The *level* serves to render the drawing board perfectly horizontal, this being indispensably necessary to an accurate measurement. It is of two forms—the tub-shaped and the tubular. The *tub-shaped level* consists of a flat cylindrical vessel with an accurately ground bottom, and covered above by a plate of glass. The vessel is filled with alcohol so as to leave only a minute bubble of air, and the top cemented on air-tight. The instrument is inconveniently sensitive when the upper plate is perfectly plane; this is, therefore, generally very slightly convex, so that there may be a tendency in the bubble to seek the centre of the surface.

The *tubular level* (*pl.* 5, *figs.* 23, 24) consists of a frame perfectly plane below, and inclosing a tube of glass closed at both ends. This tube is filled with alcohol, in which is a small bubble of air, which, when the tube is perfectly horizontal, rests in its middle. As the glass tube by itself would be liable to breakage, it is inclosed in a tube of brass, which has a piece cut out of the middle portion above from E to F, to render the bubble visible. This exposed portion of the glass is protected by the bands *aa*. To be certain of the central situation of the bubble, several concentric circles are cut on the cover of the tub level, and on the tube of the tubular level, between which the bubble must rest. Several are necessary, owing to the varying size of the bubble, produced by its expansion or contraction. A small screw is placed beneath the tube level in the socket at B, to correct any deviation from a perfect parallelism of the surfaces.

The *diopter ruler*, or *sight ruler*, is used to determine sight lines upon the plate of the tables. Its simplest construction is exhibited in *fig.* 25. A strong brass ruler, A, with bevelled edges, carries at its extremities two cross pieces, B and C, supporting hinged sight vanes, D and E. The eye diopter, or sight vane, D, contains a narrow vertical slit, or several small holes one above the other. The object vane, E, consists of a frame, in the centre of which a fine hair or wire is stretched vertically. By bringing the slit, the wire, and the axis of the point sighted at, in one line, the projection of the vertical plane determined by this line of sight, may be drawn on the table along the edge of the ruler.

In the preceding construction of the ruler, the sight line itself is not obtained, but one parallel to it. To describe this line itself, the axes of the

vanes must lie in a vertical plane with the edge of the ruler. When this edge is so adjusted, it becomes a *fiducial* edge. For the sake of sighting both forwards and backwards with the same position of the ruler, each vane contains both slit and wire, one above the other; the slit being inferior in the one, and superior in the other. A magnetic needle, F, is sometimes attached to the ruler, in which case the erecting compass can be spared, as it is only necessary to draw a sight line at the commencement of operations, and marking the direction of the needle with reference to this line, to cause the same direction to be maintained at each successive erection of the table.

The form of ruler represented in *pl.* 5, *fig.* 25, naturally serves only for short distances, as far as the unarmed eye can see clearly. To sight at greater distances, the *diopter ruler, with telescope* (*fig.* 26), has been constructed. A plate, C, is fastened to the ruler AB, carrying a telescope of feeble magnifying power. The ocular of the telescope is at E, and in its focus is a vertical cross-hair, which supplies the place of the object diopter. In the better instruments, the tube can be taken out of its bed and reversed: by this means, both forward and backward sighting can be attained. For reduction to the horizon, and sighting objects above or below the horizontal plane, the axis of the telescope can be turned, in a vertical plane, about its axis of rotation. A graduated arc, F, is often attached, which is fastened to the tube, and turns with it, while an index is fixed at G, to the frame. In this manner angles of elevation or depression may be measured. If the index point to the zero of the scale, the tube will be horizontal, and may then be used in levelling.

To illustrate the mode of using the plane table, reference must be again made to *fig.* 57. Taking for instance the figure on *fig.* 57, the first thing to be done is to determine the base *ab*, which it is of great importance to lay out properly. Its chief condition is, that as many points as possible may be determined from it, by intersections which are not too acute. This line must be measured as accurately as possible by the chain and staves. The plane table is then erected horizontally, at A, with the point *a* over the point *a* of the ground, and the north and south line marked. An assistant, with a signal, is then dispatched to *b*. A needle is now stuck in *a*, the point *b* sighted with the diopter ruler, and the line *ab* drawn. With the needle still sticking in *a*, sight lines are drawn to all the principal points visible, as C, D, E, F, G, H, I, K, L, M, N, O, P, Q, R, S, T, which are run out to the edge of the paper, and indicated by letters or numbers, for subsequent identification. In this manner are produced the sight lines *ab*, *ac*, *ad*, ... *as*, *at*. When the objects have no sharp outline, or are difficult to recognise, as at O and S, signals must be set up, and allowed to stand until the operation is completed. After this has been done, and all the sight lines again gone over, the station A is left, with a signal fixed in it, and the table carried to B, where it is again set up, after having marked on the paper the length of the base *ab*, on a reduced scale, thereby determining the point *b*. This point *b*, on the paper, is brought over the extremity *b* of the base line, first by the eye, and then by the fork,—the table erected horizontally, the ruler laid along the line *ab*, and the table turned until the sight line of the ruler

meets the signal at a. After making all necessary adjustments, as examining the compass, observing whether the point b of the paper is vertically over b of the base, &c., the objects, or signals, C, D, E,——S, sighted from a, are again sighted from b, and sight lines drawn until they intersect the corresponding lines from a. The points of intersection are indicated by the letters or numbers of the sight lines, and well marked, by the pricking of a fine needle, so as to allow the erasure of the lines. The different points thus obtained are finally connected, by straight or curved lines, as the case may require. In accurate measurements, where there is much curvature and few objects, staves, numbered in order, are first of all set up at all the points to be ascertained. In sighting, the signal-carrier, with his flag, goes to each of these stations in succession, and remains until sighted. If the operation is to extend beyond the reach of a single base line, points must be selected from which it may be continued afresh. The position of these new points must be determined from the first base. Further measurements with the chain are not necessary; if, however, certain important points can be sighted from one spot, and not from another, they may be first sighted and measured, and then transferred to the paper, according to the proper scale of reduction. For a more detailed account of the use of the plane table, we must refer our readers to special treatises on the subject.

The transition from instruments for measuring lines, to those for determining angles, is furnished by the instrument represented in *fig.* 30, *pl.* 5, and invented in 1742. This, in a less perfect form, is mentioned in Speckle's Festungsbau, 1608. Zollman has improved it so much, however, that it now bears his name. It is represented in our figure as improved by Gerstenberg, of Jena. It consists of a tripod stand, upon which is fastened the board A, as in the plane table. The erecting compass C serves to set up the instrument properly. There is a pivot at the centre B, upon which the diopter ruler, the alidade D, with the two sight vanes, E and F, can turn. The board is covered with the drawing paper, and upon it is placed the frame seen on the exterior of the board. This frame is graduated to degrees of the circle, and marked accordingly. The instrument is now set up at a station, at which a certain number of angles is to be ascertained, and the legs of these angles determined by sighting through the diopters, afterwards to be measured by a protractor, or by the graduation in the frame. The advantage of this instrument consists in its giving the angles themselves, and not, as in other angular instruments, their numerical values.

Of the purely angular measuring instruments, the first to be mentioned is the *astrolabe*. It is exhibited in *fig.* 31 on its tripod stand. By this is not to be understood the astrolabe of Hipparchus, used in determining the altitudes of the stars, but the common astrolabe, used by surveyors for hundreds of years, and which even now maintains its place, when well constructed, as an excellent means of measuring angles. It consists, in the semi-astrolabes, of a large semicircle, D, divided to 180°, and in the full astrolabes. of a large circle divided to 360°, and generally graduated also to quarter degrees. A strip, A, is attached in the direction of the diameter, which passes through 0° and 180°; this strip has a tongue at H to enable it to be

placed perfectly centrally upon the stand. This strip carries two fixed diopters, or sight vanes, B and C, as well as the centre. Another strip, E, turns about the centre, one end of which in the half-astrolabe (both ends in the full astrolabe) traverses the graduated limb and carries the sight vanes G and F. The middle line of this alidade coincides with the axis of the sight vanes and the centre, and is marked upon the bevelled edge of the alidade as an index. The diopters are both ocular and objective, for fore and back sighting. The limb of semi-astrolabes is doubly marked, first from 0° to 180°, and then from 180° to 360°, the 180° of the second corresponding to the 0° of the first. A small compass is often attached at the centre, and the tongue H fitted up with nut and screw as in *fig*. 21, so as to permit the circle to be brought from the horizontal to the vertical position, thus allowing a measurement of altitudes. To measure an angle with the astrolabe, it is placed with its centre over the vertex of the angle, and turned until the fixed diopters sight in the direction of one leg. The movable strip with its diopters is then to be sighted in the direction of the other leg, and the angle contained between the two strips, read off. To measure several angles from the same station, the first diopter may be left fixed, and the alidade moved successively to the different angles. Thus if the first angle measured between the fixed strip and the alidade amount to $35\frac{1}{4}°$, and the second to $97\frac{1}{2}°$, the angle contained between the two positions of the alidade will be $97\frac{1}{2}°—35\frac{1}{4}° = 62\frac{1}{4}°$. The astrolabe may, with a little practice, be made to perform much of the work of the plane table. Telescopes are sometimes attached, instead of the alidades. The instrument in this case falls rather in the class of the theodolite and graphometer, to which we shall shortly refer.

The *compass*, represented from above (*fig*. 33, *pl*. 5), and from the side, in *fig*. 34, is another instrument for measuring angles. It depends upon that property of the magnetic needle by which its direction is always parallel to that of the magnetic meridian, called north and south line. Even if the magnetic meridian of several places be not parallel, strictly speaking, yet the difference within a degree of longitude is so slight, as to be zero for all ordinary purposes. The compass consists of a round or square box, AB, in whose centre is a pivot, C, upon which a magnetic needle, DE, plays freely. The relative position of the latter may be read off on the limb, graduated to half and quarter degrees. To prevent the needle from playing when not in use, whereby both it and the pivot would be injured, a stop, or arrester, F, is attached: by this the needle can be lifted from the pivot, and pressed firmly against the glass covering of the box. The plate of the compass is dressed truly square, with two edges parallel to the north and south line, and three of the edges bevelled. In this way the compass itself may be used for laying off angles, by which means numerous errors may be avoided. In using the compass, it is attached to a tripod stand, by the socket I. This socket has a contrivance at G, with nut and screw for fixing, and a rim, *abcd*, is screwed upon the plate AB, which serves to carry the telescope IH. This is made fast by the screws *ef*, but may be turned, in a vertical plane, about the point G. The connexion with the socket, I, takes

place by means of a plate, with screws. To use the instrument, it is placed upon the stand, which is set over the vertex of the angle to be measured, and turned, until the sight line of the telescope falls in a vertical plane with one leg of the angle to be measured. The position of the needle is then to be noted. Suppose A to be $36\frac{1}{4}°$: the compass is again turned, until the axis of the telescope lies in the vertical plane of the second leg. As the needle, DE, retains its parallel position, it will now intersect a second point on the limb, which must also be read off. Suppose this to be $120\frac{3}{4}°$: then the angle measured will be $120\frac{3}{4}° - 36\frac{1}{4}° = 84\frac{1}{2}°$. Any number of angles may be thus measured from a single station, and their legs measured with the chain.

Although the compass has received various constructions, and is in general use, it is considerably behind the plane table in the accuracy of its results. The latter gives its results directly, instead of comparing them by the comparison of several measurements. In the compass, errors may occur in reading off the angles, which can never be determined to a greater degree of accuracy than $\frac{1}{10}°$; this defect, added to several others, may introduce false results of no inconsiderable magnitude. As an illustration of this contingency, let us refer to *pl.* 4, *fig.* 11, where, by a slight error in determining a single angle, the defective figure A′B′C′D′E is obtained instead of the correct one ABCDE.

The *prismatic compass* of Schmalkalder (*pl.* 5, *figs.* 35, 36), improved by Major von Decker in 1810, is well calculated for rapid military surveying. It consists of a plate, upon which is a small compass of about three inches in diameter, upon whose needle, *o*, a disk of pasteboard with a graduated limb is so fastened as to turn with it. The arrester or stop, *b*, serves to lift this slightly from its pivot, when not in use. The sight vanes are attached in the direction of the diameter; the objective at *h*, and the ocular at *f*, with their axes in the same vertical plane. A three-sided prism, *ade*, is attached to the ocular diopter, having a small mirror in its hypothenuse, which looks towards the graduation of the limb. To use this instrument, it is to be held horizontally before the eye in the vertex of the angle to be measured, and a sight taken through the upper part of the two vanes along one leg of the angle. Then, by the turning of the needle, a number on the limb will come under the prism, which, reflected by the mirror, can be read off through the lower part of the slit *g*. The second leg is then to be sighted in the same way, and the angle itself determined as in all compasses. The arc *m* serves for the rectification of the instrument, with reference to the variations of the needle. The graduation on the limb is inverted, so as to be seen directly by reflection from the mirror. The diopters can be made to turn down when the instrument is to be packed up, in which case the stop *b* is to be set, the point *n* for regulating *m*, fixed, and then the whole may be packed in a box four inches square, and one and a half inches deep. Its indications are sufficiently accurate for such purposes as military reconnoissance, &c.

The theodolite, as represented in *fig.* 38, *pl.* 5, is a perfected form of the full circle astrolabe. This instrument is calculated, not only for operations

of the lower geodesy, but for trigonometrical surveys, and even for the most delicate astronomical measurements. It rests upon a tripod, K, with which it is fixed on a special stand, or upon the plate of a plane table. At the end of the feet are the setting screws, *a*, *b*, and *e*, by means of which a perfectly horizontal position of the instrument can be attained. At the junction of the three feet there is erected a shaft, which carries the entire upper portion. Over this is slipped a socket, with a foot plate, L, which can be turned about the shaft by the action of a male screw, *c*, upon an endless female screw cut in its periphery; the plate may be fixed by the clamp *d*. This socket carries a correcting telescope, IH, which may be turned about the vertical shaft, independently of any motion of the foot or of the upper portion of the instrument. Over this socket rests a pierced circular plate, A, which can be fixed to the shaft, and upon which turns a circle provided with a limb, and capable of being fixed by a clamp screw, G. On the inner edge of this circle rests the accurately fitting alidade ruler, B, which, in some instruments, may form a full circle. This ruler, or alidade circle, carries an index provided with a vernier, as also the tube level C, by means of which the horizontal position of the instrument, and in particular of the circle A, may be insured. The bearer, F, of the telescope, DE, stands vertically over the centre of the alidade, the telescope supplying the place of the sight vanes, and being capable of motion in a vertical plane. A graduated arc, with index and vernier, is attached, for the purpose of measuring angles of elevation and depression. In using the theodolite, it is placed upon the stand, or the plane table, and fixed in a perfectly horizontal position by means of the setting screws, *a*, *b*, *e*, and the level C. The lower telescope is now to be directed to some fixed object, or, if none such present itself, to a temporarily fixed movable one, as a signal flag, whose centre is intersected by the vertical cross hair. This adjustment is made by means of the spindle *c* acting on the female screw in L; when completed, the clamp is to be applied, and the telescope fixed. It must now remain in this position throughout the operation, to insure the immobility of the whole apparatus. The clamp screw G is then loosened, by which means the circle A is set free: this, with the alidade, is to be turned until the cross hair in the telescope DE meets one of the two objects whose angular distance is to be measured, consequently lying in a vertical plane with one leg of the angle. The limb is now to be fixed by the clamp screw G, and the alidade B turned until the cross hair in the telescope DE meets the second of the objects, the telescope thus lying in a vertical plane with the second leg of the angle. The interval traversed by the index, in shifting from one leg to another, will represent the angular separation of the objects, and will give the angle required. If the objects lie in different horizontal planes, the telescope must be elevated or depressed to meet this case. The vertical limb will give the angular value of this elevation or depression.

When observations are conducted in a certain manner with this instrument, it becomes a *repeating or compensating circle*. In reading off the angles, slight errors may creep in, even with the greatest care taken to avoid them: to compensate for these, the operation must be repeated. For

this purpose, when the angle has been measured, other things remaining the same, the clamp G is loosened, and the alidade telescope is brought again to the first object, without displacing the index: the clamp is to be applied, and the measurement gone over again. This operation may be repeated several times. The mean of these several measurements is then to be taken, which will in general be more accurate than any single one. In the common alidade there are two indices with verniers, and in the circular alidade four, at right angles to each other, so that at each single measurement, the mean from two or four observations can be taken.

The *Graphometer* (*pl.* 5, *fig.* 37) is essentially only a simplified theodolite, applicable to the minor geodetical operations. It is fixed on a stand by means of a socket and screw, K, and has the nut and screw arrangement, I, together with the correcting telescope, GH. Instead of the full circle, there is only a semicircle, A, upon which, besides the level, C, is attached an erecting compass, *a*. The clamp *b* serves to fix the instrument, as in the theodolite, and the alidade B has only one index with vernier. The upright, D, carries the telescope, EF, which has no graduated limb attached. It is used like the theodolite, although repetition is only practicable in the case of very acute angles.

Reflecting instruments, as a means of measuring angles, are next to be mentioned. In these, only one leg of the angle to be measured is observed directly, the signal of the other being attained by reflection into the field of view of the instrument. Reflecting instruments were first invented by Hadley, in 1740, for use at sea, where a fixed stand, or several telescopes, could not be employed; thence they were transferred to astronomical and geodetical operations.

Hadley's sextant (*fig.* 32) consists of a sector, containing the sixth part of the circle, or 60°. This arc, AB, forms the base of the instrument, and to it are attached two radii, and several cross pieces for the sake of additional strength and stability. Upon one of these radii is a post with a ring for attaching a small telescope, in the prolongation of whose axis on the other radius is attached the objective, H. The objective is divided into two equal parts, of which the lower, H, is a mirror, and the upper, G, a transparent plate of glass. A vertically central line passes through both halves. The central piece, C, consists of two superincumbent plates, turning on a common axis, and in the prolongation of this axis a plane mirror stands perpendicularly, so that when the index D, which moves over the limb on an arm in the prolongation of the mirror, stands at 0° of the scale, the two mirrors are parallel to each other. E is a small frame with colored glasses, which can be turned up to protect the eye during sunshine, or in observations in the sun. To use the sextant, the observer stations himself in the vertex of the angle to be measured, and directs the instrument in such a manner that one of the objects is seen through the telescope and the upper part of the objective. The arm with the index is then turned until the second object is reflected from the mirror C to the mirror in the lower part of the objective, and the fine adjustment made by means of the tangent screw, *b*. This adjustment consists in causing both objects to be

bisected by the vertical line passing through the objective. According to optical principles, the angle thus obtained and read off on the limb, is just half of the actual angle of separation, so that the limb, instead of a graduation of 60°, is actually divided into 120 divisions, each one of which represents a degree.

Mayer of Göttingen, and after him, Borda, have improved the sextant by employing a full circle instead of the sextant. This forms the *reflecting circle* of Borda, and is represented on *pl.* 5, *fig.* 39. Here B is the graduated circle, placed upon the stand, A, similar to that of the theodolite, and capable of receiving a correcting telescope. K is a movable alidade, frequently a full circle, as in the theodolite, and provided with a vernier, N, and a correcting and clamping arrangement, I. On the alidade is a telescope carrier, FH, with a vertically movable telescope, G, and the objective, M, constructed as in the sextant. The dark glasses already mentioned are at O. The central-piece, C, is constructed as in the sextant, and carries the mirror, L, with the index, C, which has also a vernier, and a correcting and a clamp arrangement, D. E is a lens for reading off the graduation. P and Q are verniers for repeating. This reflecting circle, besides admitting the measurement of larger arcs than the sextant, has the advantage of being a repeating instrument.

To make use of the reflecting circle, the alidade, K, is so adjusted that one of the two objects is visible through the upper part of the objective, and the alidade, C, moved until the mirror, L, reflects the second object into the lower half of M. The angle at C is read off, and doubled for the true result. The principle of repetition is here the same as in the case of the theodolite.

It becomes necessary to add a word or two in explanation of the vernier (*pl.* 5, *figs.* 49, 50). The vernier, so called from its inventor, Peter Vernier (1600), is an arrangement for reading off small quantities on a scale, with great accuracy. Owing to the small size of mathematical instruments, the graduations upon them cannot be very minute, and it is rarely that quantities so small as $\frac{1}{4}$ of a line, or $\frac{1}{6}$ of a degree, can be indicated. A minuter division, so necessary, is attained by the use of the vernier. If a certain length, *am* or *an*, be supposed to be divided first into 10 and then into 9 equal parts, one part of the first division will be $\frac{9}{10}$ of a part of the second. If both divisions are placed one over the other, then, calling the first *a*, and the second *b*, the first part of *b* will project $\frac{1}{10}$ of its length beyond the first part of *a*, the second part $\frac{2}{10}$, &c., until the ninth and tenth will again coincide. Other numbers besides 9 and 10 may be employed, or an arc may be divided instead of the straight line. Suppose the limb of an instrument to be divided into quarter degrees, then each such part = 15 minutes; take 14 of these parts and divide them into 15, then each new division will be $\frac{14}{15}$ of the old, and each old one $\frac{1}{15}$, or one minute, greater than the new. By taking the axis of the alidade as the centre or zero point, and describing the given graduation to the right and left of this centre, the vernier will be capable of indicating single minutes. Suppose that in measuring an angle, the zero of the vernier has been found to be between $36\frac{1}{2}°$

and $36\frac{3}{4}°$, or that the angle is greater than the former and less than the latter. To determine the differences, find what division of the vernier accurately coincides with a division of the limb; let this be the tenth on the right side. We know that each part of the limb is one minute greater than any part of the vernier, consequently, for 10 divisions of the limb, we must add 10 minutes; then, $36° 30' + 10 = 36° 40'$. If the divisions thus met, had been to the left of the centre, the number of minutes thus ascertained would have to be subtracted instead of added. The same reasoning applies in the case of the rectilineal vernier.

It still remains to mention *levelling instruments*, used to determine the direction of a horizontal plane. Of these, the simplest is the common *mason's level* (*pl.* 5, *fig.* 56). It is known that every line hanging plumb must be perpendicular to a horizontal plane, and that a line from the vertex of an isosceles triangle to the middle of the base, will be perpendicular to the base. Upon these principles rests the determination of a horizontal position by the *foot level*. This is an isosceles triangle, *abc*, from whose vertex depends a plumb line, *cf*. If this be placed upon a board, and one end elevated or depressed until the plummet hangs opposite to the middle of the arc, *de*, the base of the triangle will be horizontal. This instrument can only be used for short distances—12 feet at the most—and for greater lengths, other means must be employed. The first to be mentioned is the *water level* (*fig.* 40). This consists of a simple stand, A, upon which, by means of a socket, B, is placed a tin tube, CD, bent at right angles at both ends. In each end, two glass tubes, E and F, are cemented water tight. If this water level be filled with water until it enters and partially fills the glass tubes, the upper surface of the water in both of these will stand in the same horizontal plane, independently of the position of CD. The horizontal plane thus indicated may be prolonged at pleasure, by sighting forwards at a given signal. An adjustable objective, IK, placed exactly over the middle of the instrument, is sometimes employed to furnish to the eye a more convenient point of reference, and to enable it to do with but one surface. The *movable diopter* (*fig.* 41) is, however, better calculated for this purpose. B is the bent part of the level, E the glass cylinder, cemented in the collar, *a*; the support, *b*, is attached to this collar, and in it a rack, *cd*, may be moved up and down by means of the pinion, *k*. The diopter is seen at *f*; its aperture, *g*, is divided into two parts by the horizontal thread, *hi*. At the commencement of the operation, the two diopters are brought to an equal height with the surface of the water, in which case sights may be taken over the threads, instead of over the surface of the water. In some cases the tubes inclose floats, supporting diopters, with a horizontal cross hair in each; these hairs must both be at precisely the same height above the water.

The *mercurial level* of Keith (*figs.* 42–44) is much more complete than the water level. This consists of a wooden box, AB, in which is a canal, EF, filled with mercury to a certain height, and then covered tight. At the two ends of the canal are rectangular boxes, C and D, into which mercury flows from the canal. GH is a bottom, separating the upper part, IK, of

the box from the canal. OK is an entirely inclosed space, cut off by the partition P. The cubes, S and T, fill the boxes C and D, so as to move freely up and down in them without shaking. These cubes carry above the diopters U and V, shown in *fig.* 44, having an aperture divided by a cross hair, X. *Figure* 43 is an under view of the instrument. The left hand side of *fig.* 42 exhibits it in section, and the right hand side in a lateral view. The entire instrument is connected by means of the strip *ab* (*fig.* 44), and the screw *c*, with the socket *d*, with which it is placed upon a stand: it can be fastened by the screw *e* (*fig* 42). To use the instrument, the canal EF is filled with mercury, whose surface naturally assumes a horizontal position. The two equally heavy, and in all respects similar cubes, S and T, are placed upon the mercury, which has flowed into the boxes C and D, and the cross hairs of the diopters will be in a horizontal plane, which can be produced by the sighting of signals. When not in use, the bottom G is pushed from under the bottom H, and the instrument is so constructed, that the mercury enters into the spaces M and N, which then are closed by sliding back the bottom G. The whole may then be transported without shaking the mercury. On account of the influence of the mercury upon brass and copper, these metals must be avoided. The cubes, with the diopters, should be constructed principally of ivory.

All these forms of levels being constructed with diopters or sight vanes, are applicable only to short distances; for long distances telescopes must be employed. The tube level is employed in this case for securing a horizontal position of the axis of the telescope. The tube level is preferable to the tub level, on account of its greater length and sensibility, and to the mercurial and water levels, for its greater convenience in use, as also for its superior delicacy.

Fig. 46 represents the simplest form of levelling telescope. This is placed on the plate of the plane table, by means of the tripod A, and the level rendered horizontal by means of the setting screws, *a, a, a.* The foot A is hollow, and in it turns a pin with a plate, B, upon which rests CD, the carrier of the level and telescope. The level, EF, rests in a peculiar frame upon this carrier, one end on a spring which is compressed by the pressure screw G. This spring serves to correct the level in case its axis should not be parallel to that of the telescope. In the supports erected at C and D, the telescope KL rests; it is held there by the clip springs H and I, which can easily be thrown back, to allow the telescope to be taken out and reversed. In the focus of the ocular there is a cross hair of human hair or spider's web, which is so fastened in a frame as to be capable of being placed exactly in the axis of the telescope, by means of the screws *b, b.* In this instrument the axis of the telescope answers to the sight line of the diopters in the water level. When it is once set up horizontally, levels may be taken in every direction, by turning the level and telescope bearer on the foot A.

The *level and compass* (*pl.* 5, *fig.* 45) is a more complicated instrument, possessing the advantage of being able to measure the angle formed by one leg levelled with another. This level has its own stand, with three feet, P,

Q, and R, upon which the plate L′ rests. This is so connected with the plate L, as that the setting screws, *e, e, e,* can render the latter horizontal, and with it the whole apparatus. The screw, N, serves in addition to turn the instrument horizontally about its axis, and the screw, M, is a clamp to hold it in any position desired. The compass, I, lies in the middle of the arm FG, and at one end is the support E for the telescope; the other support, D, stands upon a screw, by means of which it can be placed somewhat higher or lower in the arm FG, for the sake of ascertaining certain necessary corrections in the parallelism of the whole instrument. The supports are called Y's, from their shape, which affords a steadier position to the telescope than if they were semi-cylindrical. The collar bands, *f*, are attached to the Y's by hinges. The level *cd* is suspended from the telescope AB, by special collars, and is firmly fixed to A; by this the parallelism is more perfectly secured. This telescope has a cross hair which can be brought out of the axis by the screws, *b, b; a* is the handle for managing the telescope when it becomes necessary to reverse its position.

The *levelling compass* (*pl.* 5, *fig.* 47) has for its object, in addition to the purposes of the preceding instruments, that of measuring altitudes. It is fixed on a tripod stand by the socket A and screw *a,* and has an arrangement, by means of the screws, *b, c,* for producing a horizontal position and a rotation. It consists of a compass, I, to whose border the level D is attached. By means of the clamps *d* and *b,* the circular limb E is attached near the level, and perpendicular to the plane of the compass. About its axis the telescope C, fixed to the alidade GH, rotates vertically up and down. The alidade has a vernier at G, which can be fixed at the zero of the scale on the limb E, by the clamps at F and H. This will be the case when the axes of the telescope and level are parallel. To measure altitudes, the clamp screw, H, must be loosened, and the tube directed towards the object while the instrument still remains horizontal.

The *levelling circle* (*fig.* 48), which is actually only a modification of the repeating circle, may be used as a universal measuring instrument. It has a foot, A, with the screws, *a, a, a,* for horizontal adjustment. The correcting telescope, CD, is attached to the socket B, and is capable of being fixed by the screw contrivance, *c, b.* The telescope carrier, H, with the vernier alidade, G, moves on the limb EF, being managed by the screw *c.* The telescope IK rests upon the support H, and can be fixed by means of a special arrangement, at any position in a vertical plane The level, L, rests upon a plate, M, with two small processes, which pass between the beds *e* and *f* of the telescope.

In addition to the levelling apparatus already mentioned, there still remain the *levelling staves* (*figs.* 51, 52), and the *sight vanes,* or *targets* (*figs.* 53, 54). The staves are divided into feet, inches, &c., and the height is read off from the ground to the point where the sight line of the diopters or telescope intersects the staff. *Pl.* 5, *fig.* 51, exhibits one of these staves divided on the right hand side into feet and inches, and on the left into decimetres. A vane or target moves up and down the staff, until the central horizontal line of the vane falls in the horizontal plane of the level, the corresponding

point in the scale is then to be read off. This upward and downward motion takes place, as shown in *fig.* 52, by means of ropes and pulleys. Pulleys, *a* and *b*, are attached at the top and bottom of the beam AB, over which the endless string *cd* passes. E, the carrier of the vane or target, is fastened to the string at F, with which it must move. The sight vane may have very different constructions; the great object of all, however, is to show the horizontal line with the greatest possible distinctness at considerable distances. Thus in *fig.* 51 there is a broad black stripe at AB, through whose centre runs a white horizontal line. The vane has an opening through which the point of elevation of the central horizontal line can be ascertained. Other vanes are four cornered or circular, and divided into quadrants which are alternately red and white, or black and white; others are arranged as in *fig.* 53, where AB, the central line, is drawn black on a white ground. Here *abcd* is the arrangement for fastening the target to the staff. The same is seen at NOPQ (*fig.* 54), on the hind side of the target. The target is screwed to the sliding piece. In levelling with targets of the construction just described, the central line is marked on the hind side, and the graduation of the staff is also posterior. The perforated vanes (*fig.* 51) are better, however, as the leveller can frequently read off through his telescope, and thereby control the operations of the vanesman.

ASTRONOMY.

PLATES 6–15.

ASTRONOMY is that science which teaches the distribution and arrangement of the various heavenly bodies, their true and apparent motions in space, their magnitudes and distances, their physical structure, so far as known, and their mutual influences, so far as these influences are indicated by observations and established by induction.

The high importance of this science is evident to every reflecting mind. Its elevated object needs only to be mentioned to awaken feelings of dignity and grandeur in the human breast; for the conceptions and ideas it arouses of the infinity of the universe, and of the power, wisdom, and goodness of the Creator, excite in otherwise insensible dispositions, feelings of astonishment, admiration, and reverence. The benefits which human society (man) has derived from Astronomy, particularly with regard to the more accurate determination and division of time, the perfection of distant navigation, the fixing of the geography of places, &c., have been of infinite importance. To every cultivated mind, the exact knowledge of the true connexion and relation of our planet to the universe, if not absolutely indispensable, is yet most useful and attractive: it elevates the reflecting mind above an undue estimate of the size and importance of the earth, by showing its insignificance in respect to the great whole; it enlarges the circle of ideas, calls to mind the infinite, the unchangeable, and awakens longings and hopes in the soul, whose realization and continuation beyond the stars may form one portion of future blessedness.

Astronomy is divided first of all into three parts. 1. *Spherical Astronomy,* which teaches the knowledge of the various points and circles of the celestial sphere, the constellations, the position of the stars with respect to these points and circles, as also the phenomena occurring in the sphere of the Heavens. 2. *Theoretical Astronomy,* which enables us to determine from observation the true paths of the heavenly bodies, particularly of the planets. 3. *Physical Astronomy,* which gives the laws by which the motions of the heavenly bodies are regulated, shows how these motions are to be calculated according to the rules of mechanics, and finally combines all that is known up to the present time, of the physical characters of the heavenly bodies.

Practical astronomy is to be considered as the basis of these three divisions, which together form *theoretical* astronomy. This may be divided into: 1. *Observing Astronomy,* which treats of the apparatus (instruments),

and the observations made with it: 2. *Calculating Astronomy*, which teaches the method of obtaining from the observations the calculated results. For a well-grounded study of astronomy, a knowledge of pure mathematics, of mathematical physics, and of the sciences of optics and mechanics, is necessary. To become a practical astronomer, talent for observing, and a readiness in calculating, are required.

Astronomy, from its very nature, gives quite an abundant opportunity for pictorial representations, which have for their object either to explain theoretical propositions, or to make a visible exhibition of the objects and phenomena of the starry heavens. It might, however, be a not uninteresting preliminary, to take a historical survey of this science, showing how the investigating disposition of man has been occupied in its endeavor to obtain a more accurate and perfect knowledge of the Universe.

It is highly probable that the earliest nations, incited by the beauty of the starry heavens, and the necessities of life, gradually learned the changes of the days and seasons, the course of the sun, the moon, and some planets. The Chaldæans introduced the twelve signs of the zodiac, the sun-dial, and the clepsydra or water clock; and also laid the foundations of Astrology. The Persians and Babylonians determined the sun's year at 365¼ days; the Phœnicians applied Astronomy to navigation; and that the Egyptians possessed astronomical knowledge is abundantly shown by characters on their obelisks and temples. The Chinese seem to have known and used the gnomon from the most remote antiquity, and the Emperor Yao (2367 B. C.) determined the moon's year to be 354 days. At the time of Hoang Ti, they were acquainted with the equinoctial and solsticial points. The Greeks, however, first raised Astronomy to a higher level: Thales (640 B. C.) knew the cause of eclipses of the sun and moon; Anaximander constructed a geography and the first map; and Meton introduced the moon's cycle of nineteen years, to reconcile the moon's year with the course of the sun. Aristarchus (born 267 B. C.) was the most celebrated astronomer among the Greeks; he even guessed at the motion of the earth around the sun. He was followed by Hipparchus, who, by means of the equinoxes, determined the sun's year to be 365 days, 5 hours, 55 minutes, 12 seconds; and also constructed a catalogue of fixed stars. Ptolemy likewise sketched out a catalogue of fixed stars, as also a system of the universe, called after him the Ptolemaic system. The Romans had too little taste for mathematics to become good astronomers: Seneca alone, with a few other Roman philosophers, had just ideas of the heavenly bodies; the Romans were, however, so much the more given to Astrology. Julius Cæsar introduced the calendar invented by the Greek, Sosigenes, and called, after him, the Julian. Later than this, when learning fled to the newly established cloister, Astronomy was cultivated almost exclusively by the Arabians, particularly from A. D. 650. Albatari prepared astronomical tables; and in the time of Almansor, the moon's year was introduced, which is still in use among the Turks; but after the death of Almansor (in the 10th century), the study of Astronomy among the Arabians died away, and during the middle ages scarcely anything was esteemed but Astrology. Charlemagne, Gerbert

(Pope Sylvester II.), the Emperor Frederic II., King Alphonso of Castile, and some others, make the only honorable exceptions by their active participation in the study. Alphonso the Tenth held at Toledo, in 1240, an astronomical convention, whose fruit was the Alphonsine tables. At a later period appeared the first European astronomers, properly so called, Vitello, Bonatus, Purbach, John of Gamundia (first rectifier of the calendar), and Regiomontanus (calculator of ephemerides). At the end of the fifteenth century, Savonarola and Picus of Mirandola earnestly contended against Astrology. All were, however, eclipsed by Copernicus, who, in 1508, presented to the world his theory, and its proofs, of the true arrangement of the planets in our system. Nevertheless he found many opponents, particularly Tycho Brahe, one of the greatest of practical astronomers. This latter individual defended the immobility of the earth, and presented another system of the planets, known under his name, which was, however, very soon overthrown by the celebrated laws of Kepler (1571–1630), who constructed the first tables (the Rudolphian), calculated according to the Copernican system. A contemporary of Kepler, Galileo, was in a measure the martyr of the Copernican theory, for he was obliged to renounce at Rome his belief in the double motion of the earth; this, however, could not hinder the spread of truth, since at this time the telescope was invented, by means of which an entirely new view of the universe was gained. The mountains of the moon were discovered, the phases of Venus, the moons of Jupiter, the spots of the sun, &c., all of which testified to the truth of the Copernican theory. Huyghens, the inventor of the pendulum, discovered a moon, and the true shape of the ring of Saturn. In the seventeenth century, Halley, Flamsteed, Hevel, and others, examined the heavens incessantly and accurately, and Newton was the immortal creator of Physical Astronomy, by his discovery of the law of gravitation. The delusion of Astrology now vanished, and there remained alone the fear of great comets. In the eighteenth century, Euler, Clairaut, and D'Alembert, worked out still further Newton's Mechanics of the Heavens. Dollond invented the achromatic telescope, and, somewhat later, Herschel brought the Newtonian reflecting telescope to a wonderful degree of perfection: the discovery of the planet Uranus and his six moons was the result. A little before this, Mayer constructed his accurate tables of the moon. Bradley discovered aberration and mutation, and Lacaille, at the Cape of Good Hope, mapped out the southern hemisphere. Maupertuis, in Lapland, and La Condamine, in Peru, carried on measurements of a degree, in order to a more accurate determination of the size and figure of the earth; this end was, however, first obtained during the end of the last and the beginning of the present century, by the well known great French measurement of a degree. After Laplace and Lagrange had worked out in a masterly manner the theory of planetary perturbations, Bürg, Burckhardt, Zach, Carlini, Lindenau, Bouvard, Damoiseau, and others, were enabled to construct their sun, moon, and planetary tables, which agree within a few seconds with the actual positions of those bodies. At the commencement of the present century, not a few comets were discovered by Olbers, Pons, Mechain, Huth, and others, as also the

planets Ceres, Pallas, Juno, and Vesta, by Piazzi, Olbers, and Harding. Lalande, Bode, but especially Maskelyne and Piazzi, prepared fuller and more accurate catalogues of the fixed stars. Gauss taught new and very ingenious methods of accurately computing the planetary orbits, while Bessel attained the reputation of one of the greatest theoretical and practical astronomers that ever lived. During this time, however, the art of constructing astronomical instruments and achromatic telescopes was not behindhand, as the names of a Dollond, Ramsden, Troughton, Reichenbach, Fraunhofer, Repsold, and others, can satisfactorily testify. Finally, in later times, the appropriate and well arranged observatories at Altona, Berlin, Göttingen, Greenwich, Helsingfors, Königsberg, Ofen, Paris, Pulkowa, Seeberg, Vienna, &c., have been erected, and three new planets discovered —Astrea, Neptune, and Iris—and the existence of a central sun has been indicated by Mädler. Our century can point to a mass of accurate observations which have already been employed by theoretical astronomy to such account, that the science of the stars has been raised to a giant height, almost to entire perfection—a perfection which hardly any other branch of human knowledge can boast.

I. SPHERICAL ASTRONOMY.

The Armillary Sphere; the most important Points, Circles, and Terms in the Celestial Sphere.

THE ARMILLARY SPHERE.

1. The ancients at an early period imagined the existence of certain points, circles, &c., on the sphere of the heavens, by means of which they might the more readily comprehend, and be the better able to follow the various celestial phenomena. They also invented an instrument, the *armillary sphere*, partly with the view of giving an intelligible exhibition of the mutual relation of these points and circles, and of the axis of the heavenly motions, and partly to make actual observations by means of this sphere. The *armillary sphere* (*pl.* 6, *fig.* 1) consists of a frame, with a horizon on which are represented the 360 degrees, the regions of the heavens, the calendar, and the height of the sun for every day in the year. Two notches in the horizontal circle, and corresponding to its north and south points, receive the *fixed meridian*, whose plane is perpendicular to, and centre coincident with that of the horizontal circle. This meridian, within which the other circles as well as the small terrestrial globe may all be rotated together on the common axis of the heavens and earth, can be moved in these notches, still remaining in the original vertical plane; in this manner the general axis may be placed at various angular distances with the horizon. The centre of the small terrestrial globe is coincident with that of the general armillary sphere, the names and position of the

other circles are evident from the figure without further explanation. The hour circle fastened to the north pole of the fixed meridian has a movable index, which, when fastened, revolves with the axis. The artificial sphere known as the celestial globe has the advantage over the armillary sphere in allowing the representation of the stars; but the latter exhibits to the senses far more clearly the relation of the most important points and circles of the celestial sphere to the inclosed terrestrial globe.

2. *Fig.* 2 gives likewise an explanatory representation of the most important points of spherical astronomy. The circle EHZT is the *fixed meridian* or *noon circle;* if its surface represent the western hemisphere of the celestial globe, then HRT is half the horizon, H its north, R its west, and T its south point. Z is the *zenith*, the visible highest point above the horizon, standing perpendicularly above the centre of the sphere, while N, the *nadir*, is the invisible lowest point below the horizon; the straight line ZN connecting these points is called the axis of the horizon, and corresponds to the direction of the plummet. The arc ACQ represents the *semi-equator*, ECK the *semi-ecliptic* (path of the sun). The equator ACQ passes through the east and west points (R) of the horizon. The point C, where the ecliptic and equator intersect, is called the *vernal equinox*. The spherical angle ACE, or KCQ, gives the amount of inclination of the ecliptic to the equator, that is, the obliquity of the ecliptic, 23° 27′, measured also by the arc AE or QK. The visible point N, everywhere 90° distant from the equator ACQ, is the *north pole*, the invisible one N″, directly opposite, is its *south pole;* the visible point, P, distant from N about 23° 27′, and 90° distant from every point of the ecliptic, is the north pole of the ecliptic, P′ its corresponding and invisible south pole.

Let S′ be the place of any star in the celestial sphere; and from the zenith Z, draw through the star S′ the arc ZS′T′ of a great circle, perpendicular to the horizon HRT, then the circle of which ZS′T′ is only the fourth part is the *vertical circle* of the star S′; and the arc T′S′ the *altitude* of this star, which is expressed in degrees, reckoning from the horizon; finally the arc ZS′ is the *zenith distance.* In the horizon the altitude is 0° and the zenith distance 90°, while in the zenith the altitude is 90°, and the zenith distance 0°. The arc TT′ of the horizon lying between the meridian and the quadrant ZS′T′ of the vertical circle passing through the star S′ is called its *azimuth*, also measured by the spherical angle T′ZT. The azimuth is reckoned from 0° to 180°, positively from the south point T, eastwardly as far as the north point H, and negatively from T to H, in the opposite direction by the west; as is very evident, the azimuth and altitude of a star completely fix its position in the celestial sphere with respect to the horizon.

If from the north pole, N, of the equator an arc, NS′Q′, be drawn through the star S′, perpendicular to the equator, the circle of which NS′Q′ is the quadrant is called the *declination circle* of the star, and the arc Q′S′ the *declination.* This is called *north* or *south* as the star is north or south of the equator, consequently as it stands in the northern or southern hemisphere of the heavens. The declination is estimated in degrees from the

equator; at the equator being 0°, and at either of its poles 90°. The part CQ′ of the equator ACQ, lying between the vernal equinox C, and the declination circle NS′Q′ of the star S′, is called the *right ascension* of this star; it is reckoned from the vernal equinox C, around the equator from west to east, varying from 0° to 360° (or from 0 to 24 hours), and is expressed in these degrees (or hours). By means, then, of the right ascension and declination of a star, we fix its position in the heavens for a long interval of time, with respect to the equator.

Draw from the pole, P, of the ecliptic, ECK, through the star S′, an arc PS′K′, cutting the ecliptic at right angles, then the great circle of which PS′K′ is only the quadrant, is called the *circle of latitude* of the star S′; and the arc K′S′, the *latitude* of this star. This is *north* when the star is above the ecliptic, as in the figure, and *south* when it is below. The latitude is estimated in degrees from the ecliptic; at the ecliptic it is 0°, at either pole 90°. The part CK′, of the ecliptic ECK, lying between the vernal equinox C, and the circle of latitude PS′K′ of the star S′, is called the *longitude* of the star. It is estimated on the ecliptic from west to east, and commences with the vernal equinox, expressed in degrees from 0° to 360°, or in terms of the 12 signs of the zodiac. The latitude and longitude of a star completely determine its position on the celestial sphere with respect to the ecliptic.

The arc HN of our figure represents the height of the pole N above the horizon HRT, that is, the *altitude of the pole;* and the arc TQ the height of the equator ARQ (on the meridian), above this horizon HRT, or the *altitude of the equator*. The altitude of pole and equator for the same place of observation, are together equal to 90°. The spherical angle Q′NQ, having the north pole, N, for its vertex, or the corresponding arc QQ′ of the equator, is called the *hour angle* of the star S′.

3. The following is a very satisfactory proof among many well known ones, of the spherical shape of the earth. Suppose an observer (*fig.* 8) stationed at a particular point, S, from which a ship sails off in a straight line. At a short distance the whole of the vessel will be visible to the water-line; with increasing distance the ship decreases in apparent height, but is visible to the water's edge. After reaching the horizon at B, there is not only a still further decrease in apparent size, but a disappearance of part of the vessel itself, beginning with the hull. At C only the sails and masts are visible; the appearance presented is represented by *c*. From a higher point T, however, whose horizon passes through D, the hull of the ship will be again visible. The distance still increasing, the lower sails seem just to sink into the water, as at *d*, and finally to disappear entirely. The distinctness with which the summits of the masts are observed, just before their disappearance, must carry home the conviction, that but for the intervening segment, ABCDE, of the sea, the actual distance, SE, is not so great as to prevent an equally perfect view of the whole.

The most important Points, Circles, and Terms of the Terrestrial Sphere.

4. Spherical astronomy determines certain points and circles, as well on the terrestrial as on the celestial sphere. If, for instance, in *fig.* 12, C represent the centre of the earth, and NCS its axis of rotation, then, N,S, are the poles of the earth, QE the terrestrial equator, and AB the parallel of latitude of the place of observation, A, on the surface. Consequently the straight line, AP, parallel to SCN, is that direction in which the visible pole, P, of the heavens, is seen from the place of observation, A. The line AZ, a prolongation of a radius of the sphere, is the direction of the zenith from the observer at A. Furthermore, let NAES be the meridian of A; NGS, a fixed meridian, as the meridian of Paris; then, GE, or the spherical angle, GNE, is the (geographical) longitude of A, and EA, or the angle ECA, the (geographical) latitude. (For further particulars respecting geographical longitude and latitude, see Section 10.) Finally, if *ns* be a plane, tangent to the earth's surface at A, it will constitute the visible apparent horizon of the place; and the straight line, *n*A*s*, produced by the intersection of this plane with the meridian, will be the meridian line of A, so that for A, *n* will be the north, and *s* the south pole of the horizon.

Miscellaneous Considerations respecting the Apparent Rotation of the Celestial Sphere, and the attendant Phenomena.

5. A careful examination of the phenomena exhibited in the apparent daily rotation of the starry heavens, shows that in respect to this rotation, the size of the earth may be considered as entirely insignificant, that is, the observer can be supposed to be situated in the centre of the earth; an assumption very allowable when we reflect on the immense distance of the fixed stars from the earth. Let *pl.* 6, *fig.* 13, represent the celestial sphere, *i*, the observer, Z, his zenith, N, his nadir, then will the circle, H*w*O*e*H (whose poles are N and Z), be the celestial horizon; P*p* represent the poles of the heavens, the circle, HZONH, the meridian, and HP, the altitude of the pole for the observer at *i*. This will be readily seen by referring to what was said on the subject in § 2. The circle, E*w*Q*e*E, perpendicular to the axis, P*p*, will be the equator, in which the vernal equinox occurs at ♈. Then, as already explained in *fig.* 2, the arc, ♈T (*fig.* 13), will be the right ascension, TS the declination, and PS the polar distance of the star, S, projected in the equator by the declination circle, PST*p*; BD will also be the diurnal circle (parallel of declination) described by the star in its apparent motion about the pole, P. The circle, ZM, perpendicular to the horizon, is the vertical circle; the arc, HM, the azimuth; MS, the altitude, and ZS the zenith distance of the star, S. Finally, the points, H, *w*, O, *e*, are respectively the north, west, south, and east points of the horizon.

If H*h* and O*o* represent small parallels of declination, touching the north point of the horizon at H, and the south point at O, then H*h* will be the circle of *perpetual apparition*, between which and the visible pole P the

stars never set; and O*o*, the circle of *perpetual occultation*, between which and the invisible pole *p* the stars never rise. All stars situated between these two circles will be sometimes visible and sometimes invisible. Thus, the star S will be seen when in that part, AB*a*, of its diurnal circle above the horizon, and will be invisible when in the portion AD*a*. Furthermore, the same star will, in the diurnal rotation of the heavens, come back to the same meridian every twenty-four hours, and consequently as the daily rotation of the heavens is uniform, the interval of sidereal time between the arrival of the star at the meridian of two different places, may be expressed by the difference of longitude of the two places. On the other hand, the interval expressed in the sidereal time between the arrival of two different stars at the same meridian, is measured by their difference of right ascension, so that here we find one reason for dividing the equator into both degrees and hours. We find, also, from an inspection of *figs*. 13 and 2, *pl*. 6, that the altitude of the pole at any place is its geographical latitude, and that the sum of the altitudes of the pole and the equator is always for the same place equal to 90°. Likewise we find that every star attains the greatest height above the horizon at its *culmination*, and that all stars lying within the circle of perpetual apparition, cross the meridian twice above the horizon, once above the pole, and once below it; the one being the upper culmination or transit, the other the lower.

The Apparent Course of the Superior and Inferior Planets.

In the Copernican system, the *inferior* planets, Mercury and Venus, which are nearer the sun than the earth, are distinguished from the *superior*, Mars, Vesta, Astræa, Juno, Ceres, Pallas, Jupiter, Saturn, Uranus, and Neptune, which are more remote. This distinction is very proper, as the *phenomena* attending the courses of the inferior and superior planets, are in many respects essentially different from each other. The reason is, that we view the planets from our earth, which itself revolves around the sun at a different rate from the rest, and therefore we see them at very different distances. We observe therefore not the true, but the apparent courses of the planets, which will now be explained. It must ever be kept in mind, however, that the fixed stars being at almost an infinite distance from our earth, their rays must always reach it in parallel lines. In the first place, to illustrate the *apparent* course of an *inferior* planet, let ACEG, in *fig*. 24, represent the orbit of Venus, *acegi* that of the earth, and S the sun. Since the distance of Venus from the sun is about three-fourths that of the earth, and since she traverses her orbit in 7½ months, then, if in *fig*. 24 we divide her orbit into 5, and that of the earth into 8 equal parts, one of these will represent the space traversed by each in 1½ months. If, when Venus is at A, the earth is at *a*, the former is said to be in *inferior conjunction* with the sun (♀ ☌ ☉), that is, on the same side of the sun with the earth. Her apparent diameter is here the greatest, although actually invisible (*fig*. 21); at the expiration of three-fourths of a month, Venus is at B, and the earth at *b*.

The former has consequently *retrograded*, since, taking the fixed direction of the star *s* as a line of reference, Venus to the observer on the earth appears to the right of the star, which is seen in the direction *bs*, parallel to A*s*. It is also evident that Venus has become a *morning* star, since when the observer at *b* looks towards the sun, he sees the planet on the right of the sun. Hence Venus is seen in the eastern horizon before the rising of the sun. *Pl.* 6, *fig.* 21, represents her at this time, crescentic, and with two digits illuminated.

A month and a half after inferior conjunction, the fixed star in whose vicinity Venus was seen when the earth was at *b*, will be seen in a direction parallel to B*b*, from the earth at *c*; consequently Venus will be seen in the direction C*c*, to the left or eastwardly of this star. Hence, it follows that the apparent motion of Venus has again become *direct*. It will also be seen from *fig.* 24, that the angle *sc*S, formed by lines drawn from the earth to the sun and the planet, is larger than the former angle, S*b*B; we accordingly say that the *elongation* of Venus has increased. The illuminated part of her disk has (as seen in *fig.* 21) increased to about four digits; her apparent diameter, however, sensibly diminishes. This elongation must be greatest at the time when the earth (*fig.* 24) is at *d*, and Venus at D, which is the case about $2\frac{1}{2}$ months after inferior conjunction; and this greatest elongation is equal to the angle D*d*S. Venus then (*fig.* 21), like the moon in her last quarter, has half of her disk, or six digits, illuminated.

Half a year after inferior conjunction, when the earth is at *f*, Venus will have reached F, consequently the elongation of Venus has again become less, since the angle, S*f*F, expressing this elongation, is evidently less than the angle S*d*D. Venus, then, during this time has approached the sun, has become fuller (*fig.* 21), but nevertheless smaller, although still the morning star (*fig.* 24). Three months after, the earth is at *h*, and Venus at H; the latter has therefore reached *superior conjunction*, that is, is on the opposite side of the sun from the earth. Her apparent diameter is now the least, and her entire disk (*fig.* 21) illuminated, although invisible. At a later period she appears in the evening sky as the *evening* star, and exhibits the same phases (*fig.* 21) as before, but in an inverted order. Her apparent diameter also increases.

To illustrate the course of a superior planet, let S (*fig.* 25) represent the sun in the centre of the earth's orbit, *abcdefgh*. Let that of Mars be ACEGJLN. The earth revolves in twelve, and Mars in about twenty-three months, about the sun. If, therefore, the original positions of the earth and Mars were at *a* and A, at the expiration of 1, 2, 3, 4, &c., months, the earth will have reached the points *b*, *c*, *d*, *e*, &c.; and Mars B, C, D, E, &c., respectively. When the earth was at *a*, and Mars at A, the latter was in *opposition* to the sun (♂ ☍ ☉), and the stars in the vicinity of Mars must show themselves in the direction *az*. The motionof all the planets from A to B is called direct; consequently Mars has a *direct motion* when advancing from the stars observed in the direction *az*, towards the left, and a *retrograde* motion when moving towards the right. *Pl.* 6, *fig.* 25, shows also that when the earth goes towards *b*, and Mars towards B, the motion of the latter must be retrograde,

because he is observed to the right of the stars at *z*, seen in a direction, *by*, parallel to *az*. Therefore Mars, at the time of opposition, retrogrades.

After the earth has arrived at *b*, Mars is at B, and consequently no longer exactly opposite to the sun, S; but as the stars seen in the direction *by* culminate about midnight, he will then have passed the meridian and will be near his setting, and consequently will be seen in the evening sky.

Two months after opposition, the earth being at *c*, and Mars at C, the retrograde motion of the latter will have ceased, since the straight lines connecting *b*, B and *c*, C, are nearly parallel; consequently to an observer at the earth, the planet will appear *stationary* for some days. At a later period the motion of Mars will be direct. Two months later, Mars, still moving direct, will be at D, and the earth at *d*, so that now the lines *c* C, *d* D, form the angle C*d*′D. It is evident that the directions *d*S, *d*D, from the earth to the sun and to the planet, form nearly a right angle to each other, and consequently that about this time Mars at sunset must be near the south, and must set about midnight.

At the end of the tenth month, when the earth is at *g*, Mars has not completed half his apparent course in the heavens, as shown by his not having reached a point opposite to the stars seen in the direction *gx*. The distance of Mars from the sun, however, is apparently only the angle G*g*S; as at this time an observer at *g*, looking towards the sun and planet, sees the latter to the left or eastward of the former. Mars must appear in the evening sky, after sunset, and evidently much smaller than at time of opposition.

At the expiration of a full year, the earth being at *h*, and the planet at H, the straight line, *h*H, shows that Mars has completed rather more than half the apparent circuit of the heavens, since the stars in the direction *z* stand nearly opposite to him. Consequently the *conjunction* of Mars with the sun (☌ ♂ ☉) has not yet taken place, as, from an inspection of *fig*. 25, it will be seen that the sun still appears to the right of Mars or H.

Two months later Mars is seen early in the morning sky, for since from *i*, the position of the observer, the planet is seen to the right of the sun, at I, he must evidently rise before the sun. By continuing this consideration, with the assistance of *fig*. 25, we shall soon find that Mars, shortly before his new opposition, again becomes retrograde, and that the phenomena before observed must all succeed each other again in the same order.

From all that has preceded it follows, without further explanation, that the inferior planets have an inferior and a superior conjunction, but no opposition; that the superior planets have a conjunction and an opposition, never two conjunctions; and, finally, that while the inferior planets are never visible in the heavens at midnight, the superior may be seen at any hour of the night.

The Moon; her Revolution around the Earth; Phases of the Moon; the Moon's Nodes.

7. Our earth, in her yearly course about the sun, is accompanied by a satellite, the moon, which revolves around the earth in about a month, and in little more than four weeks wanders through the whole zodiac. We need only observe the moon for a few hours, on several successive clear evenings, to satisfy ourselves that while, with the other stars, she follows the diurnal motion from morning till night, she has a peculiar motion of her own, from west to east, advancing daily a little over 13 degrees among the fixed stars of the zodiac. This peculiar motion of the moon is the result of her revolution about the earth; and for the same reason, being an opaque body illuminated by the sun, she is exhibited in all possible shapes (phases). The four principal of these are, the *new* moon, the *first quarter*, the *full moon*, and the *last quarter*. The new and full moon are known as the *syzigies;* the first and last quarters as the *quadratures*. *Pl.* 6, *fig*. 19, offers an intelligible illustration of the various phases of the moon, depending on her different position with respect to the sun and earth. Let *abcd* be the earth placed in the middle of the moon's orbit, NEVLN, and S the sun, whose distance, S*a*, from the earth is supposed to be so great that all his rays are parallel to the line SNV. Let the moon be at N$\beta\gamma\alpha$, or between the earth and sun, her dark side, $\beta\gamma\alpha$, will then be turned to the illuminated side (*dac*) of the earth. At this time the moon is *new*, and being above the horizon in the day time, is invisible. Compare *pl.* 10, *fig*. 5. Two or three days after this time, the moon, moving in her orbit in the direction of the arrow (*fig*. 19), is seen soon after sunset, in the evening sky, as a narrow crescent, which soon sets. This crescentic shape of the moon becomes broader, and she sets later every day, and removing constantly from the sun, she shines through the first hours of the night. In about seven days the moon will have reached L$\pi\nu\mu$, or the *first quarter*, and to an observer at *d*, or the boundary between day and night, will be seen to the left of the sun, as a semicircular disk, with the straight edge to the left or east: the moon now culminates about six o'clock in the evening (see *pl.* 10, *fig*. 5), and sets about midnight. From this time, the outline which from new moon to the first quarter was concave, becomes convex; the moon shines longer, and sets after midnight. In about seven days she will have reached V$\lambda\kappa\eta$, and become full, standing directly opposite the sun, behind the earth; her illuminated half, $\lambda\kappa\eta$, consequently, to the dark side, *dcb*, of the earth, appears as a full circle, which rises in the east as the sun sets in the west, culminates at midnight, and sets in the west at sunrise. The moon now rises about an hour later each night, and gradually loses the illumination of her right or westerly side, so that the circular disk becomes oval, until, in seven days after full moon, she will have arrived at E$\zeta\varepsilon\delta$, or the last quarter. To the observer at *c*, the boundary between night and day, her illuminated side, E$\varepsilon\delta$, appears as a half disk, to the left or west of the sun, and the line separating the dark and the illuminated portion will be on the west side, while in the first quarter it was on the east. At the last quarter, she rises

about midnight, and crosses the meridian at about 6 o'clock in the morning. Her semicircular disk now begins to become narrower, the straight edge concave, and the moon again assumes, as she approaches the sun, a crescent shape, which is smaller as the interval of time between the rise of the moon and that of the sun diminishes. About seven days after the last quarter, the moon entirely vanishes, again reaching $N\beta\gamma\alpha$, her position about four weeks before. She now becomes new moon afresh, rising and setting with the sun, and her phases follow in the same succession.

8. Since our earth is also an opaque globe illuminated by the sun, we must present the same alternation of phases to an observer at the moon, that the moon presents to us, only in an inverted order. At the time, therefore, of new moon, first quarter, full, and last quarter, our earth must be *full earth, last quarter, new earth,* and *first quarter.*

The moon, during a revolution around the earth, describes an orbit, cutting the apparent path of the sun (the ecliptic), or the orbit of the earth, to which it is inclined about 5° 8′ 48″, in two points called the moon's *nodes* (*pl.* 6, *fig.* 20). The point in which the centre of the moon cuts the ecliptic in passing from the south to the north, is called the moon's *ascending node*, ☊, or the head of the dragon; and the one produced in passing from north to south, the moon's *descending mode*, ☋, or the dragon's tail. The distance of the moon from the ecliptic is called the latitude, which may be north or south. At the nodes, the latitude is 0°; hence it follows that the sun, moon, and earth (the sun and earth being in the same plane), must lie at all times nearly in the same plane (*fig.* 20), and when the moon is in one of her nodes, that is in the ecliptic itself, then the three bodies—sun, moon, and earth—have their centres in the same straight line.

The ancients observed that the moon's nodes did not remain in the same part of the ecliptic, but continually moved backwards, or from east to west (indicated in *fig.* 20 by the dotted lines). This retrogression takes place in the following manner: one of the nodes of the moon's orbit—the ascending, for instance—retrogrades in such a manner that if it had coincided with the new moon at starting, this coincidence would again happen after an interval of 18 years and 11 days. The moon's node will then be still 11 degrees distant from the position assumed 18 years and 11 days before. But the sun has in the meantime advanced 11 degrees, and consequently stands again in the node; therefore, since the moon is again in the new moon, an eclipse of the sun must occur, just as it did 18 years, 11 days before. However, the coincidence of the new moon with the node does not take place exactly in the same manner. These periods of 18 years and 11 days, supposed to have been called Saros by the Chaldæan astronomers, are known as the periods of Halley, and were employed by the ancients in the prediction of solar and lunar eclipses. The retrogression of the moon's nodes is a consequence of the secular perturbations of the moon's orbit, and at a mean, with reference to the fixed stars, amounts annually to 19° 20′ 29″ (*pl.* 6, *fig.* 20).

The Seasons; Daily and Yearly Motion of the Sun.

9. The illustration on *pl.* 9, *figs.* 1, 2, 3, serves to elucidate the theory of seasons, and of the daily and annual motion of the sun. The following may be taken in connexion with what is said further on (sect. 26) respecting the annual motion of the earth round the sun.

The *visible horizon* is that circle in whose centre we are supposed to stand, and which bounds our vision, and upon which the heavens appear to rest. The sun is said to *rise* in the morning when it appears above the horizon, as at C, *pl.* 9, *fig.* 2, and to *set* in the evening when sinking below the horizon, as at C′. The sun appears each day to describe a greater or less circular arc, CMC′, above the horizon, and this arc is constantly inclined towards the horizon. *Noon* of a place is that time of day when the sun at M has arrived at its greatest height above the horizon. The *vertical* of a place (Paris in *fig.* 2) is the straight line VT, parallel to the direction of the plumb line. The line RSO is the *meridian line* of Paris, or the direction of a shadow at time of noon at Paris; the four points, O, G, R, L, are the four cardinal points of the horizon. The sun appears to traverse a part, CMC′, of a circle by day, completed at night in C′NC. beneath the horizon, and the entire apparent circuit of the sun about the earth lasts 24 hours, or an entire day. A careful examination will, however, disclose the fact that the points of rising and setting of the sun, as also those in which he cuts the meridian, vary from day to day, with respect to the horizon of one and the same place. Thus, on 21st December, CMC′ is the day circle of the sun; GQL that of the 20th March, greater than the preceding, and KUA that of the 21st of June, which is greatest of all. Hence the sun appears to remain stationary for a time, and then to return towards the south, describing anew the arc GQL to the 23d of September, and CMC′ on the 21st of December, as before. Here he appears stationary for a time, and then returns to the north as before. These arcs described by the sun are all parallel to each other: the greatest of them, KUAK (towards the north) is called the *tropic of Cancer*, the least, CMC′NC (to the south) the *tropic of Capricorn.* The two periods of the year in which the sun describes the tropics are called the solstices. The circle described on the 23d September and the 20th March is called the *equator*, and the two periods when the sun describes the equator are called the *equinoxes.* Consequently the sun seems to move north for six months in the year, viz. from December 21st to 21st June, and south for six months, from 21st June to 21st December.

There is still another motion observable in the case of the sun, viz. a daily progression eastward of about a degree, while the fixed stars retain their relative positions unchanged. Since now 1° in space answers to four minutes in time, the sun will return to the same point of the heavens four minutes later each day, which in 90 days will amount to six hours. The imaginary straight line, PF, passing through the centre, S, of the horizon, and about which the sun appears to describe his circles, is called the *axis of the heavens.* Since all parallel circles of the sun are inclined to the

horizon, and the axis of the heavens is perpendicular to their planes, the axis itself will be inclined to the horizon at the angle PSO, or the arc PO. Their two extremities, P, F, are called *the poles*, P being the north pole, F the south. From the preceding it follows, that for all planes between the equator and the poles of the earth, the celestial sphere is *oblique* (*pl.* 9, *fig.* 3), since the equator, the parallels of declination, and the axis of rotation, have an oblique situation with respect to the horizon.

If the observer be situated at the north pole (*fig.* 4), O, then the celestial sphere becomes to him *parallel*, since there the equator and the tropics, as seen in *fig.* 4, are parallel to the horizon. The axis P*p* of the horizon coincides with the axis of the sphere P*p*, and the horizon H*h* with the equator E*e*. Should the sun be situated in the plane of the equator, he will describe the circle SH, or the horizon, his disk being half above and half below it. When, after the expiration of three months, the sun has reached the tropic of Cancer at S″, he will describe the circle S″N, 23° 27′ above the horizon. He will now be evidently above the horizon, and, in fact, so remain for three months longer, gradually returning to the equator. Thus at and about the north pole the day is six months long. As a matter of course the region about the south pole of the earth must have an equal length of night. During the other six months of the year, these conditions are reversed for the two poles. The fixed stars, however, of these regions never set, that is, those of the northern heavens at the north pole, and those of the southern heavens at the south pole, since they describe their circles parallel to the horizon.

Fig. 5, *pl.* 9, shows the position of the points and circles of the celestial sphere to the observer at the equatorial regions of the earth. There the two poles, P, *p*, lie in the horizon, H*h*, and consequently the axis, P*p*, zenith, E, and nadir, *c*, in the equator. The planes of the equator and tropics are perpendicular to the plane of the horizon, whence the celestial globe forms a *right sphere*. The fixed stars all rise and set perpendicularly to the horizon; each one remaining visible for 12 hours, and invisible for a like portion of time. It is very evident that, in the region of the equator, all the fixed stars must gradually come in sight, and that whether the sun describe the circles, S, S′, or S″, they will all be divided into two equal parts by the horizon. In consequence of this, throughout the whole year at the equator, the days and nights will be equal, and of 12 hours each.

The Geographical Latitude and Longitude of a place; Determination of these.

10. An accurate knowledge of the geographical latitude and longitude of a place, that is, its geographical position upon the earth, is of vast importance to mathematical geography and navigation. The *geographical latitude* of a place is the shortest distance of the place from the equator, expressed in degrees, minutes, and seconds. It will, therefore, be northern or southern, as the place lies north or south of the equator. If, through the given place and the axis of the earth, we pass a great circle, cutting the

equator at right angles, it will be the circle of latitude of the place, and that part of the circle intercepted between the place and the equator, will be its geographical latitude, which may consequently range from 0° to 90°. Any place at the equator has a latitude of 0°; at the poles the latitude will be 90°. Now, as the altitude of the poles is equal to the latitude of the place of observation, we know the one when we know the other. For the determination of the altitude of the pole, a knowledge of the mutual situation of the zenith and pole, with reference to the horizon and equator, is necessary, and this can only be obtained by measurement of the altitudes of certain stars. In practice, therefore, the determination of the altitudes of the pole will depend upon the accuracy with which this measurement can be effected. Various methods have been devised to meet the wants of observation, as also the uncertainties which result from the declination of the star employed, its parallax, and refraction, so that the following eight methods of determining polar altitudes have been suggested and followed.

1, By the meridian altitude of the star; 2, by circummeridian altitudes; 3, by two culmination altitudes at the superior and inferior transits across the meridian; 4, by two meridian altitudes in the southern and northern parts of the meridian; 5, by the altitude of the polar star; 6, by equal altitudes of the circumpolar stars; 7, by altitudes of two stars and the observed interval of time; 8, by the observation of a star in its passage through the eastern and western part of the prime vertical.

The *geographical longitude* of a place is the arc of the equator intercepted between the circle of latitude of the place, and the circle of latitude of another place, assumed as a fixed point of reference. The latter circle of latitude is then called the *first meridian*, and the former the meridian of the place whose longitude is to be determined. Since nature indicates no definite line or circle of departure for the determination of longitude, as she has done in the equator for the determination of latitude, it is evident that the position of the prime vertical, and consequently the amount of the longitude, must be entirely arbitrary. This uncertainty has had for its consequence, the assumption of various standards, inconvenient in practice, and very often producing mischievous effects upon the interests of navigation, which, although now palliated, have not been entirely removed. The difference in the estimate of longitude has, however, been hitherto still greater among astronomers than among navigators, although a difference in estimating the longitude among the former, where calculation is appealed to for assistance, is much less troublesome than among the latter. Almost every astronomer counted the geographical longitude from his own observatory. The meridian of Paris has, however, more recently been selected by the astronomers of the continent of Europe, to which almost all observations and geographical longitudes are referred. It is generally customary in astronomy, at the present time, to regard only the meridian difference of two places of observation, or the angle which the meridian or noon circles of the two places bear to each other, at the pole lying nearest to them. This angle is, of course, measured by the arc of the equator lying between the two meridian circles. This difference of geographical longitude is often called the

meridian difference of two places, since it is equal to the difference of time given by two clocks keeping true time, at the respective places of observation. This difference expressed in hours, minutes, and seconds, is converted into degrees, minutes, &c., by being multiplied by 15, and then will be equal to the above-mentioned angle formed by the two meridian circles at the pole, or the arc of the equator contained between these circles.

The measurement of the distance of two meridian circles, however involves greater difficulties than the determination of the altitude of the pole. The meridian difference of two places is determined by the time required by a star, in the course of its apparent daily revolution, to pass from the meridian of the eastward place to that of the western. Should this star be the sun, we can only compare the times which the clocks in the two places show at one and the same moment. For since at each place the clocks are set 0, or 12, at the time the sun passes his meridian, these, if they keep accurate time, must always differ by the same number of hours, minutes, and seconds. There is therefore only needed some means by which the clocks and instruments used at the different places of observation may be compared with each other, and this may be done in two ways; first, by carrying one clock to the other, without changing their rate in the least; or, secondly, by observing the clock time at which some phenomenon visible from both places, and determinable to the seconds, takes place. The difference in time obtained by either of these methods is then the desired meridian difference, or the difference in longitude, of the two places. We may also remark that the first method consists in the employment and application of portable clocks or chronometers. The second, however, consists in the observation, *a*, of eclipses of the moon and of Jupiter's satellites; *b*, of occultations; *c*, of eclipses of the sun; *d*, of corresponding culminations of the moon and neighboring fixed stars; *e*, of lunar distances; and *f*, of gunpowder or other artificial signals. It is not our place here, to go over these different methods separately and circumstantially, nor to explain the mode of calculating the meridian difference desired. It must suffice to examine a little more closely one, and indeed a very simple method, namely, to show how from observations of the eclipses of one of Jupiter's satellites in two places, the difference of their geographical longitude can be obtained. Let, for instance, in *pl.* 10, *fig.* 1, E be the place of one observer, V that of another, both points being on the surface of the earth. Let both observers note by their clocks the time of their place at which the eclipse of any one of Jupiter's satellites takes place, that is, the time when the satellite begins to enter the cone of shadow ending at x. We will now assume that the time at E is 8 o'clock and the time at V is 10 o'clock, the difference of these two periods amounts to 2 hours 30°, therefore V is 30° east of E.

Unfortunately, the moments (beginning and end) of an eclipse of one of Jupiter's satellites cannot be so readily determined as that of the moon, on account of the unequal illumination and magnifying power of the different telescopes, as also the different acuteness of vision of different observers. Consequently, the meridian difference deduced will not be accurate, or at any rate not reliable; while the other methods admit of greater precision.

The Fixed Stars; their Size, Number, Arrangement, and Distances.

11. By *fixed stars* is to be understood all those stars which are neither planets, nor moons, nor comets, deriving the name from the fact of never changing their relative positions when viewed with the naked eye. The fixed stars are the most numerous objects in the heavens, and are divided into eight classes, according to their various apparent sizes and brilliancy. We speak, for instance, of stars of 1st, 2d, 3d, 4th, 5th, 6th, 7th magnitude, so that those of the 1st magnitude possess the greatest brilliancy, and those of the 7th are only visible to the very acute naked eye. To the 8th class, that is, to those of the 8th, 9th, 10th, &c., magnitudes, belong all those millions of stars which are only visible through the telescope, hence called *telescopic stars.* The color also, especially in the double stars, is very various.

Although the fixed stars do not change their relative positions to each other, yet they have a common apparent motion, produced by the actual daily rotation of the earth on its axis; in addition to this, the annual revolution of the earth about the sun produces various differences in their position with respect to the sun. Finally, there are other although very minute variations in the position of the stars with respect to the horizon, the equator, the ecliptics, &c., produced by refraction, parallax, precession of the equinoxes, nutation, &c., which will be referred to hereafter.

As the number of the stars is apparently infinite, it must necessarily be impossible for us ever to estimate it. Nevertheless, many astronomers have attempted to determine this approximately, at least with respect to the visible stars. The number of those of the first magnitude is 14, of the second, 70, of the third, nearly 300; that of stars of the 4th and succeeding magnitudes is much greater. The fixed stars stand so close together in many parts of the heavens, that any enumeration of these would be absolutely impossible. We need only examine the milky way through a good telescope, to discover that it consists of an innumerable number of fixed stars. Herschel at one time saw more than 50,000 stars cross the fixed field of his great reflector in a space of 30 degrees, near the club of Orion: at another time, he observed 258,000 to pass his 20 foot reflector in 41 minutes.

Lambert, in his "*Kosmologische Briefe,*" was the first to treat of the distribution and arrangement of stars in the heavens. It is in more recent times, however, since the discovery by Mädler of the central sun of our system of fixed stars, that the first tolerably satisfactory explanation of the arrangement of the stars has been given. With respect also to the distances of the stars, nothing more definite was known than that the nearest of them was so remote as to require six years for its light to reach our earth. Bessel and Struve, however, by means of observations of the particular motions exhibited by the double stars 61 Cygni, and Vega (α) Lyræ, have determined these distances with approximate accuracy. According to Bessel's investigations, the distance between our sun and star 61 Cygni would be traversed by light within ten years.

12. Besides the division of stars according to their apparent size, they are classified as *single, double, and multiple stars, variable and temporary stars,* and *nebulous stars.* These will all be referred to hereafter. Groups of stars which are visible, partly to the naked eye, partly by means of the telescope, are called constellations; while those in which, even with the aid of the most powerful telescope, the individual stars cannot be made out, the whole appearing as a light nebulous cloud, are called *nebulæ.* Finally, that belt of light formed by innumerable stars, and encompassing the whole heavens, varying in breadth and concentration, exhibiting sometimes one branch, and sometimes two, is called the *milky way.* This passes through the constellations (*pls.* 12, 13) Cassiopeia, Perseus, Orion, Gemini, Argo, Centaurus, Ara, Scorpio, Sagittarius, Ophiuchus, Aquila, Cygnus, and Cepheus. Its two branches unite in Ara and Cygnus.

THE CONSTELLATIONS.

13. For the sake of assisting the memory in recollecting the general distribution and relations of the stars in the sky, they have been divided into *constellations.* Those of the northern hemisphere are represented in *pl.* 12, those of the southern in *pl.* 13. These constellations are groups of fixed stars, whose outlines have been supposed to represent figures of men, animals, and other objects, and to which corresponding names have been given With the arrangement of new constellations, it becomes necessary to have new names, which may either be derived from the animal kingdom, as done by the ancients, *e. g.* the Giraffe and the Lizard, or names may be selected commemorating important discoveries and inventions in the arts and sciences, *e. g.* the compass, the air pump, and the pendulum clock.

The names of the separate constellations, as given by Hipparchus, are the following:—

a. The Twelve Constellations of the Zodiac.

♈ Aries,	(Ram,)	*Widder.*
♉ Taurus,	(Bull,)	*Stier.*
♊ Gemini,	(Twins,)	*Zwillinge.*
♋ Cancer,	(Crab,)	*Krebs.*
♌ Leo,	(Lion,)	*Löwe.*
♍ Virgo,	(Virgin,)	*Jungfrau.*
♎ Libra,	(Balance,)	*Wage.*
♏ Scorpio,	(Scorpion,)	*Skorpion.*
♐ Sagittarius,	(Archer,)	*Schütze.*
♑ Capricornus,	(Goat,)	*Steinbock.*
♒ Aquarius,	(Water Bearer,)	*Wassermann.*
♓ Pisces,	(The Fish,)	*Fische.*

b. The Twenty-one Constellations of the Northern Heavens.

Cassiopeia,	(Cassiopeia,)	*Kassiopeia.*
Andromeda,	(Andromeda,)	*Andromeda.*
Triangulum,	(Triangle,)	*Nördliches Dreieck.*
Perseus,	(Perseus,)	*Perseus.*
Auriga,	(Charioteer,)	*Fuhrmann.*
Ursa Major,	(Great Bear,)	*Grosser Bär.*
Draco Borealis,	(Dragon,)	*Nördl. Drache.*
Bootes,	(Bootes,)	*Bootes.*
Corona Borealis,	(Northern Crown,)	*Nördl. Krone.*
Cepheus,	(Cepheus,)	*Cepheus.*
Pegasus,	(Pegasus,)	*Pegasus.*
Ursa Minor,	(Lesser Bear,)	*Kleiner Bär.*
Hercules,	(Hercules,)	*Hercules.*
Ophiuchus, or	(Serpentarius,)	*Ophiucus.*
Serpens,	(Serpent,)	*Schlange.*
Lyra,	(Lyre,)	*Leier.*
Aquila,	(Eagle,)	*Adler.*
Cygnus,	(Swan,)	*Schwan.*
Sagitta,	(Arrow,)	*Pfeil.*
Delphinus,	(Dolphin,)	*Delphin.*
Equuleus,	(Little Horse,)	*Kleines Pferd.*

c. The Fifteen Constellations of the Southern Heavens.

Cetus,	(Whale,)	*Wallfisch.*
Canis Major,	(Great Dog,)	*Grosser Hund.*
Canis Minor,	(Little Dog,)	*Kleiner Hund.*
Hydra,	(Hydra,)	*Grosse Wasserschlange.*
Crater,	(Cup,)	*Becher.*
Corvus,	(Crow,)	*Rabe.*
Lupus,	(Wolf,)	*Wolf.*
Orion,	(Orion,)	*Orion.*
Centaurus,	(Centaur,)	*Centaur.*
Argo,	(Ship,)	*Schiff Argo.*
Corona Australis,	(Southern Crown,)	*Südliche Krone.*
Piscis Australis,	(Southern Fish,)	*Südlicher Fisch.*
Lepus,	(Hare,)	*Hase.*
Ara,	(Altar,)	*Altar.*
Eridanus,	(The Po,)	*Fluss Eridanus.*

To these forty-eight constellations of the ancients, there have been added in modern times the following fifty-eight :—

d. The Fifty-Eight Constellations discovered in modern times in the Northern and Southern Heavens.

Antinous,	(Antinous,)	*Antinous.*
Coma Berenicis,	(Berenice's Hair,)	*Haupthaar der Berenice.*
Robur Carolinum,	(Charles's Oak,)	*Karlseiche.*
Columba,	(The Dove,)	*Taube.*
Crux,	(Cross,)	*Kreuz.*
Scutum Sobieskianum,	(Sobieski's Shield,)	*Sobieski's Schild.*
Monoceros,	(Unicorn,)	*Einhorn.*
Camelopardalis,	(Giraffe,)	*Giraffe.*
Sextans,	(Sextant,)	*Uranischer Sextant.*
Canes Venatici,	(Greyhounds,)	*Jagdhunde.*
Leo Minor,	(Little Lion,)	*Kleiner Löwe.*
Lynx,	(Lynx,)	*Luchs.*
Vulpes et Anser,	(Fox and Goose,)	*Fuchs und Gans.*
Lacerta,	(Lizard,)	*Sterneidechse.*
Triangulum,	(Little Triangle,)	*Kleines Dreieck.*
Musca,	(Fly,)	*Fliege.*
Cerberus,	(Cerberus,)	*Cerberus.*
Anser Americanus,	(American Goose,)	*Amerikanische Gans.*
Phœnix,	(Phœnix,)	*Phönix.*
Hydrus,	(Hydra,)	*Kleine Wasserschlange.*
Dorado,	(Swordfish,)	*Schwertfisch.*
Piscis Volans,	(Flying Fish,)	*Fliegender Fisch.*
Chameleo,	(Chameleon,)	*Chamäleon.*
Avis Indica,	(Bird of Paradise,)	*Paradiesvogel.*
Triangulum,	(Southern Triangle,)	*Südliches Dreieck.*
Pavo,	(Peacock,)	*Pfau.*
Indus,	(Indian,)	*Indianer.*
Grus,	(Crane,)	*Kranich.*
Mons Mænalus,	(Mount Mænalus,)	*Berg Mänalus.*
Cor Carolinum,	(Charles's Heart,)	*Herz Karl's II.*
Cervus,	(Reindeer,)	*Rennthier.*
Pica Indica,	(Indian Bird,)	*Indianischer Vogel.*
Taurus Poniatowski,	(Poniatowski's Bull,)	*Stier Poniatowski's*
Quadra,	Square,	*Mauerquadrat.*
Officinum Sculptorum,	(Sculptor's Workshop,)	*Bildhauerwerkstatt.*
Fornax Chemica,	(Chemical Furnace,)	*Chemischer Ofen.*
Horologium,	(Clock,)	*Pendeluhr.*
Reticulus Rhomboidalis,	(Rhomboidal Net,)	*Rhomboidisches Netz.*
Cœlum Sculptorium,	(Graver,)	*Grabstichel.*
Equuleus Pictoris,	(Painter's Easel,)	*Malerstaffelei.*
Pyxis Nautica,	(Mariner's Compass,)	*Seecompass.*
Machina pneumatica,	(Air Pump,)	*Luftpumpe.*
Octans,	(Octant,)	*Seeoctant.*

Watchman,	*Erntehüter.*
Sceptre of Brandenburg,	*Brandenburgisches Scepter.*
Honors of Frederick,	*Friedrichsehre.*
George's Harp,	*Georgsharfe.*
Herschel's Telescope,	*Herschel's Teleskop.*
Balloon,	*Luftballon.*
Printing Press,	*Buchdruckerwerkstatt.*
Electric Machine,	*Electrisir Maschine.*
Log Line,	*Log Leine.*
Compasses,	*Zirkel.*
Ruler and Square,	*Lineal und Winkelmaass.*
Telescope,	*Astronom. Fernrohr.*
Microscope,	*Mikroskop.*
Table Mountain,	*Tafelberg.*
Level,	*Setzwage.*

There are consequently **106** constellations in the heavens, forty-eight old, and fifty-eight new.

Maps of the Stars; Planispheres; Application of the Method of Alignments in Learning the Stars and Constellations.

14. For the sake of more readily learning the constellations and their particular stars, as also for the more certain guidance of astronomers, those delineations on paper of the starry heavens known as celestial maps, or maps of the stars, have been invented. These maps comprehend either the two planispheres, as in *pls.* **12** and **13**, and are then called *planispheres,* or they contain single parts of the heavens, and then together form an atlas. The celestial maps of Bayer, Doppelmayer, Goldbach, Flamsteed, Bode, Harding, Schwinck, Riedig, Argelander, and others, are well known. Besides the introduction of constellations, the ancients, particularly the Arabians, ascribed particular names to the brighter fixed stars, as, for instance, in Orion (*pl.* 12), *Bellatrix* and *Betelgeux, Capella* in Auriga, *Altair* in Aquila, *Arcturus* in Bootes, *Castor and Pollux* in Gemini, *Marcab* in Pegasus, &c., and in *pl.* **13**, *Rigel* in Orion, *Fomalhaut* in Piscis Australis, *Sirius* in Canis Major, *Antares* in Scorpio, &c. Johann Bayer, however, in the beginning of the seventeenth century, introduced a much better and more complete assistant to the memory in recollecting and referring to the stars, by employing the letters of the Greek and Roman alphabets, which convenient notation has since been justly retained. It may be observed in the two charts of the stars, *pls.* **12, 13.**

A very easy means of finding and readily learning the most important stars and constellations is afforded by what is called the method of *alignments;* this consists in having straight lines drawn in the chart (*pls.* **12, 13**) connecting the single brighter stars, thus forming triangles and quadrilaterals, which are again reconstructed in the sky by imaginary lines drawn between these stars. This will be referred to hereafter.

The Double Stars; Remarkable Collections of Stars; Nebulous Spots and Stars.

15. By *double stars* is meant two stars, generally so near together that to the naked eye they appear but as a single star. There are nevertheless many double stars only visible through the telescope. These may consist of two kinds—*optical,* or apparent, and *physical,* or actual double stars. The first are such as, not related to each other, happen to fall nearly on the same line of vision; the latter are those which, connected in one system, revolve the one about the other. One of these is frequently larger than the other, although sometimes their size is nearly equal. Their colors, however, are always different. The single fixed stars and the optical double stars shine only with a whitish light, verging sometimes on yellow, sometimes on red. The physical or actual double stars, of which alone we here treat, have only been studied within the last few decades. The peculiar motions of these remarkable stars appear to occur according to the Newtonian laws of gravitation, and Savary, Encke, John Herschel, and Mädler, have already determined the orbits of many double stars, as 70 p, Ophiuchus, polar star, γ Andromedæ, ζ Ursæ Majoris, ζ Herculis, &c., as also their periods of revolution, which latter in some double stars amount to a few years, in others to many centuries. Since one of two double stars, as before stated, revolves around the other, it may readily happen that with respect to our earth one may pass before the other so as to cover it completely. This has actually been confirmed by observation. Thus, for instance, stars which once were double are now single, and others which were once single are now seen as double. Consequently the apparent distances of the double stars cannot be otherwise than variable. Struve has published a catalogue of 3112 double and multiple stars, arranged in order, which is thus far the most complete and accurate.

Since the double stars generally present very slight points of light of various distance and distinctness, their observation may serve as the surest test of the excellence of a telescope. Achromatic telescopes, which, for instance, merely exhibit ζ, Ursa Major, or Mizar (*pl.* 12, *fig.* 3), and γ, Andromeda, Alamak (*fig.* 6), as double stars, are only of ordinary power. Those, however, are much better which show as double stars Castor and Pollux, or α Gemini (*pl.* 12, *fig.* 1) and the pole star, or α Ursæ Minoris (*fig.* 9), as also Mesarthim, or γ Arietis (*fig.* 8), and Cor Caroli in Canes Venatici (*fig.* 10). An instrument that shall show γ Virginis (*fig.* 2), α Arietis (*fig.* 5), β Orionis, or Rigel (*fig.* 7), and *Ras Algithi,* or α Herculis (*fig.* 4), as double, is one of extraordinary excellence. The double star Vega, or α Lyræ (*fig.* 11), is probably not really but only optically double, the smaller of the 12 magnitudes being distant about 43 seconds from the principal stars.

16. Among the spots of the northern heavens richest in stars belong the groups figured on *pl.* 12; the *Pleiades,* or the seven stars, *fig.* 12, in the back of Taurus, the *Hyades,* or rain stars, *fig.* 13, in the forehead of Taurus, the little group of stars (*fig.* 14) between the tips, β and ζ, of the

horns of Taurus; the rich region about Vega and Lyra; the so called Lucida Lyræ; the numerous stars about Arcturus (*fig.* 16), in Bootes, and the vicinity of the great remarkable nebula (*fig.* 17) in Orion.

With respect to the nebulous spots and stars, *pl.* 13 (*figs.* 1 to 20) represents twenty of the largest and most beautiful. *Fig.* 1 is a double nebula in Gemini (108° 45′ right ascension, and 29° 49′ declination), consisting of two round nebulæ touching each other, which shine almost like stars. *Fig.* 2 exhibits the double nebula in Coma Berenicis (right ascension 187° 0′, declination 12° 8′), of great brilliancy. *Fig.* 3 gives a view of a small double nebula of right ascension 158° 15′, declination south 17° 55′. *Fig.* 4 is a curiously shaped nebula in Ophiuchus (Serpentarius). *Fig.* 5 represents two nebulous spots touching each other nearly at right angles, of tolerably elliptical shape, to be found in the constellation of Canes Venatici. *Fig.* 6 represents the remarkable annular nebula in Lyra, rt. asc. 281° 45′, N. dec. 32° 49′, whose opening is filled with a second ash-grey nebula. the whole appearing like a veil drawn over an almost circular hoop. At 33° 0′ of right ascension and 41° 34′ North declination, in Perseus, is seen a distinct and very eccentric nebula (*fig.* 7) of 4′ length and 40″ breadth, in whose midst is a concentric, also elliptical, space, at whose two extremes two little stars are seen. Still more remarkable is the nebulous spot (rt. asc. 268° 0′, S. dec. 23° 1′) in Sagittarius (*fig.* 8), seemingly split into three pieces; a double star is seen in the midst of the dark interspace. *Fig.* 9 gives a view of the extremely remarkable and tolerably large nebula in constellation Robur Caroli of the southern hemisphere, consequently not visible in the northern. A number of minute stars will be observed to shine out from it.

The following may be particularly mentioned among the number of irresolvable planetary nebulæ. The well known great nebula in Andromeda (*pl.* 13, *fig.* 10), visible to the naked eye, of peculiar feebly glimmering light, 30′ in diameter; the stars standing in its vicinity do not appear to belong to it. A nebula (*fig.* 11) occurs in Cetus, similar to the one in Perseus (*fig.* 7), only longer and broader. A curiously shaped elongated nebula (*fig.* 12) is met with in Cygnus, while planetary, entirely round, and brilliant nebulous spots exhibit themselves in Sagittarius (*fig.* 13) and the hand of Andromeda (*fig.* 14). A spot (*fig.* 15) similar to these last is shown in Orion, and a granular nebula, with a very bright spot, in Ursa Major (rt. asc. 127° 45′, N. dec. 50° 49′). But the most remarkable, perhaps, of all is the great nebula in Orion (*fig.* 17), under the middle of the (so called) Jacob's Staff, near the star ⊖; distinguished above all the others by its peculiar shape (not unlike the opened jaws of a wild beast), by the curious variety in the distribution of its light, as well as by its great extent. Even the fixed stars in and about it are remarkable for their lustre, and the positions of some of them appear to have a particular relation to the nebula itself. It has been supposed, from a comparison of older figures of this nebula with its present appearance, that it has undergone a decided change, although this is by no means absolutely certain. An almost equally remarkable object is found in the constellation Vulpes (rt. asc. 298° 0′ and

N. decl. 22° 17′), in the shape of a large oval nebulous spot (*fig.* 18), whose major axis is to its minor as 4 to 3. In the two foci are found two circular nebulæ, much darker, and equally illuminated in all parts. *Fig.* 19 represents a curious nebulous figure found in the head of the northern Canis Venaticus. It is a round, bright, central spot, surrounded by a nebulous ring split on one side. Further improvements in the telescope will probably exhibit more clearly the true character of these wonderful objects. The Magellanic Clouds, observed in the southern hemisphere and invisible to us (*fig.* 20), are in like manner highly remarkable appearances of the starry heavens, consisting of isolated clear spots, like separated portions of the Milky Way. According to Horner, who has communicated some observations on these Magellanic Clouds, the greater of them is about as bright as the Milky Way in its brightest part, as near Cygnus, while La Caille was not able to find a single star in them with his 14-foot telescope.

Multiple, Variable, and New Stars.

17. In conclusion, it remains to state that, besides the double stars, there are threefold and fourfold, or *multiple* stars, as also *changeable* and *new stars*. Threefold stars are found, for example, in Orion, under $72\frac{1}{4}°$ rt. asc., and $14\frac{1}{4}°$ N. decl., in ζ Cancer, ξ Libra, 7 Taurus, ψ Cassiopeia, π Monoceros, ψ Libra, as also in Lynx, under $97\frac{1}{2}°$ rt. asc. and $59\frac{3}{5}°$ N. declination. Of the fourfold stars θ Orion is perhaps the most distinguished; it stands very near the darkest part of the great nebula in Orion; ε Lyra is also a fourfold star. Among multiple stars, ζ Orion is known as the most remarkable; it was known by Schröter as a 12-fold, but by Struve as a 16-fold star. The variable stars are also remarkable, that is, those stars whose apparent magnitude does not remain the same. These stars shine within a certain period with various degrees of brilliancy, and it is said that they have a certain period, as, for instance, Algol in Perseus, α Hercules, β Lyra, γ Antinous, δ Cepheus, &c. Nevertheless the periods of several variable stars appear subject to many irregularities. Other stars often vanish entirely, and reappear at a later period, as Mira in Cetus, χ Cygni, and seven stars in Leo, Virgo, Hydra, Corona, and Aquarius.

In addition to all these, stars have sometimes appeared suddenly in regions of the heavens tolerably free from stars, which could not have been there before; such appeared in Aquila in 389, between Cepheus and Cassiopeia in 945, again near the same place in 1264, in Cassiopeia (Tycho's star) in 1572 and 1573, in Ophiuchus, 1604 and 1605, and in Cygnus (Anthelm's star), 1670. Astronomers have not yet been able to frame a satisfactory hypothesis to explain the phenomena which the variable and new stars exhibit in so remarkable a manner.

II. Theoretical Astronomy.

The Circle and Ellipse.

18. In the study of Astronomy a knowledge of the circle and ellipse is absolutely necessary. If a straight line, CD (*pl.* 6, *fig.* 3), make a complete revolution in the same plane around one extremity, C, the other end, D, will describe the *circumference*, DHAGFD, in which each point is equally distant from the *centre*, C. The plane surface inclosed by this circumference is called *the circle.* Every straight line drawn from the centre, C, to the circumference is a *radius;* and every straight line connecting two points of a circumference and passing through the centre is a *diameter* (AB). All radii are equal to each other, so also are all diameters. Any straight line connecting two points of the circumference, not passing through the centre, is called a *chord* (EF). It divides the circle into two unequal parts, EAHDBF and EGF, of which the latter, as the smaller, is called the *segment.* The diameter, AB, on the contrary, divides the circle into two equal parts, AHDB and AEGFB, called *semicircles.* The angle, DCB, formed by two radii, DC and CB, at the centre of the circle, is called a *central angle;* the surface, BCD, inclosed between a central angle and the arc of the circumference inclosed between the radii, is called a *sector.* Finally, a straight line outside of the circle and touching the circumference in only one point, is called a *tangent,* JK.

An ellipse, ADBEA (*pl.* 6, *fig.* 4), is a complete curve, possessing the peculiarity that, if two straight lines be drawn from certain points, SS′, in its area, called the *foci,* to any point in the circumference, their sum will be equal to the sum of similar lines drawn to any other points. Thus $SP + S'P = Sp + S'p$. This same sum will also be always equal to the length of the greater axis, AB. The lines SP and S′P (or also Sp and S′p) are called the *radii vectores* of the point P (or p). The straight line, DC, passing through the centre, C, at right angles to the major axis, AB, is called the *minor axis.* CS or CS′ is called the eccentricity of the ellipse. Should this ellipse represent the orbit of a planet in one of whose foci the sun is situated, then if the planet be situated at P or p, the line SP or Sp will be the *radius vector* of the planet.

Parallax; Horizontal Parallax and Parallax in Altitude; Parallax of a Place.

19. *Fig.* 5 will serve for the explanation of what astronomers mean by the word *parallax.* Let ABD represent the meridian circle of a place of observation, A; C the centre of the earth; let HJ be a part of the infinitely distant sphere of the heavens; finally, let the moon be at M. The line HCD represents the true horizon, aAa' the apparent horizon of the place, A. If the moon be supposed to stand in the horizon at M, it will be

referred by an observer at A to that part of the celestial sphere occupied by the star *a*. From the centre of the earth it would be referred in the direction CM to the celestial sphere at *b*. The angle AMC, or *a*M*b*, will represent the difference of the two directions ; and this angle, AMC, is called the *parallax* of the point M, or its *horizontal parallax,* since M is situated in the horizon. Let the moon now stand higher in the heavens, as at M′, then from A she will be seen in the direction AM′*c*, consequently referred to the point *c* of the heavens; and from C, the centre of the earth, in the direction CM′*d*, referred consequently to the celestial sphere at *d*. The angle CM′A = *c*M′*d* will be the difference of the two directions CM′ and AM′; and CM′A will be the parallax of M′, more definitely its *parallax in altitude,* because M′ is situated a certain distance above the horizon. Let the moon now be situated at M″, or in the direction of the zenith, *e*, of A, then the moon, M″, will be referred to the heavens at *e;* the two lines of sight will coincide, and they will form no angle, so that for heavenly bodies situated in the direction of the zenith, the parallax in altitude vanishes or becomes zero. We find, also, by examining the figure that the parallax is at its maximum in the horizon, diminishing with the angle of elevation until in the zenith it is zero.

Suppose now a second place of observation, E, lying in the same meridian, and the moon to be situated at M′; then from A she will be seen in the direction AM′*c*, consequently referred to *c*, and from E in the direction EM′*d*′, and referred to *d*′. The angle EM′A gives the difference of the two directions EM′, AM′. This angle EM′A, or the corresponding arc *cd*′, is called the *local parallax* of the star at M′ for the two places A and E. Hence it is clear, that for the same altitude of the same star, its local parallax will be less as the distance of the two places of observation on the same meridian is less.

The Heliocentric and Geocentric place of the Planets; their Commutation and Annual Parallax.

20. The motions exhibited by the planets are not so simple as the apparent daily motions of the fixed stars, as above considered. This results from the fact of their having two motions, of which one is diurnal, in common with the fixed stars, from east to west, and the other an orbitual motion from west to east. Add to this, that the planets are not, like the fixed stars, almost infinitely distant from us, and that therefore it makes a material difference whether their revolutions be observed from the sun or the earth. The place of a planet, as seen from the sun, is called its *heliocentric place;* its position as seen from the earth, is the *geocentric place. Fig.* 14, *pl.* 6, is intended to illustrate these terms. Let S be the centre of the sun, the circle described through T the orbit of the earth, that through P the orbit of any superior planet, and the extreme circle, the ecliptic ; the exterior and interior circles lying, of course, in the same plane, which may be the plane of the paper. Furthermore, let the earth be at T, the planet at P, and let

♈ be the vernal equinox. The arrows give the direction of the motions of the earth and planet, as also the order of the signs of the ecliptic. The planet P will now be seen from the sun, S, in the direction Sp; the angle ♈Sp, or the arc ♈p, will consequently be the *heliocentric* longitude of the planet, and the angle ♈ST, or the arc ♈t, the *heliocentric longitude* of the earth. From the earth T, the planet P will evidently be seen in the direction Tp'. It is further allowable, on account of the almost infinite distance of the fixed stars, to consider the line TA parallel to S♈ as meeting in the same point of the celestial sphere, and consequently A and ♈ as coinciding. The angle ATp' then, or the arc ♈p', is the *geocentric longitude* of the planet; and since the plane of the planet at P has a certain inclination to the common plane of the inner and outer circles, the angle PSQ, or the arc PQ, will represent the *heliocentric latitude* of the planet, if the arc PQ be supposed drawn from the planet P, perpendicular to the plane of the earth's orbit. Finally, the angle PTQ will be the *geocentric latitude* of the planet. The angle PTS, at the earth, will be the *elongation* of the planet, or its apparent angular distance from the sun, this elongation being (*pl.* 6, *fig.* 14) equal to the heliocentric longitude of the earth, + 180°, diminished by the geocentric longitude of the planet. From the preceding it follows that p will be the *heliocentric*, and p' the *geocentric* place of the planet, with respect to the ecliptic. We can speak in the same manner of the heliocentric and geocentric position of a planet with respect to the equator, and consequently of the heliocentric and geocentric right ascension and declination.

We have still to speak of the *commutation* and the *annual parallax* of a planet. The commutation is the angle PST (*fig.* 14), at the sun, S, obtained by deducting the heliocentric longitude of the earth from the heliocentric longitude of the planet. The annual parallax is the angle TPS, at the planet P, obtained by deducting the heliocentric longitude of the planet from its geocentric longitude. It is very evident that we can never speak of the geocentric place of the earth.

Some other Important Elements in the Theory of the Planetary Motions.

21. The *mean anomaly*, the *true anomaly*, the *perihelion distance*, and the *aphelion distance*, are far more important to the theory of the planetary motions than the commutation and the annual parallax. If, for instance, m (*fig.* 15) be the *mean place* of a planet, p the *true place*, then rays, Sm, Sp, from the sun to these two positions will form, with respect to the major axis AP, of the elliptical planetary orbit, the angles mSP′, pSP, which are respectively called the *mean* and the *true anomaly* of the planet. Their great importance consists in this, that the mean anomaly expresses the uniform or the *mean motion in the circle*, while the true anomaly expresses the *true motion in the ellipse*. The true anomaly, or the angle PSp, consists of two parts, angles mSP′ and mSp. This latter angle, mSp, however, which evidently is the difference between the true and mean motions of the planet, is called the

equation of the orbit, and is one of the most essential elements in the theory of the planetary motions. For, knowing the equation of the orbit, the sum of the mean anomaly and the equation of the orbit gives the true anomaly, and consequently the true—that is, the elliptic longitude of a planet in its orbit. In conclusion, P is the *perihelion* and A the *aphelion* of the planet, in which points it will be respectively at its least and greatest distances from the sun.

Eclipses of the Sun and Moon.

22. We come now to eclipses of the sun and moon (*pl.* 6, *fig.* 18). The full moon at times loses gradually its light in a manner just as if a blackish grey disk were drawn slowly over its face, moving from left to right, and passing off on the opposite side. An eclipse of the moon is then said to take place, which, as it only occurs when the moon is full, and, indeed, when the full moon is in or near the straight line connecting the centres of the sun and earth, is of easy explanation. The earth, as an opake globe illuminated by the sun, must throw a shadow into space on the side opposite to the sun, this shadow being conical, and longer than the distance of the moon from the earth. The moon must also, as a body likewise illuminated by the sun, lose partly or entirely her light by passing partly or entirely into the shadow of the earth. If the moon's orbit lay in the plane of the ecliptic, it is clear that an eclipse would occur at every full moon, which experience shows not to be the case. We learn above, section 8, that the path of the moon is inclined 5° 8′ 48″ to the ecliptic; the full moon passes for the most part, therefore, above or below the earth's shadow, and it is only when the full moon happens in or near one of the moon's nodes, that the moon can encounter the earth's shadow. As the moon's nodes do not remain in the same part of the ecliptic, it is very evident that the eclipses of the moon (and sun) must take place in different years and months, so that definite periods arise, after whose lapse the eclipses again occur in the same months.

Astronomy teaches the conditions necessary for the occurrence of an eclipse. Should the full moon be situated at a distance from one of its nodes of less than 12½°, the moon may be partially eclipsed. Should the distance be less than 9½°, and greater than 5½°, a partial eclipse must take place. The eclipse will be *total*, that is, the moon will be entirely obscured, when the full moon takes place at less than 5½° from one of the nodes. A merely partial eclipse of the moon cannot last more than 2 hours and 18 minutes; the time of a partial, and at the same time total eclipse, may amount to 4 hours and 24 minutes. Astronomers are accustomed to determine beforehand the particular circumstances of an eclipse of the moon, as the beginning, the middle, and the end of the obscuration, the countries in which it will be visible, as well as the size of the eclipsed portion. The latter is given in digits (1 digit = $\frac{1}{12}$ of the diameter of the disk). Unfortunately, however, the period of beginning and ending of an eclipse cannot

be exactly determined, as the earth's shadow falling on the moon (*pl.* 6, *fig.* 18), like any other shadow, is not bounded by sharp outlines, but fades off into light, and in every eclipse there is also distinguished a full shadow or *umbra*, and a half shadow or *penumbra*. From the preceding it is clear that an eclipse of the moon is not merely an apparent, but a real occurrence. Consequently, the inhabitants of all those portions of the earth in which the moon shines at the time of the eclipse, see every particular of the eclipse in the same manner and at the same time, even if the local times of the places should be different.

Sometimes a black disk is seen to pass gradually from right to left before the sun. But as this phenomenon is not seen alike at all places above whose horizon the sun may be at the time, since in some countries the sun is covered more or less than in others, or not at all, it is evident that this phenomenon cannot be the result of an actual obscuration of the sun. It is rather produced by the moon (*pl.* 6, *fig.* 18, and *pl.* 14, *fig.* 56), since it only happens at the time of new moon. It has been observed that these uncommon phenomena, called eclipses of the sun, occur only when the new moon happens to be in or near one of its nodes. By its intervention between the earth and sun, it then hides the view of the latter entirely or partly from the former. The phenomenon exhibited, is precisely as if the sun in a cloudless sky were covered for a time by a little black cloud passing over his disk. As the shadow of the cloud moves along in the plain beneath, in the direction in which the cloud is driven by the wind, and conceals from the observer whom it overtakes the sight of the sun, while others out of the bounds of the shadow still see that luminary; just so the shadow of the moon moves along over the earth's surface from west to east in the direction of the moon's motion around the earth, conceals from the countries traversed by it the view of the sun, and produces the phenomenon of eclipse. All regions not thus traversed see the sun as usual.

Eclipses of the sun are distinguished into *partial, total, central*, and *annular*. An eclipse is *partial*, when the moon covers the sun only in part; *total*, when this covering is complete. When the moon's disk is apparently smaller than that of the sun and stands directly before him, the eclipse is annular. A total or annular eclipse is central when the centres of the sun and moon's disk coincide. For the entire earth's surface, a partial eclipse can last about 7 hours; one, partial and total, 4 hours and 48 minutes; but for a given place on the earth's surface, a total eclipse cannot continue more than $4\frac{1}{2}$ hours at the very utmost. The calculation of the separate circumstances of an eclipse of the sun becomes, therefore, more difficult and circumstantial than those of the moon, since, as before mentioned, the former are not actual occurrences like the latter, but only apparent phenomena, whose shape, size, and duration depend upon the place of the observer on the earth's surface.

An eclipse of the sun cannot take place when the new moon is more than 1° 35′ distant north or south of the ecliptic: 1° 24′ is the minimum distance that the moon can pass near the sun without causing an eclipse. A north or south latitude of the moon less than 1° 24′ at time of new moon always

produces an eclipse of the sun, whose extent, as in the case of the lunar eclipses, is given in digits (12 digits = diameter of the sun's disk). Hence it follows that the limits of possible occurrence of eclipses are much more extended in the case of the sun than in that of the moon. Consequently it follows that, for the whole surface of the earth, the former are much more frequent than the latter, for on an average there may happen within 18 years 41 eclipses of the sun and only about 29 of the moon. At least two eclipses of the sun must occur annually, because the sun every six months comes in the neighborhood of the moon's ascending or descending node; while eclipses of the moon may be wanting for an entire year. But for any particular place on the surface of the earth, as, for instance, Leipzig, the visible eclipses of the moon are thrice as numerous as those of the sun. It may be assumed that every part of the earth's surface may expect a partial eclipse of the sun within every two years, and a total within 200. Sometimes, though but rarely, the number of solar and lunar eclipses for the entire earth may amount to seven in a year, occurring then in January, July, and December.

By the actual observation of a solar eclipse, its beginning and end can be determined much more accurately than the same circumstances of a lunar eclipse. For this reason observations on the former are much more certain assistants in determining geographical longitude.

The course and extent of the moon's shadow over the surface of the earth during a total or annular eclipse of the sun is shown by *fig.* 56, *pl.* 14. It represents also the manner in which the principal circumstances of such a phenomenon are usually delineated on a map.

The Planetary Systems of Ptolemy, Copernicus, Tycho, and the Egyptians. Tabular Exhibition of the Most Important Features of our Planetary System.

23. By a *planetary system*, the ancients understood the disposition and course of seven planets with respect to our earth. Since the time of Copernicus, however, by the *solar system*, in the ordinary acceptance of the term, is meant the disposition and mutual arrangement of 14 primary and 19 secondary planets about our sun, which system is commonly termed the *Copernican*. By a solar system, taken generally, is understood any fixed star of the heavens, as a sun, with the spheres revolving about it as planets. Since astronomy has been pursued as a science, four planetary systems have had the greatest share of attention, being, in order of time, that of *Ptolemy*, the *Egyptians*, *Copernicus*, and *Tycho*.

According to Ptolemy (an astronomer of Alexandria, living about A.D. 150), the earth (*pl.* 7, *fig.* 1) stands immovably in the centre of 12 circles. From the earth outwards the seven first circles represent the paths of the following bodies as planets, and in the following order:—the *Moon; Mercury; Venus;* the *Sun; Mars; Jupiter;* and *Saturn.* The eighth circle, *e*, represents the path of the fixed stars; the ninth, *d*, and tenth, *c*, called

the first and second crystal heavens, were intended to serve in explaining the phenomena produced by precession (precession of the equinoxes); the eleventh circle, *b*, called primum mobile, was supposed to carry along the ten circles inclosed by it, in its daily rotation from east to west, while each planet traverses its ascribed path from west to east about the earth; the twelfth and last circle, *a*, Ptolemy indicated by the name of Empyræum, or the abode of spirits and the blessed. This Ptolemaic system explains the heavenly appearances only imperfectly, provides in no way for the varying distances of the planets, and is, on the whole, very unnatural; it endured, however, with little change until the time of Copernicus.

Certain of the Egyptian astronomers easily perceived that the arrangement of the orbits of Venus and Mercury, according to Ptolemy, could not be the true one, since it could not explain the superior conjunctions of these two planets. They, therefore, allowed the moon, and then the sun, to revolve (*pl.* 7, *fig.* 2) round the earth, but supposed Venus and Mercury to revolve round the sun in minor orbits, accompanying it in its revolution round the earth. Mars, Jupiter, and Saturn moved in great circles about the earth, as in the Ptolemaic system. This system, termed the *Egyptian*, is as false as the Ptolemaic, and could not be maintained so long in authority. It was reserved for the great Copernicus (1472—1543) to teach the world the true theory of the arrangement of the primary and secondary planets, afterwards confirmed by the laws of Kepler, and the discovery by Newton of the law of universal gravitation. According to this theory the sun ☉ is a fixed star, occupying the centre of as many circular orbits as there are primary planets. These latter occur in the following order from the sun; Mercury ☿, Venus ♀, Earth ♁, Mars ♂, Jupiter ♃, Saturn ♄; the Moon ☽, a secondary planet, revolves about the earth, and with it around the sun. All these motions take place in the direction from west to east (shown by the direction of the arrows, *fig.* 5).

Tycho de Brahe, who lived in the second half of the sixteenth century, certainly recognised the correctness of the Copernican system at an early period, but his ambitious vanity, and perhaps still more his religious prejudices, urged him to oppose it. In Tycho's opinion the earth could not move around the sun, because the Bible would be thereby falsified. He preferred to represent the earth (*fig.* 3), like the earlier theorists, as placed immovably in the centre of the universe, the moon revolving round it first, and then the sun; around which latter the other planets, Mercury, Venus, Mars, Jupiter, and Saturn, revolve as their centre. This system of Tycho, however, could not longer maintain any stand when the true Copernican system had been discovered. The Copernican system received a very essential confirmation by Kepler, who showed that the planets revolve in orbits that are ellipses, but which differ very little from circles, the sun being situated in one focus common to all (*pl.* 10, *fig.* 2). This is also approximately exhibited in *pl.* 7, *fig.* 5, by the eccentric position of the circular planetary orbits.

Since the invention of the telescope, the following primary and secondary planets have been discovered as members of our solar system; the four

moons of Jupiter, the ring and seven satellites of Saturn, the planet Uranus ♅ (March 13, 1781, by Herschel), his six moons also by Herschel, Ceres ⚳ (Jan. 1, 1801, by Piazzi), Pallas ⚴ (March 28, 1802, by Olbers), Juno ⚵ (Sept. 1, 1804, by Harding), Vesta ⚶ (also by Olbers, March 29, 1807), Astræa ⚳ (Dec. 8, 1845, by Henke), Neptune ♆, his ring and moon (Sept. 23, 1846, by Galle), and the planet provisionally called Iris (July 1, 1847, by Henke). The orbits of all these planets and moons, except Astræa, Iris, and Neptune, are represented on *pl.* 7, *fig.* 5, upon which are represented also the orbits of Halley's Comet 1759, 1835, and of the great comet of 1811, as well as that of Encke. The outer circle of the figure represents the ecliptic with its division into the 12 signs, and on it are indicated by corresponding signs, in what parts to look for the following points:—the ascending node, ☊, of the great comet of 1811; the aphelion of Mars; the aphelion of Jupiter; the descending node, ☋, of Mars; the aphelion of Juno; the perihelion of Vesta; the descending nodes of Mars and Venus; the aphelion of Saturn; the descending node of Jupiter; the aphelion of the Earth; the descending node of Saturn; the aphelion of Pallas and Venus; the descending node of the Comet of 1811, and the aphelion of Ceres. In addition to these are given the proportional diameters for the Sun, Jupiter, Saturn, and Uranus.

24. Since the planets move slower in their orbits as their distance from the sun is greater, *fig.* 6, *pl.* 7, is intended to exhibit the relative velocity of these motions. When Mercury, the planet nearest to the sun, has completed an entire revolution in 360°, Venus in the same time describes an arc of 141° 22′, the Earth an arc of 86° 44′, &c. The proportionate velocities of the recently discovered planet Astræa (between Vesta and Juno) and Neptune (almost twice the distance of Uranus from the sun) could not well be represented in the figure. Astræa, if introduced into the preceding comparison, would describe an arc of about 21°, and Neptune one of about 20′. From the measurement of these various axes, it results, that Venus moves 2½, the Earth 4, Mars 8 times slower than Mercury. *Fig.* 7 represents the inclinations of all the planetary orbits (except those of Astræa, Iris, and Neptune) to the plane of the ecliptic or the earth's orbit.

25. We shall now present a tabular view of the most important elements of our planetary system, principally with regard to those points which could not be represented on *pl.* 7, without affecting the distinctness of the figures. The estimates are given in English geographical miles, according to the most recent observations and calculations. The different values ascribed to Neptune may possibly require rectification whenever his elements are better known than they can be now.

Distance, Period of Revolution, and Eccentricity of the Planets.

Planets.	Mean distance from the Sun.	Eccentricity.	Sidereal period in mean solar days and Julian years.					
Mercury .	32,000,000	6,580,000		87*d*	23*h*	15′	46″	
Venus . .	65,392,000	412,000		224	16	49	7	
Earth . .	82,664,000	1,388,000		365	5	9	10	7‴
Mars . .	125,956,000	11,740,000	1*y*	321	17	30	41	
Vesta . .	195,212,000	17,296,000	3	229	17	38	0	
Astræa . .	212,864,000	18,716,000	4	49	6	8	50	
Juno . .	220,672,000	56,400,000	4	132	1	36	0	
Ceres . .	229,052,000	17,564,000	4	223	17	38	0	
Pallas . .	229,192,000	55,468,000	4	225	7	19	0	
Jupiter . .	430,084,000	20,732,000	11	314	20	2	7	
Saturn . .	788,516,000	44,232,000	29	166	23	16	32	
Uranus . .	1,185,692,000	73,892,000	84	5′	19	41	36	
Neptune .	2,900,584,000	25,524,000	167½					

Rotation, Light, Gravitation, Density, &c., of Planets.

Planets.	Mass. The Sun = 1.	Density. Earth = 1.	Gravitation. Earth = 1.	Light & Warmth. Earth = 1.	Rotation in Mean Solar Time.		
Mercury .	1.2025810	2.94	1.15	6.67	1*d*	0*h*	5′
Venus . .	1.401847	0.92	0.91	1.91	0	23	21
Earth . .	1.354936	1.00	1.00	1.00	0	23	56.1
Mars . .	1.2680337	0.96	0.50	0.43	1	0	37.3
Jupiter . .	1.1054	0.24	2.69	0.037	0	9	55.5
Saturn . .	1.3500	0.14	1.26	0.011	0	10	29.3
Uranus. .	1.17918	0.24	1.05	0.003			

Surfaces and Volumes of the Planets in Geographical Miles.

Planets,	Surfaces in Square Miles.	Volumes in Thousands of Cubic Miles.
Mercury.	22,272,000	10,210,816
Venus	144,048,000	163,373,248
Earth, ...	148,512,000	170,180,672
Mars, ...	40,096,000	23,825,280
Vesta, ...	148,500	832
Juno, ...	4,448,000	850,880
Ceres, ...	5,936,000	1,361,472
Pallas, ...	10,400,000	2,893,056
Jupiter, ...	18,710,496,000	230,635,198,720
Saturn, ...	12,088,880,000	124,082,652,800
Uranus, ...	2,802,416,000	13,954,799,360
Sun, ...	1,865,312,000,000	239,460,953,408,000
Moon, ...	10,350,400	3,024,812

Velocity and Fall of the Planetary Orbits; their position with respect to the Sun's Equator.

Planets.	Mean Velocity in a Second.	Fall towards the Sun in a Second.	Right Ascension of the External Node of the Orbit.
Mercury . . .	26.8 Eng. miles.	8.46	316° 51′
Venus . . .	19.6	2.42	242 45′
Earth	16.4	1.27	248 0′
Mars	13.6	0.55	254 21′
Vesta	10.8	0.20	180 33′
Juno	10.4	0.20	197 3′
Ceres	10.0	0.20	208 43′
Pallas	10.0	0.20	182 19
Jupiter . . .	6.8	0.047	242 5′
Saturn . . .	5.2	0.014	231 12′
Uranus . . .	4.0	0.003	247 30′
Neptune . . .	2.8		

Inclination and Motion of the Planetary Orbits.

Planets.	Inclination of the Orbit to the Sun's Equator.	Arc of the Retrograde Motion.		Duration of the Retrograde Motion.
Mercury . . .	2° 54′	8° 33′	16° 18′	20 and 24 days.
Venus	4 9	15 20	16 31	41 " 43 "
Earth	7 30			
Mars	5 50	11 8	19 30	62 " 81 "
Vesta	4 28			
Juno	16 28			
Ceres	3 43			
Pallas . . .	37 8			
Jupiter . . .	6 24	10 0		119 "
Saturn . . .	5 57	6 48		137 "
Uranus . . .	6 44	3 36		151 "

The most important items respecting the moons of Jupiter, Saturn, and Uranus, will be found further on, principally in section 33. Sections 30 and 31 will contain the principal points in the history of our moon.

Annual Revolution of the Earth around the Sun, and Various Phenomena resulting from this Revolution.

26. The central figure on *pl.* 8 is intended to exhibit clearly, with many other phenomena, the annual revolution of the earth about the sun; nevertheless, for the better knowledge of the whole subject, it will be necessary to premise a few general observations. It is, in the first place, evident that bodies cannot themselves change their condition, and that thus a body once set in motion can never stop—that it will continue to move in the same direction and with the same velocity as when it set out, unless some other external force changes its direction or velocity. This peculiarity of

bodies is called inertia. It is, moreover, evident, by reference to what is said, sec. 46, of the parallelogram of forces, that if the paths of the planets, and consequently of the earth, also be ellipses about the sun, two forces must combine to their production. The one is the *attractive force* of the sun, varying with the distances; the other a continuous *tangential force,* originating in an impulse. *Pl.* 10, *fig.* 2, will show how an elliptical orbit for each planet is produced by the co-operation of these two forces. We have to remark, finally, that gravitation communicates to all freely falling bodies a tendency towards the centre of the earth.

We will now direct our attention to the orbit of the earth, with the help of the figure in *pl.* 8, representing its motions. By measuring the distances of the earth from the sun, at different times of the year, the shape of its orbit has been ascertained. These distances, as they were unequal, could not, of course, be semi-diameters of a circle, but they corresponded, taken together, to the radii vectores of an ellipse (*pl.* 10, *fig.* 2). The mean distance of the earth from the sun, is about 95,103,000 (statute) miles; it moves in its orbit at the average rate of $16\frac{4}{10}$ (Eng. geog.) miles in a second; the eccentricity of its orbit amounts to 0,016,784; the least or perihelion distance from the sun, to 81,276,000 (Eng. geog.) miles, or 33,917,997 hours; while the greatest or aphelion distance is 84,052,000 (Eng. geog.) miles, or 35,085,379 hours. The straight line connecting the perihelion and aphelion, passing through the centre of the sun, is called the line of *apsides.* The inclination of the earth's orbit to its equator, or the so-called *obliquity of the ecliptic,* amounts to 23° 27′. The velocity of the earth is greatest at the perihelion and least at the aphelion. It is further to be observed, that the mean distance of the earth from the sun is equal to half the major axis of the earth's orbit, and the line of apsides is itself the major axis. There are four noteworthy points in the earth's orbit; they are those which mark the beginning of the four seasons. Two of these points are called the *solstices*—they mark the beginning of winter and summer. The straight line (*pl.* 8) uniting them, passing through the centre of the sun, is called the *solstitial colure.* The two other points are the *equinoxes,* vernal and autumnal, marking the commencement of spring and autumn. The straight line connecting these points, passing through the centre of the sun, is the *equinoctial colure.* It still remains to observe, that the axis of the earth being always parallel to the axis of the heavens, may also be conceived to coincide with it; for, in consequence of the great distance of the fixed stars from our sun, the diameter of the earth's orbit (of more than 190,000,000 [statute] miles), as well as the whole orbit itself, would be seen as a mere point at the stars.

The representation on *pl.* 8, shows the position of the earth on the first day of each of the twelve months of the year, the solar distances corresponding to these twelve positions, and the shape of the earth's orbit. The horizontal projection has been chosen, in order to represent to the eye the increase and diminution of days, and the variation of illumination about the pole of the earth. The deeper circle surrounding the pole at a short distance, is intended to represent the parallel of latitude of Paris, or the hour

circle of that place divided into twenty-four hours. Although at the end of December the earth is nearest the sun, yet at that time in the northern hemisphere, the heat is less than at any other. The reason of this lies in the fact of the short days and long nights, as well as that the sun's rays fall very obliquely on the earth, traversing a longer path through the atmosphere, and consequently losing much of their heating power. At the beginning of July, on the contrary, although then the earth is at its greatest distance, the temperature of the northern hemisphere is greatest, on account of the long days and short nights, and the great altitude of the sun at noon. This, of course, depends upon the declination of the sun north or south from the equator. This declination of the sun for the first day of every month, is given in *pl.* 8. The great inner circle contains the division of the year into days and months, and enables us, by drawing a straight line to any point of this circle, to find the situation of the earth on the day corresponding to the point. The external circle, on the contrary, is divided into twelve equal arcs, of which each one answers to a *sign* (30 degrees) of the ecliptic. It must not be forgotten, however, that owing to the precession of the equinoxes, these signs no longer correspond to the constellations of the same name, so that now the sign Pisces corresponds to the constellation Aries, the sign Aries to the constellation Taurus, &c. It is further evident from an inspection of the *plate*, that if the earth at the beginning of spring, summer, autumn, and winter, should be in the signs Aries, Cancer, Libra, and Capricornus respectively, then the sun, as being always directly opposite in the ecliptic, will be in the signs Libra, Capricornus, Aries, and Cancer.

By properly combining the preceding with sections 27, 28, which are devoted principally to an explanation of the theory of the seasons, and particularly of the daily and yearly motion of the sun, it will not be difficult to obtain a perfect idea of all the phenomena occurring in the course of a year, over the whole surface of the earth. On account of the great importance of this subject, it may be remarked further, that the changes of days and nights, as also of the seasons, may be very easily represented by means of a *Tellurium*, or by means of a terrestrial globe, with a little simple additional mechanism.

The inner space of *pl.* 8, is employed to represent the orbits of the two inferior planets, Mercury and Venus, in their proportional size, shape, and eccentricity. The accompanying figures are readily intelligible without further explanation.

27. The sun is stationary! It is incredible that so enormous a body as the sun should have three different motions at the same time. Accurate observations have shown that the earth moves about its axis from west to east, once every day, by means of which the daily apparent motion of the heavens, as well as of the sun, from east to west is produced. It is furthermore only apparently, not really, that the sun in the course of a year moves around the earth from west to east in an ellipse termed the *ecliptic* (*pl.* 9, *fig.* 3, LCD), making an angle with the equator of 23° 27′. This inclination, termed the *obliquity of the ecliptic*, combined with the revolution of the earth, explains the apparent motion of the sun, both towards the north and

towards the south. If, for instance, we suppose the sun on June 21st to have reached the point L, it is evident that while the earth rotates about her axis, *ax*, from west to east, or in the direction from Q to U, it will seem as if the sun in the same time, but from east to west, had described the parallel circle, VOLV (or the tropic of Cancer), about the earth; wher the sun is at R, he will appear to describe the equator CRFC; and on December 21st, the tropic of Capricorn or the circle DHMD. The planes of the equator and tropics intersect the earth in circles, which are respectively the terrestrial equator and the terrestrial tropics. It is further to be observed that the poles, N, K, of the ecliptic, during the apparent daily rotation of the heavens, describe two small circles, of which the northern, AN, is called the *arctic*, the southern, BK, the *antarctic*. The polar circles of the earth corresponding to these are *u*, *y*, and *k*, *b*.

The rotation of the earth on its axis, and the inclination of the ecliptic to the equator, explain without any difficulty the inequality of days and nights, and the succession of the seasons. For when the sun on the 21st June traverses the tropic of Cancer, he remains much longer above the horizon *zz* than when describing the tropic of Capricorn on Dec. 21. In this latter case we readily perceive that the sun remains much longer below the horizon, and that consequently the nights are much longer than on the 21st of June. It is further evident, that about the 20th of March and 23d of September, when the sun is on the equator, an equality of days and nights must take place. *Figs.* 1 and 3, *pl.* 9, also show that during the six months that the sun is north of the equator, spring and summer must take place in the northern hemisphere, autumn and winter in the southern; but during the six months that this luminary is south of the equator the case must be reversed, spring and summer now happening in the southern hemisphere, autumn and winter in the northern.

28. All that has hitherto been said with regard to phenomena occurring in connexion with the earth, can only be explained by the assumption of a rotatory motion of the earth about an axis at the same time with a revolution about the sun in an elliptical orbit (*pl.* 9, *fig.* 1). The inequality of the seasons is a necessary consequence of the elliptic motion of the earth, and of the inclination of its axis of rotation to the plane of the ecliptic. If, for instance, the axis of rotation were perpendicular to the plane of the ecliptic, then there would be no change of seasons, but rather a single continuous torrid, temperate, or frigid zone. In this case also the planes of the equator and ecliptic would coincide, and as the sun would remain constantly in this plane, the days and nights would be equal the whole year through, and the poles of the earth be illuminated by only half the sun's disk. The equatorial regions would have a burning summer continually, the temperate zones would have an eternal spring, and the polar lands would experience without intermission intense cold, in which ice would never melt. But the axis of the earth, ever parallel to itself, is inclined to the plane of the ecliptic 66° 33′, and thence follows,—1st, a progressive difference in the length of the days and nights for all points of the earth's surface from the equator to the poles, and from the first day of the year to the last; 2dly, an increase and diminu-

tion of temperature for the northern and southern hemispheres, in proportion as these are turned more or less to the sun. *Pl.* 9, *fig.* 1, shows that the earth (revolving in the direction of the arrows), wherever in her orbit she may happen to be, always has her axis constantly parallel to itself and at the same angle of inclination to the plane of her orbit. The earth is found at A, on Dec. 21, in the beginning of winter, the shortest day of the year for our northern hemisphere. The sun is then at his greatest distance south, describes the tropic of Capricorn, and at noon stands in the zenith to all inhabitants of the earth whose latitude is 23° 27′ south. The north pole, nevertheless, lies in the middle of its six months' night. The boundary of the earth's shadow will fall in the north polar circle, whose inhabitants will then have a night of 24 hours. On the other hand, this will be summer to the southern hemisphere, and its inhabitants will have the longest days. This point, A (*fig.* 1), of the earth's orbit is the winter solstice; the earth, which now, seen from the sun, stands in 0° of Cancer, will traverse the part AB of its orbit in 89 days, from Dec. 21 to March 20, thus marking the duration of the winter.

B is the position of the earth on the first day of spring (autumn for the southern hemisphere): the sun then describes the equator (see the direction of the equinoctial line), and as the shaded portion of the earth divides the parallel circles into two equal parts, the days and nights will at this time be equal all over the world. At the north pole the long day of six months is just commencing. The earth, standing in 0° of Libra, now traverses the part BC of its path in 93 days, from March 20th to June 21, marking the duration of spring.

C is the position of the earth on June 21, the first day of summer (of winter in the southern hemisphere); the sun then describes the tropic of Cancer, and the north pole lies in the middle of its day of six months. The inhabitants of the north polar, or arctic circle, see the sun for 24 hours, and all other dwellers on the northern hemisphere have longer days than nights. The south pole lies in the middle of its night of six months, and the inhabitants of the southern hemisphere have longer nights than days. C is the summer solstice, and the summer lasts 94 days, that is, the time from June 21st to Sept. 23, during which the earth passes from C to D. The earth itself, on the 21st June, as seen from the sun, stands in 0° of Capricornus.

Finally, on the 23d Sept., the first day of autumn (spring for the southern hemisphere), the earth is at D, the sun standing again in the equator, the days and nights are again equal all over the earth. The sun now becomes invisible to the north pole, and visible to the south pole, the autumnal equinox occurring on Sept. 23d, the vernal on March 20. Autumn lasts 89 days, that is, the interval from Sept. 23d to Dec. 21, in which time the earth traverses the distance DA of its orbit.

From the preceding explanation it is evident that the four seasons are not of equal duration, for spring and summer together embrace 187 days, while autumn and winter last only for 178 days. There is thus a difference of nine days between the times occupied by the earth in traversing BC, CD, and DA, AB. This difference of nine days is a consequence partly of the

eccentricity of the earth's orbit, partly of the varying velocity with which the earth moves around the sun.

The Transit of Mercury and Venus across the Disk of the Sun.

29. When the inferior planets, Mercury and Venus, during their revolution around the sun, come into inferior conjunction, and at the same time into or not far from the imaginary straight line drawn through the centres of the sun and earth, they will, if examined through a telescope, be seen as dark spots passing over the sun's disk (*pl.* 14, *fig.* 55). These *transits* of Mercury and Venus belong to the rarer celestial phenomena, since they evidently can only take place when inferior conjunction occurs near one of the nodes of their orbits. It is plain that with regard to the origin and progress of these transits, the conditions are precisely the same as in eclipses of the sun, which latter might be called with equal propriety, transits of the moon over the sun's disk. Astronomy teaches that Venus at her inferior conjunction must be within 1° 49′ of one of her nodes, and Mercury within 3° 28′, for a transit to occur to any observer on the earth's surface. These two limits, then, determine the periods of these occurrences, which for Venus are 8 and 113½ years, and for Mercury 6, 7, 10, and 13 years. Thus, transits of Venus happened on June 5, 1761, and June 3, 1769; the next will take place December 9, 1874, and December 6, 1882, June 7, 2004, and June 5, 2012. *Pl.* 9, *fig.* 6, exhibits the 13 transits of Mercury during the present century, with its direction each time over the sun's disk. It is to be remembered that Mercury as well as Venus will enter on the left (eastern) limb of the sun, and emerge on the right (western) limb, for the reason that at the time of their inferior conjunction these planets are retrograding. The reason of the transits of Venus occurring in the beginning of June and December, and those of Mercury only at the beginning of May and November, lies in the fact, that at these times the earth is in the line of nodes of each planet respectively. On account of the small apparent diameters of Venus and Mercury, these phenomena could not be detected before the discovery of the telescope. The first transit of Mercury was observed by Gassendi at Paris, November 7, 1631; the first transit of Venus by Horrox, at Hoole, in England, December, 1639.

The observation of the transit of Venus is to the astronomer of vast importance, as it is almost the only certain way of obtaining the sun's parallax, and hence of finding the mean distance of the sun from the earth. Halley first recognised this fact, and recommended these transits to the observations of astronomers. His suggestion was followed out when the next transits occurred in 1761 and 1769, and at these times observations were made in many places. The transit of 1769 was observed in the South Seas, in California, as also in the northern regions of Asia. Venus at the time of her inferior conjunction is very near the earth, and consequently seen from different places on the earth's surface, will be referred to very different points on the sun's disk, so that first of all the parallax of both Venus and

the sun can be obtained. According to Encke, who has fully and most accurately carried out the calculation, the mean horizontal parallax of the sun at the equator amounts to $8\frac{5776}{10,000}$ seconds, and consequently the mean distance of the earth from the sun to 95,103,000 (English) miles. That the transits of Mercury are not available in determining the sun's parallax, is readily intelligible, when we know that the parallax of Mercury at its inferior conjunction is not very different from that of the sun, while the parallax of Venus at inferior conjunction is almost four times as great as that of the sun.

Pl. 9, *figs.* 7 and 8, represents the individual phenomena of the transits of Mercury for the whole earth, as they occurred May 4, 1786. The obscure portions in *figs.* 7 and 8 cover those regions in which the transit was visible, the bright portions answer to the countries in which it was invisible. Those places lying in the boundary of the obscure and light parts, observed, as indicated in the figures, either an entrance or emersion at sunrise only, or an entrance or emersion at sunset only.

Additional Remarks on the Course of the Moon.

30. We have already said all that is essential with respect to the origin of the moon's phases, the inclination of the moon's orbit to the ecliptic, the retrograde motion of the moon's nodes, as also the causes of solar and lunar eclipses. There still remain a few additional considerations respecting the moon's course. The principal figure on *pl.* 10, *fig.* 5, contains the phases of the moon, or the various aspects under which she presents herself to us. The earth being in the centre of the external circle, the moon's orbit is so placed, with reference to this circle, as readily to exhibit its eccentricity. The moon revolves in this orbit from west to east around the earth, and the figure represents her in her proper proportion to the earth and in the eight principal points of her course. The sun may be supposed to be stationed at a distance to the right of the earth in *pl.* 10, exceeding 410 times that of the moon from the earth. In order to exhibit the phases of the moon in their fullest conditions, they are represented within the moon's orbit in much larger proportion. The orbit of the moon is properly an ellipse, the earth standing in one of the foci. Its eccentricity amounts to 0.0548442; the greatest distance of the moon from the earth (that is at time of the apogee), to 63.842 semi-diameters of the earth, and the least, or that at time of perigee, to 55.916 semi-diameters. Both perigee and apogee retrograde from evening to morning about 40° 42′ annually. The moon appears to revolve in 24 hours from east to west around the earth, which, with the apparent daily motion of the heavens, is produced by the rotation of the earth on her axis. Again, the moon, as full moon during the nights of summer, appears to describe a very small arc, and during the nights of winter a very large arc above the horizon, which is explained in the following manner: Let us suppose the night to be that of the winter solstice (*pl.* 9, *fig.* 1), on December 21, consequently the longest night in the year. Let now the

earth (*pl.* 10, *fig.* 4) be opposite the sun, situated in the tropic of Capricorn, which it describes on this day, and the moon, as full moon at M, in the tropic of Cancer, which she describes; our horizon, HR, shows that the diurnal arc of the sun is very small, while the moon traverses by night a very large arc. The contrary of this must take place on the night of the summer solstice, June 21, the shortest night of the year. Should the moon, as full moon in spring and autumn, stand in the equinoxes as the sun, she would then be as long above as below the horizon.

31. If we suppose the moon to be at L′ (*pl.* 10, *fig.* 3), in conjunction or between the earth and sun, then the centres of these three bodies will be in the straight line RE. While the moon is completing a revolution around the earth, moving daily about 13° 10′ 35″, the earth will have passed forward over a certain part of her orbit, about to *t*. The moon will then be at *n*, in a direction, *xz*, which is parallel to the former, RE. The moon has consequently a periodical or tropical revolution completed within 27 days, 7 hours, 43 minutes, $4\frac{7}{10}$ seconds. For the moon to return to conjunction, however, that is, again to become new moon, it must in addition traverse the arc *n*L, equal to the arc T*t* or R*m*, described by the earth in its orbit, which arc amounts to about 27°. To accomplish this the moon requires somewhat more than two days; she consequently returns to conjunction in 29 days, 12 hours, 14 minutes, and 3 seconds. This period is known as the lunar month (in its proper sense), and the revolution itself (from conjunction to conjunction) is the synodic revolution, or simply the lunation, which may also be counted from one full moon to another. *Pl.* 10, *fig.* 10, shows that the course of the moon projected on the plane of the earth's orbit, must form a kind of serpentine. The moon has yet to pass over the arc *nv* (*fig.* 3), to come back to the same fixed star; this return, accomplished in 27 days, 7 hours, 43 minutes, and 12 seconds, is called the *sidereal revolution* of the moon. Finally, the revolution of the moon, with respect to the nodes of its orbit, occupies 27 days, 5 hours, 5 minutes, and 36 seconds.

The moon revolving about our earth is forced to accompany her in her course round the sun, so that the path traversed by the moon in space is properly an epicycloid. This compound motion of the moon is, consequently, the source of the various phases represented in *fig.* 5, as already explained in sections 7 and 29. *Pl.* 10, *fig.* 5, shows, in addition, the first, second, third, and fourth octants, as also that the moon, when full, is in opposition to, and when new, in conjunction with the sun. The inclination of the moon's orbit to the earth's equator is very variable, ranging in 19 years from 18° 19′ to 28° 36′. The inclination of the moon's equator to the ecliptic (1° 28′ 25″) never varies. The time of the rotation of the moon about her axis corresponds precisely with that of her mean revolution around the earth, consequently equal to 27 days, 7 hours, 43 minutes, and 12 seconds. Hence the moon always turns the same side to us, and the opposite side is constantly concealed, except the small part of it revealed by *libration*. For this reason the conclusion was early formed, though too hastily, that the moon had no rotation.

The Primary Causes of the Elliptical Orbits of the Planets; Kepler's Laws.

32. In the explanation of the yearly course of the earth about the sun (by means of the figure on *pl.* 8), it was mentioned that the paths of the planets, and consequently those of the earth and moon, are ellipses, which are produced by the co-operation of two special forces. How that takes place will be explained hereafter. Let us suppose that any body (as ♏, *pl.* 10, *fig.* 2) once set in motion is impelled by two forces; let the line ♏P represent the direction and intensity of the one force, the line ♏V the direction and intensity of the other. It is evident that the body ♏ will not move towards S, the sun alone, nor towards x alone. It must rather (see what is said, section 26, about compound motion) follow the direction ♏o, and pass to the point ♐; the force represented by ♏P is the attractive force of the sun at S, and the force represented by ♏V, is the tangential force produced by an impulse. As the ever-varying central force, namely the attraction of the sun, is constantly acting upon this tangential force, this must also vary. The curvature ♏ ♎, ♏ ♌, of a planet's orbit, produced by the co-operation of these two forces in the first, and in all following moments, must manifestly depend upon their relative proportion. The central force again depends upon the distance of the planet from the sun. Should the original velocity of the planet in the first second be exactly equal to the planet's fall towards the sun in the same second, then the ratio will be 1 : 1, and the orbit will be a circle. It is perfectly plain, however, and much more probable, that the original velocity may have been a little greater or less than what would be necessary to the production of a circular orbit. Then the planet would move in an ellipse (*pl.* 10, *fig.* 2), in which the point ♉, where the planet started, would be the perihelion if the projectile force had been the greatest, for it would recede from the sun from the very beginning of its motion. But if the planet had started in the point ♋, the projectile force must have been the lesser of the two, and the point ♋ would be the aphelion. The higher mechanics shows that ellipses arise when, beginning at the perihelion, the original velocity amounts to from $51\frac{96}{100}$ to $73\frac{36}{100}$ English geographical miles in a second, and that ellipses likewise arise when, beginning in the aphelion, the original velocity amounts to from $\frac{4}{100}$ to $51\frac{88}{100}$ geographical miles in a second. It has actually been found that the planets (and their moons), whether starting in their perihelion or aphelion, must have had initial velocities falling within the above limits, and consequently must describe elliptic orbits.

If the tangential force operate on the point ♏, in the direction V♏, and the attractive force of the sun in the direction P♏, the point ♏ will move in the direction ♏ ♎ to ♎. At ♎ the tangential force operates in the direction B♎, and the central force in the direction p♎, therefore the point ♎ now moves in the direction ♎♍ to ♍. At ♍ the tangential force acts afresh in the direction D♍, and the central force in the direction k♍, consequently the point ♍ now moves in the direction ♍♌ to ♌, &c. It is hence evident that the planet must describe an ellipse, not, however, the broken

line ♏♎, ♍♌. For we may suppose the parallelogram of forces to be constructed anew every successive moment, and consequently infinitely small, so that instead of the broken line, a continually curved one, namely an ellipse, will be produced, since both forces, the central and tangential, operate incessantly upon the planet. It is only for the more intelligible illustration of the subject that the single parallelograms in *fig.* 2 are represented on so large a scale; the eccentricity, CS, is, for the same reason, assumed tolerably great, as the real eccentricity of the planetary orbits is much less.

The proposition that *the planets describe ellipses, the centre of the sun being in one of their foci*, is the first of the three celebrated laws of Kepler, upon which the whole theory of the planetary motions depends. The second law (discovered, however, first) is, that *any two areas (sectors), SQq and SQ′q′, described by the radii vectores, SQ* and *Sq, SQ′* and *Sq′, are proportional to the times* (*pl.* 6, *fig.* 4). This law may also be expressed in this manner: The different velocities of a planet are as the squares of its different distances from the sun. Suppose the planet to describe the distances Q*q*, and Q′*q*′ of its orbit in equal times, then the elliptical sectors, SQ*q* and SQ′*q*′, will have equivalent areas. Hence it follows, that if the areas of the sectors at the perihelion B and at the aphelion A are to be equivalent, the arc described by the planet at the perihelion must be greater than that described in equal time at the aphelion. Thus the planet must move with the greatest velocity at the perihelion, and the least at the aphelion, and these two different velocities are as the squares of the distances SA and SB. In general, the velocity of a planet's motion must be greater as it approaches the sun, and less as it recedes from it.

The first law of Kepler expresses the character of the curve described by the planets; the second, the varying velocities of the planetary motion; while the third law is a bond of union connecting the different planets together. This third law is expressed as follows: *the squares of the times of revolution of two planets are as the cubes of their mean distances from the sun.* The great value of this law consists in its presenting a geometrical proportion, so that knowing three of the four elements, the mean distances from the sun, and the periods of revolution, the fourth can always be obtained. In conclusion, the laws of Kepler are true laws of nature, since Newton has demonstrated that they are only consequences of that single and supreme law discovered by him—*the law of universal gravitation.*

The Moons of Jupiter, Saturn, and Uranus.

33. The planet Jupiter is accompanied by four moons in his journey of 11½ years around the sun. Immediately after the discovery of the telescope, Simon Marius (at Ansbach), in November, 1609, observed four small stars very near to Jupiter, which, almost always in a straight line with him, appeared sometimes to the right, sometimes to the left, never separating

far from him. Marius observed them carefully, and in March, 1610, was convinced that those four small stars were moons of Jupiter. Galileo observed them for the first time on January 10th, 1610.

Indicating the satellites of Jupiter by I., II., III., IV., their mean distances from the centre of Jupiter are as follows:—232,000 (English) geographical miles for I.; 372,000 for II.; 592,000 for III.; and 1,040,000 for IV. The eccentricities of their orbits (*pl.* 10, *fig.* 6) are inconsiderable, as also their inclinations. The sidereal period of I. is 1 day, 18 hours, 28 minutes; of II., 3 days, 13 hours, 14 minutes; of III., 7 days, 3 hours, 43 minutes; and of IV., 16 days, 16 hours, 32 minutes. These are uncommonly short, and consequently eclipses of Jupiter's moons occur with remarkable frequency. We often see one or another moon vanish suddenly and re-appear on the eastern side after the lapse of some hours. A tolerably attentive examination soon shows that such an eclipse of the moon is produced by the shadow of the primary. This also shows incontestably that Jupiter and his four moons are opake bodies, deriving all their light from the sun.

By the assistance of a good telescope, it will frequently be observed that these moons enter Jupiter's disk on the eastern border, moving towards the western border, accompanied by circular dark spots, going in the same direction and with the same velocity. These spots are evidently nothing else than the shadows of the moons cast from them upon the surface of Jupiter. These phenomena are consequently eclipses of the sun to Jupiter, produced by his moons. The maximum duration of the eclipses amounts for satellite I., to 2 hours, 16 minutes; for II., to 2 hours, 52 minutes; for III., to 3 hours, 34 minutes; and for IV., to 4 hours, 45 minutes. In one year of Jupiter, that is in almost 12 of our years, 4,400 eclipses of the moons, and as many of the sun, may be observed. The beginning and ending of the same eclipse of I. and II. are never both seen, as before the opposition to Jupiter only the beginning, and after it, only the ending are observed. On the other hand, both beginning and ending in III. and IV. may be perceived. With respect to eclipses of the sun, it is to be remarked that at the time of visible beginning, the shadows follow the satellites, and precede them at the time of ending. In conclusion, one moon of Jupiter may sometimes, though rarely, eclipse another.

Observations of the so frequently occurring eclipses of Jupiter's satellites offer an exceedingly ready means of determining geographical longitude. Unfortunately, the moment of such an eclipse, just as in the case of an eclipse of our moon, will be observed very differently at different places, owing to the unequal illumination and magnifying power of telescopes, and the different acuteness of sight of the several observers. It was the observation of the eclipses of Jupiter's satellites that led the Danish astronomer, Olaus Römer (in the latter half of the seventeenth century), to the discovery of the velocity of light.

When Jupiter is at his mean distance from the earth, the diameters of his moons appear to us respectively at angles of $1''.02$, $0''.91$, $1''.49$, $1''.27$. Hence the apparent diameters of his moons to Jupiter will be $31' \, 11''$,

17′ 35″, 18′ 0″, and 8′ 54″. The true diameters are for I., 2,120; for II., 1,880; for III., 3,120; and for IV., 2,640 English geographical miles; their densities are $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{10}$, and $\frac{1}{4}$ that of the earth.

34. The planet *Saturn*, on his 30 years' journey around the sun, is accompanied by seven moons, revolving round him from west to east. Huyghens discovered one of these moons, namely, the sixth, March 25, 1655. Cassini found the seventh and most distant on October 25, 1671; the fifth on December 13, 1672; as also in March, 1684, the third and fourth. One hundred years after, August 28 and Sept. 17, 1789, Herschel discovered the two satellites nearest the planet. The orbits of the six inner moons (*pl.* 10, *fig.* 7) are nearly circular, and lie almost entirely in the plane of Saturn's ring; the orbit of the seventh, however, lies nearly in the plane of the ecliptic. Their periods of revolution are 0 days, 22 hours, 38 minutes; 1 day, 8 hours, 53 minutes; 1 day, 21 hours, 18 minutes; 2 days, 17 hours, 45 minutes; 4 days, 12 hours, 25 minutes; 15 days, 22 hours, 41 minutes; and 79 days, 7 hours, 55 minutes. Their mean distances from Saturn amount, in geographical miles, to 76,680, 99,640, 163,880, 211,680, 295,480, 642,840, 2,098,744. With regard to the true magnitudes, masses, and densities of the satellites of Saturn, nothing satisfactory is known, as these moons are among the smallest and most remote objects of the heavens. Schröter estimated the true diameter of the fifth and sixth satellites at 1,040 and 2,720 geographical miles. It is only since 1830 that the sixth moon has received a more accurate determination by Bessel.

The first and second moons had only been seen by their discoverer Herschel, until in 1836 Camont found the second, and in June 27, 1838, the astronomers at Rome were enabled to observe the first. The seventh or outermost moon revolves about Saturn at the great distance of 2,098,744 geographical miles, and has the remarkable peculiarity of almost entirely vanishing when to the east of Saturn, and of shining brightest in its western elongation. This is probably produced by the fact of its completing a rotation about its axis in the time that it is accomplishing a revolution about Saturn, presenting the same side to the earth when it comes into the same position with respect to its primary—the one side reflecting the sun's light much more completely than the other, consequently the equality of the time of rotation and revolution in the secondary planets appears to apply to these bodies also. The moons of Saturn are sometimes eclipsed, and sometimes produce eclipses of the sun to inhabitants of Saturn; nevertheless, both kinds of eclipses, which always follow closely the time of the disappearance of the ring, occur more rarely than in the case of Jupiter, a consequence of the great inclination of their orbits to that of Saturn (*pl.* 10, *fig.* 7). Yet eclipses of Saturn's moons often occur among themselves, and are also caused by the ring.

35. Uranus, on his 84 years' journey about the sun, is accompanied by six moons (*fig.* 8). Herschel, on January 11, 1787, discovered the second and fourth; on January 18, 1790, the first; on February 9, 1790, the fifth; on February 28, 1794, the sixth; and on March 26, 1794, the third satellite of Uranus. These moons are at the following successive distances from their

primary, expressed in geographical miles, 196,000, 254,000, 296,000, 339,600 680,000, and 1,360,000. Their magnitudes, so difficult to determine, must be very great to make them visible at so immense a distance from the earth. The plane of their orbits is almost perpendicular to that of the orbit of Uranus (*fig.* 8); and it is remarkable that these six moons have the unique motion from north to south. The inclination of the equator of Uranus to its orbit, or the obliquity of his ecliptic, is very nearly a right angle, whence all difference of zones must disappear; while, on the other hand, that of seasons must be very great. When near one pole of Uranus, the sun stands during summer almost immovably in the zenith, and afterwards, for almost a year, describes a very small circle about the zenith; in like manner, the satellites present themselves a very long time in the first and last quarters. New moon and full moon only take place when a pole of Uranus has the sun in its horizon, and at this time alone can eclipses of the sun and moons occur.

In conclusion, it may be remarked that *figs.* 6, 7, and 8, *pl.* 10, represent somewhat in perspective, the systems of the moons of Jupiter, Saturn, and Uranus; and *fig.* 9, the orbit of our moon. The proportional size of the orbits is represented as accurately as the *small size of the scale would allow.*

Estimates of the proportional size of the Planets.

36. To obtain a clear idea of the relative size of the planets (Astræa, Neptune, and Iris excepted) and the sun, we may compare the scale at the bottom of *pl.* 14, representing the sun's radius, with the diameters of the circles representing the planets (*figs* 1 to 11). Taking the diameter of the sun, 2AB, at 770,944 geographical miles, that of the eleven planets in geographical miles, and the ratio of the planets' diameters to that of the sun, will be as follows:—

Figs.	Planets.	True Diameter.	Ratio of Diameters of Planets to Sun's Diameter.
1	Saturn,	62,072	1 : $12\frac{3}{5}$
2	Jupiter,	77,228	1 : 10
3	Uranus,	29,888	1 : $25\frac{4}{5}$
4	Earth,	6,880	1 : $112\frac{1}{10}$
5	Venus,	6,776	1 : $113\frac{3}{5}$
6	Mars,	3,572	1 : $215\frac{4}{5}$
7	Mercury,	2,688	1 : $286\frac{4}{5}$
8	Pallas,	1,800	1 : $428\frac{3}{10}$
9	Ceres,	1,360	1 : $566\frac{9}{10}$
10	Juno,	1,200	1 : $856\frac{2}{5}$
11	Vesta,	200	1 : $3,854\frac{7}{10}$

The following table exhibits the Relative Volumes of the Sun and Planets.

Planets.	Actual Volume in Millions of Geographical Miles.	Ratio of True Volumes of the Planets to that of the Sun.
Sun,	14,966,309,588	1 : 1
Jupiter,	15,039,700	1 : 995
Saturn,	7,817,666	1 : 1914
Uranus,	872,175	1 : 17,160
Earth,	10,636$\frac{4}{10}$	1 : 1,407,122
Venus,	10,210$\frac{3}{10}$	1 : 1,465,733
Mars,	1489	1 : 10,052,599
Mercury,	638	1 : 23,458,165
Pallas,	180$\frac{8}{10}$	1 : 82,778,261
Ceres,	85	1 : 176,489,500
Juno,	53$\frac{2}{10}$	1 : 281,321,609
Vesta,	$\frac{4}{77}$	1 : 288,101,459,569

37. *Pl.* 14, *figs.* 12–15, gives the four principal positions of Saturn and his rings, with respect to our earth, as they are perceived during the 29½ years' revolution of Saturn about the sun. *Fig.* 12 gives a view of Saturn and his rings at the time this planet is situated in the sign of Cancer (♋); *fig.* 13 represents them when—14¾ years later—Saturn is found in the sign of Capricornus ♑; *fig.* 14, when in the sign of Libra ♎; and *fig.* 15, when—14¾ years later—he is found in Aries ♈.

The dimensions of the rings, as also their distances from each other and from Saturn, will be found in sec. 58. This ring system is, however, no great source of gratification to the inhabitants of Saturn, as it is only visible in the middle equatorial regions of the planet. It does not shine by night, only during the day, or a little while after sunset and before sunrise, and even this in general only during the summer. For at night the whole ring is shaded by Saturn; and in winter, instead of illuminating the planet, it casts a shadow upon the side of the planet opposite to the sun, covering an area of many millions of miles, and lasting in part almost fifteen of our years. On the other hand, the inhabitants of the surface of the ring will perceive Saturn's hemisphere in their horizon, of enormous size, and should they be on the very edge of the ring, they will perceive the planet in their zenith about 20,000 times larger than the sun. The floor itself upon which they stand, reaches to the right and to the left, visible up to the heavens, closing beyond the planet, thus affording beyond all question an entirely unique and most magnificent spectacle.

38. Every planet revolving about the sun must have at one time a period of least, and at another one of greatest distance from the earth. In the first case the planet is said to be in its *perigee*, in the second in its *apogee*. The two inferior planets, Mercury and Venus, are in their perigee at inferior conjunction, and in their apogee at superior conjunction; the superior planets are in apogee at conjunction, and in perigee at opposition

On account, however, of the different eccentricities, as well as the different inclinations of the planetary orbits to that of the earth, the perigee and apogee for each planet cannot be always the same. The consequence of this is, that seen from the earth, the planets do not always appear of equal apparent size. Each planet must appear greatest in its perigee, and least in its apogee. The difference for all the planes is approximately represented in *pl.* 14, *figs.* 16–35, according to the scale AB, whose larger divisions measure 25 seconds, the smaller $2\frac{1}{2}$. This difference at the time of perigee is the following:—

Distance of the Planets from the Sun, and apparent Diameter at time of Greatest Perigee.

Figs.	Planets.	Distance in Millions of Geographical Miles.	Apparent Diameter in Seconds.
16	Venus,	20	62
18	Jupiter,	316	46
20	Saturn,	644	20
22	Mars,	28	23
24	Mercury,	160	12
26	Uranus,	1,392	4
28	Pallas,	88	$3\frac{1}{2}$
30	Juno,	80	$2\frac{1}{3}$
32	Ceres,	124	2
34	Vesta,	92	$\frac{4}{5}$

And for the time of Greatest Apogee.

Figs.	Planets.	Distance in Millions of Geographical Miles.	Apparent Diameter in Seconds.
17	Venus,	140	$9\frac{1}{2}$
19	Jupiter,	520	30
21	Saturn,	892	$15\frac{1}{2}$
23	Mars,	216	$3\frac{1}{3}$
25	Mercury,	120	$4\frac{2}{5}$
27	Uranus,	1,696	$3\frac{4}{5}$
29	Pallas,	368	$1\frac{1}{2}$
31	Juno,	360	$1\frac{1}{5}$
33	Ceres,	328	$\frac{4}{5}$
35	Vesta,	296	$\frac{2}{5}$

39. *Pl.* 14, *figs.* 36–45, exhibits approximately the apparent size of the sun, as seen from the planets at their mean distances from him. To this we will add the apparent diameters in seconds.

Apparent Size and Diameter of the Sun, as seen from the Planets.

Figs.	Apparent Size in Order.	Apparent Diameter.
36	Mercury,	1° 22′ 49″
37	Venus,	0 40 32
38	Earth,	0 32 3
39	Mars,	0 21 2
40	Vesta,	0 13 35
41	Juno,	0 12 0
42	Ceres and	0 11 34
	Pallas,	0 11 33
43	Jupiter,	0 6 10
44	Saturn,	0 3 22
45	Uranus,	0 1 40

The figures above are also constructed on the scale AB, only the greatest divisions amount to 1500, the smaller to 150, and the smallest to 15 seconds. Hence it follows that to Uranus the sun appears 49½ times smaller than to Mercury, and to Jupiter 5½ times smaller than to the earth.

Finally, *figs.* 46–54 show the true diameter of the earth's moon, and of these planets, compared with the true diameter of the earth. If in *pl.* 14, *fig.* 46, the diameter of the earth be taken as equal to 100 parts, then will the other diameters be as follows :—

Moon of the earth, *fig.*	47,	26 parts.
Venus,	48,	98
Mars,	49,	52
Mercury,	50,	39
Pallas,	51,	26
Ceres,	52,	20
Juno,	53,	18
Vesta,	54,	3

Principal Phases of a Transit of Mercury or Venus over the Sun's Disk, and manner of observing them.

40. The general theory of the transits of Venus and Mercury has been already explained in section 29, by reference to *figs.* 6, 7, 8, *pl.* 9. It still remains to show by *fig.* 55, *pl.* 14, the mode of observing such a phenomenon. When, as is generally the case, an astronomical telescope is employed, which represents all objects inverted, the attention of the observer, before the commencement of the transit, is to be directed to the right border of the sun's disk. The straight line, passing through the figure, represents the course that Mercury, for instance, takes across the sun. When arrived at *d*, the *first external contact* of the limbs of Mercury and the sun, or the so-called *external immersion*, takes place; at this time the

transit begins. Arrived at *c*, we have the *first internal contact* of the limbs of Mercury and the sun, or the so-called *internal immersion;* Mercury has now entered altogether on the disk of the sun. When the planet reaches *b*, *the second internal contact*, or the so-called *internal emersion*, takes place; at this moment the planet begins to leave the sun. Finally, the arrival of Mercury at *a*, brings about the *second external contact*, or the *external emersion*, the transit ceasing at this moment.

In reality, or when observed with the terrestrial telescope, these four phases follow in a direction from left to right; in other words, Mercury occupies in succession the points *a, b, c, d*. The case is precisely the same in a transit of Venus. The method of obtaining the sun's parallax, and hence the mean distance of the earth from the sun, from such observations instituted at different places on the surface of the earth, cannot be here intelligibly exhibited and explained, as geometrical considerations combined with trigonometrical calculations are absolutely necessary.

The Total Eclipse of the Sun, June 4th, 1788.

41. *Fig.* 56, *pl.* 14, furnishes a representation of the extent and course of the moon's shadow over the earth's surface during the existence of a total eclipse of the sun. For our illustration we have taken the total eclipse of the sun, which was observed on the 4th of June, 1788, in the eastern hemisphere of the earth. The figure represents, in the first place, a broad curved line or zone of intense black. This zone covers all places at which the total eclipse was seen. To all places in its very central line, the eclipse was annular and of longest duration, while the localities on its external border saw a total eclipse only for a moment. Parallel to this zone, and to the north and south of it, lines are drawn with these indications, 9, 6, and 3 digits obscuration, with corresponding shading. They include all those places where the eclipse affected 9, 6, and 3 digits of the sun's disk respectively; to the north of the zone of total obscuration, the sun being eclipsed on its inferior, and to the south of it on its superior limb, and this in proportion to the proximity of the place to the central zone. The upper and lower arcs, GH and JK, cut all those places which saw, for a moment only, the beginning or the end of the entire eclipse. The arcs uniting the ends of GH and JK, cut the places which—the westerly at sunrise, the easterly at sunset—perceived just half the eclipse (or 6 digits). The other curved lines cut all those places where the beginning, the middle, or the end was perceived at sunrise, or the beginning, the middle, or the end at sunset. A glance at the chart consequently shows that this eclipse was visible in the whole of Europe and Asia, except Kamtschatka, the greater part of Siberia, and the island Celebes, as also in north and middle Africa.

42. The explanation of the origin and progress of eclipses of the sun and moon has already been given in section 22. To project any other great solar eclipse upon a map, as has been done in *fig.* 56, it will be first of all necessary to call in the assistance of a terrestrial globe, and likewise to

obtain by previous calculation, several places on the earth's surface, where, for a given time of the place, a central (total or annular) eclipse will happen. In other words, it will be necessary to know beforehand the course of the centre of the moon's shadow over the earth's surface. Cut from thin brass plate or pasteboard a circle, whose diameter is equal to the diameter of the globe, multiplied by the difference of the *apparent* radii of the sun and moon, and divided by the parallax of the moon. Next, place horizontally upon the globe a ruler, and upon the ruler the above-mentioned circle. Let A be a place on the globe, which for a given local time has the sun in its zenith, and let B represent a place which, according to the preceding calculation, is to see a central eclipse at this same time; adjust the globe in such a position that A occupies its highest point, and for this position of the globe, move the circle upon the inner horizontal ruler in such a manner that the centre of the circle shall lie perpendicularly above the point B. If we suppose straight lines to be drawn from all points in the circumference of the circle, perpendicular to its plane, then those vertically under the plane will inclose that space on the globe which, for the time in question, will be covered by the full shadow of the moon. We must now place the still horizontal ruler in such a manner that the centre of the circle shall constantly be vertically above the successive points B′, B″, B‴, &c., which for given times have a central eclipse. The globe must be turned at the same time in such a manner that its highest point is continually that which at the given times has the sun in its zenith. We can thus mark the entire path of the full shadow and its bounds on the globe, from which it may be transferred to a map. Cut now a circle whose diameter is equal to the diameter of the globe, multiplied by the sum of the apparent radii of the sun and moon, divided by the moon's parallax, and proceed with this circle as before; we shall in this manner obtain all those places which lie before the northern and southern borders of the half shadow, and which consequently only perceive a contact of the edges of the moon and of the sun.

III.—Physical Astronomy.

Rotation of the Earth; its Spheroidal Form; Centrifugal Force; Simple Proof of the Spheroidal Shape of the Earth; Local Variation of Gravity.

43. The daily motion of the starry heavens is only apparent, being a consequence of the actual turning of the earth on its axis, called its *rotation.* This rotation proceeds in a direction from west to east, since we see the apparent rotation of the heavens taking place in the opposite direction, or from east to west. Let, in *fig.* 7, *pl.* 6, the greater circle, KIH*kih*, represent the stationary heavens, and the smaller circle, the rotating earth with its centre C. The point *o*, in the upper part of the circle, will then, by its rotation, be made successively to assume the positions *o*, *o*. The consequence will be, that the horizon of *o* will be first H*oh*, then J*oi*, then K*ok*,

&c. The place *o* will consequently first see the part H*kih* of the heavens, then JH*ki*, then KJH*k*, &c. Now, since we do not perceive the motion of the earth, we are led to imagine that it is the heavens that move, which, however, is only an illusion caused by the earth's rotation.

The flattened shape of the earth is also a consequence of its rotation. It is known from the theory of physics, that all parts of any body driven round in a circle with uniform velocity, endeavor to recede from the centre of this circle, which effort is called the *centrifugal force*. Let, in *fig.* 6, *pl.* 6, AMBN represent an elastic globe with an axis AB, passing through the centre C. If now the globe be turned rapidly about AB, all its parts will move the faster, the more remote they are from the poles A and B, or the nearer they are to the equator MN; A and B moving only on themselves as poles. With a more rapid motion there will be an increase of centrifugal force, and those parts of the globe lying near the points M and N will separate more from the axis of rotation AB. Hence the spherical globe, AMBN, will finally assume the ellipsoidal shape, *m*B*n*A, and appear depressed or flattened. The earth, when first set in motion, must have been in precisely the same condition as the above globe, assuming, however, that at that time its matter was in a fluid or semi-fluid state.

44. Thus the earth is not a perfect sphere, but an elliptical spheroid, in which the curvature of a meridian section at the equator is sensibly greater than at the poles, shown by measurements of degrees of latitude. Let NABDEF (*pl.* 6, *fig.* 9) represent a meridian section of the earth, C its centre, NA, BD, and GE, each a meridian arc, corresponding to a degree of latitude or to a degree of change in the meridian altitude of a star. Finally, let *n*N, *a*A, *b*B, *d*D, *g*G, *e*E, be the direction of the plummet at the places N, A, B, D, G, E, of which N is situated in the pole, and E in the equator. If now any two neighboring vertical lines, as *n*N and *a*A, *b*B and *d*D, *g*G and *e*E, be prolonged to their intersections in X, *y*, *z*, then the angles NXA, B*y*D, G*z*E, will each amount to a degree, and consequently be all equal. Thus the small arcs NA, BD, GE, may be considered as circular arcs described about X, *y*, *z*, as centres. The points X, *y*, *z*, are called the *centres of curvature*, and the lines XN or XA, *y*B or *y*D, and *z*G or *z*E, *radii of curvature*, by which the curvature at these points is determined and measured. Geometry teaches us that the intersections of all these vertical lines do not, as in the sphere, all fall in C, but must lie in a certain curve, X*yz*, called the *evolute*. Experience has now shown that the terrestrial meridian is an ellipse, having for its major axis the equatorial diameter EF (*fig.* 9), and for its minor axis, the axis of the earth NS. This agrees also with the ratio of increase of the degree from the equator towards either pole. The radius of curvature at E is the least, that at N the greatest. The dotted lines in *fig.* 9 represent the parts of the evolute belonging to the other quadrants. It is to the celebrated Bessel that we owe the most recent and authentic results from measurements of degrees. According to him, a mean degree of the meridian is equal to 57013.109 toises (364,576 English feet); half the major axis, or half the equatorial diameter to 3272077.14 toises (20,923,624 English feet); and half the minor

axis, or half the axis of rotation to 3261139.33 toises (20,853,681 English feet). Consequently the flattening of the earth's spheroid amounts to almost $\frac{1}{300}$.

45. One important consequence of the centrifugal force is the local variation of gravity. It has actually been observed that there is a difference in the gravity or weight of the same body, when brought in succession to places of different geographical latitude. The methods by which these variations of gravity can be indicated, and their magnitude determined, are both statical and dynamical. The statical method consists in bringing the gravity of the weight in equilibrium with some natural force of entirely different character, upon which the local situation exerts no influence. Such a force is the elasticity of a spring. Let ABC (*pl.* 6, *fig.* 10) represent a strong beam of brass, standing on a firmly connected foot, AED. In this latter a flat plate of agate is inserted, and the whole foot rendered capable of being placed in a perfectly horizontal position by a water-level. To C is fastened the spiral spring G, carrying at its lower extremity the polished weight F, the length and strength of the spring being so adjusted that, even in the highest geographical latitude, the weight attached shall not at any time touch the agate plate D. If now the apparatus in perfect order be carried to a place of less geographical latitude, and again erected, it will soon be perceived that the weight F, loaded with the same additional weights as before, will no longer have power to stretch the spring to the degree necessary for bringing about contact between the weight and agate plate. It will therefore be necessary to add more weight, and this addition will evidently measure the difference of gravity in the two places, in so far as it operates on the sum of the suspended matter, that is, upon the sum of the weight F, and the half weight of the spiral spring G.

The dynamical method, on the other hand, consists in determining the velocity which is communicated in a second to a freely-falling body, by that force which draws a given heavy body to the earth. This velocity can only be determined indirectly by observations, as the oscillation of a pendulum. It is shown by the laws of mechanics, that when the same pendulum oscillates in two different places, and consequently under the influence of different forces of gravitation, the intensities of gravitation will be to each other as the squares of the numbers of oscillations made in a given time, and hence their ratio is readily determinable.

Compound Motion; Parallelogram of Forces.

46. Two or more forces acting on the same body in different directions cause it to assume a *compound motion.* If, however, these forces act on each other in such a manner that no motion can result to the body, they are said to hold one another in *equilibrium.* If any two forces act upon the same body in directions forming a known angle with each other, and with known intensities, it is evident that in its course it can obey exclusively neither the one nor the other of these forces. We must therefore investi-

gate the direction and velocity with which the body will move forward. This is easily done by representing the given directions and velocities by the two straight lines, AB, CD (*pl.* 6, *fig.* 11), completing the parallelogram ABCD, and drawing its diagonal AD. AD, by its position with respect to AB and AC, will then give the desired direction, and by its length the desired velocity of the motion of the body. This construction, so important in mechanics, is called the *parallelogram of forces;* AB and AC, the *lateral forces;* and AD the *mean force,* the *resulting force,* or simply the *resultant.* It has been previously mentioned (section 32) that in astronomy a very important application is made of the parallelogram of forces.

Refraction; Morning and Evening Twilight.

47. *Refraction* or *bending of rays* is of great importance in astronomical observations, as it causes the apparent to differ from the true altitude of a star. The atmosphere, like any other transparent body, turns an obliquely incident ray of light, SA (*pl.* 6, *fig.* 17), from its rectilineal direction—in other words, it bends it. Thus a ray of light, SA, coming from a rarer medium (the ether), and incident at a point, A, upon a denser medium (the atmosphere), is bent towards the perpendicular BAC at the point of incidence, just in proportion to the density of this medium into which the light passes. Suppose now an observer to be situated at any point, A, of the earth's surface, KA*k* (*fig.* 16); furthermore, let L*l*, M*m*, N*n*, represent successive strata of decreasing density, into which the atmosphere may be supposed to be divided, these evidently being spherical layers concentric with the earth's surface, KA*k*. Finally, let S represent a star beyond the external limits of the atmosphere. If now there were no atmosphere, the observer at A would see the star in the direction of the straight line AS. In reality, however, the ray SA begins, as soon as it reaches the atmosphere at *d*, to take a more inclined direction, *dc*, according to the above-mentioned law of dioptrics. This change of direction at first, owing to the extraordinary rarity of the outermost layers of the atmosphere, is very slight, but increases as the ray approaches the earth, entering successively into denser and denser strata, and the refraction becoming accordingly greater and greater. Thus, instead of following the rectilineal direction S*d*A, it describes a curve S*dcba*, which becomes more and more concave, finally reaching the earth, not at A, but at a point *a*, nearer to S. *This* ray consequently does not come to the eye of the observer. The ray by which the observer at A perceives the star S, is not S*d*A, but another ray, which, in the absence of an atmosphere, would have reached the point K, behind the observer. Now, however, by the refractive power of the atmosphere, it is bent into the curved line SDCBA, actually reaching the earth at A. It is a well-known law in optics, that every object is seen in the direction which the ray from the object has at the time of its entrance into the eye, the intermediate course of the ray not coming into account. The star S will therefore be seen, not in the direction AS, but in the direction of the straight line A*s*, tangent to the curve SDCBA

at A. Since the curve described by the refracted ray has its concavity inferior, the tangent line A*s* must lie above the unbroken ray AS; consequently the star S will, by means of the refracting atmosphere, appear higher above the horizon AH than it would were there no such atmosphere. Moreover, since the direction of the strata of air is the same in every direction about A, the ray cannot deviate laterally, but must always remain in the same vertical plane, SAC′, passing through the eye, the star, and the centre of the earth.

From what precedes it is evident that refraction causes all the heavenly bodies to appear *higher* than they really are. Therefore, a star actually below the horizon may, by refraction, be raised above it, and become visible, which could not occur without the refracting atmosphere. Thus, for example, the sun, when actually at P, below the horizon AH of the observer, may be rendered visible by the curved line P*qr*A, of which A*p* is the tangent, so as to be referred to *p*. The amount of the astronomical refraction (to be distinguished from terrestrial refraction), for any given altitude in the heavens, depends mainly upon the character, density, temperature, and moisture of the atmosphere; and for this reason the accurate determination of refraction for all heights, particularly for moderate ones, is one of the most difficult problems of physical astronomy. The following general considerations alone can here be mentioned: In the zenith there is no refraction, that is, it is equal to Zero; consequently, a star directly overhead will be seen in its true direction, or as if no refracting atmosphere surrounded the earth. The astronomical refraction increases from the zenith to the horizon, at first very slightly, afterwards more decidedly, so that a star situated near the horizon will appear more distant from its true place than one at a greater altitude. The mean amount of refraction for a celestial body, midway between the zenith and horizon, or at an altitude of 45°, is only 57 seconds, an amount scarcely sensible to the naked eye; but in the horizon, where refraction is greatest, it amounts to 33 minutes, which is more than the greatest apparent diameter of the sun or moon.

48. A prominent consequence of the refraction and deviation of the rays of light is the *morning and evening twilight*. Night, as is well known, does not immediately follow the day, nor the day night; after the setting of the sun, his rays still penetrate the higher regions of the atmosphere, losing themselves in space. The night thus comes on gradually. This prolongation of day is known commonly as twilight, produced partly by reflection, partly by refraction. Let SO (*fig.* 22, *pl.* 6) be a ray of sunlight entering the atmosphere at O; then, instead of following the original direction, and leaving the atmosphere at M, it will be diverted from its course or become broken. This deviation will be the greater the further the ray penetrates into the lower strata, which are denser as they are situated nearer the earth, so that the ray will describe the curve OG. In like manner, the ray SZ will become a curve from Z to D. Since, as has already been shown in *pl.* 6, *fig.* 16, this refraction is most considerable in the horizon, the sun appears to rise earlier and set later than is actually the case, by which means the day is lengthened and the night shortened. Before the rising of the sun, a

part of his light will be reflected from the atmosphere to the surface of the earth. Thus, in *fig.* 22, let MGBD be the earth, and G a point on its surface for which the sun is just about to rise. It is now evident that the horizon HGR, of the point G, receives light from every part of the heavens, whether direct or reflected; consequently the point A, for which the sun has not yet risen, has above its horizon the indirectly illuminated part, KLR, of the heavens. The morning redness is more brilliant at K, becoming gradually feebler towards R. The point B, on the contrary, has no light at all, and it is there midnight. The cause of evening twilight may be explained in the same manner, and it will be perceived that for the point D, where the sun is just setting, the whole horizon is still bright; while for the point C, the part TNX of the heavens is only indirectly illuminated. A circle in the heavens, parallel to the horizon, and at a distance of 18° below it, is called the *crepuscular* circle, or circle of twilight.

Let the point M (*fig.* 22) represent Leipzig, and the time be exactly noon; for the place G, situated 90° west, it will be past 6 o'clock A.M.; for the place Y, 45° west, it will be 9 o'clock A.M.; and to this latter the sun, on account of refraction, will appear at J. It will moreover be seen that for a place III., situated 45° east, the sun will appear at E, and that it will there be already 3 o'clock P.M. Finally, it will be readily understood, that while the earth moves about its axis, whose north pole is P, from west to east, or in the direction of the arrows, it must seem as if the sun, in an inverse order, attained successively the points F, J, S, E, U. *Fig.* 22 has been drawn to represent the time of the equinox, when the sun appears to describe the equator.

The morning and evening dawn or twilight is, in conclusion, not only more or less different in the same place at different seasons, but also different at places of different latitudes for the same season.

The Tides.

49. Another very remarkable phenomenon, produced by the attraction of the sun and moon upon the surface of the earth, is the *ebb and flow of the tide*, that well-known and generally regular motion of the sea, which results in a considerable variation of its height twice every day. On the coast of a great and open sea, as the North Sea, the phenomena of ebb and flow will take place in the following manner:—At the time when the water is highest, or at *high tide*, no change will be perceptible for some minutes. Gradually the water begins to run off westward, slowly at first, then with a continually increasing velocity, which reaches its maximum in about three hours. After this the fall continues for three hours with decreasing velocity, so that in a little more than six hours from the time of highest tide, *low tide* takes place. The sea, after remaining at this stage for some minutes, again begins to rise for six hours, and indeed in the same manner as it fell, so that in a little more than six hours from low tide, high tide again prevails. The rise of the water is called the *flow* of the tide, and

the falling the *ebb*. In this manner the whole phenomenon is incessantly repeated in periods of 12 hours and 25 minutes. The difference of elevation of the water at high tide and at low tide is not the same in all places at the same time, nor in one and the same place at all times. This difference, for example, on the German coast of the North Sea, amounts to 13 feet; while at the western end of the Straits of Dover it is sometimes more than 40 feet. The position of the coast, and the direction of the wind, change not a little the regular course of the whole phenomenon. Apart, however, from these local and temporary influences, a *monthly* and a *yearly period* are plainly evident. There is a greater difference between high and low water at the time of new and full moon, than at the time of the quadratures; and furthermore, this difference is more considerable when the sun and earth are nearest to each other, than when most remote. From this there cannot be the slightest doubt that the sun, and more particularly the moon, produce the ebb and flow of the tide.

For the proper elucidation of this phenomenon, suppose the earth's surface to be covered equally with water, and let us inquire what shape this watery surface will assume when the earth, on account of the attraction. begins to fall towards the moon; we will here have reference only to the influence of the latter, as being the most important. It is evident that those portions of the water will be attracted the strongest, which lie nearest the moon M (*pl.* 6, *fig.* 23), and consequently the water surrounding the earth will be heaped up highest at that place, O, which has the moon in its zenith. Here, then, where a mountain of water has arisen, the height of the water will be greatest, decreasing, however, more and more in every direction, reaching the minimum at those points, Z and Z′ (at time of full moon V and V′), which have the moon in the horizon; at the point O′ of the earth, which has the moon in its nadir, there will also be an elevation of the water: this point will be attracted least, and consequently will remain further behind the other points. Hence it follows that there will be a rise of the water at the two points, O, O′, of the earth, distant a whole diameter from each other, and lying in the straight line connecting the centres of the earth and moon. From these two points, the height of the water will decrease according to a certain law, until finally it will be lowest in the points of that great circle which has the two points of highest water for its poles; that is, as before mentioned, in all those places which have the moon in their horizon. Now, although the earth is not a globe entirely surrounded by water, yet by far the greatest portion is covered with water; and the waters of the sea will thus be heaped up in those points which have the moon in the meridian, whether at the inferior or superior culmination. Since the moon, on account of its own motion and the rotation of the earth, culminates every 12 hours and 25 minutes for one and the same place, the phenomenon known as the ebb and flow must continually return within this period. Nevertheless, the time of flood does not exactly coincide with that of the culmination of the moon, which at first may appear strange; but when we reflect that on account of the inertia of the material, the mass of water cannot immediately follow the apparent motion of the moon, that apparent anomaly will be

explained. That the flow in different places of the earth follows sometimes sooner, sometimes later, the culmination of the moon, is to be ascribed to local causes, as has been satisfactorily ascertained by means of many accurately conducted experiments.

Not only the moon, but also the sun, exercises an attraction on the water, and the attractive forces of the sun and moon must co-operate at time of new and full moon, and act against each other in the quadratures. That at full moon a high tide must occur, might at first appear singular, until explained by the fact that, on the side of the earth opposite to the moon, an elevation of the water necessarily occurs. Thus is explained the *monthly period of ebb and flow*. The tides occurring at the time of the syzigies in O and O′, are commonly called *spring tides;* those at the quadratures E, E′, *neap tides*.

Finally, there are several causes of the *yearly period* of ebb and flow, but we shall here only mention those which depend on the varying distance of the sun from the earth. Higher tides will take place in the winter than in the summer months. Since the moon can never separate more than 30° from the celestial equator, it is evident that within that terrestrial zone included between 30 degrees of north and south latitude, the tides must be greatest. This is confirmed by observation, since in the polar seas the entire phenomenon disappears. From what has been said, the conclusion is readily deducible—that the ocean alone, with the open and large seas in connexion with it, can have tides; for supposing the moon to be above the Caspian sea, for instance, then its waters will be attracted; yet, on account of the small extent of surface, this lunar attraction will be everywhere equal, so that an elevation of any particular part cannot take place.

The Resistance of the Ether; its Influence on the Motion of Comets.

50. The apparent course of the inferior and superior planets has already (*pl.* 6, *figs.* 24, 25) been explained. Some attention will here be directed to a circumstance of great importance to the planets, and particularly to the comets. For a long while it was believed that the spaces between the heavenly bodies were absolutely empty, or that a perfect vacuum existed there. This supposition, however, does not seem to be confirmed, at least with respect to the interspace of our planetary system. It is well known that all bodies fall with equal velocity in a perfect vacuum: moreover, the denser the air, and the rarer the body moving in it, the more readily the latter loses its original velocity, since it must experience a greater resistance than another body of greater density moving in the same medium. This must be true with regard to the planets. As these have shown no diminution in their velocity produced by the resistance, we must suppose one of two things: either that the interspaces of our planetary system are absolutely empty, or that if a medium really exists, it is much too rare, in comparison with the density of the planets, to produce any retarding

influence on their motions. The comets, however, which are known to be bodies of very slight density, may experience some retardation, even if it be very slight, from the rare ether existing in space. The continual abbreviation of the period of revolution of Encke's comet, already several times returned, observed, and calculated, indicates conclusively enough that space is filled by a medium, and consequently is not absolutely empty. This abbreviation of the period of the above mentioned comet, is evidently a consequence of the resistance which the ether opposes to the course of the comet. The comet itself may by this means experience, one day or other, a very destructive catastrophe. Since it meets a resistance in its course around the sun, it naturally will not be able to retain its original orbit, but must by degrees move in arcs which lie nearer to the sun than those previously described in similar times. The orbit will, therefore, remain no longer an ellipse or a closed curve, but must become a spiral, terminating in the sun itself, since the comet will be more and more affected by his attraction, and consequently approach nearer and nearer (*pl.* 6, *fig.* 26). The immediate consequence of this must be a gradually diminishing period of the comet, which will accomplish its course with greater and greater velocity, until finally it will be lost in the sun.

The Sun's Spots.

51. *Pl.* 9, *figs.* 9-13 represent the black spots seen sometimes, with the assistance of the telescope, on the sun's disk, of greater or less size, irregular shape, and surrounded by an ash grey border, generally of uniform breadth. The solar spots appear frequently to change not only their shape, as shown by *figs.* 10 and 11, but also their position on the sun's disk. They are sometimes very large, and their constant occurrence in connexion with the *solar faculæ* (*fig.* 9), as also their ash grey border, plainly indicate a common origin with these. Thus, for instance, their black spots may be seen to break out in the midst of these faculæ, or, inversely, faculæ arise in the places whence spots have just vanished. These *faculæ* are streaks, which, by their dazzling light, are distinguishable from the rest of the disk, and resemble, so to speak, veins of light.

It is very rare that the spots on the sun are seen at a distance of more than 30° from his equator on each side. At their entrance on the sun's disk they appear very small, and when they come near to the border of the sun, they are seen as black lines, becoming broader the nearer they are to the centre of the disk; they moreover seem to move in almost parallel lines from east to west over the sun's disk: their true motion, however, is from west to east, as they would appear to an eye at the centre of the globe of the sun. A spot generally occupies from 12 to 13 days in crossing the visible disk of the sun. It is then invisible for a period of 14–15 days, but at the expiration of this time it appears in the same place in about 27 or 28 days after the first appearance, to commence its second revolution. The paths of the spots appear towards the 10th of June and 10th of December as

straight lines; on all other days of the year as ellipses, whose convex sides are turned for half a year towards the north, and for the same length of time towards the south, and whose greatest curvature takes place shortly before March 10th and September 10th. Observations on the sun's spots have enabled us to ascertain a rotation of the sun on its axis in a period of $25\frac{1}{2}$ days.

Herschel's hypothesis with respect to the nature of the spots on the sun, appears to be the most probable. He assumes a threefold concentric envelope of the obscure body of the sun proper. This first envelope is the photosphere, or atmosphere of light; beneath it the second, a transparent and very elastic medium; and beneath this layer the third, a cloudy obscure envelope, illuminated on its outer side, and reflecting the light to our eyes. In this manner it forms an ash grey border, which is seen sometimes on the sun without a central spot, whenever an opening may exist in the first, or first and second layers. Whenever this fissure or opening extends, as is generally the case, through the third layer, the dark nucleus of the sun is then perceived, and about it the above mentioned grey border, which is, accordingly, nothing else than the light passing into the opening from the outermost layer, and reflected back from the inner atmosphere to our eye.

The group of spots represented in *fig.* 9, was discovered May, 1799, by Fritsch of Quedlinburgh. The western spot, very near the border of the sun, appeared as a black nucleus of oval shape, with an equally oval nebulous inclosure. Eastward of the oval spot, Fritsch observed another circular one, both united by a so called valley or mountain way, having the appearance of a ring mountain of our moon. From this mountain way run lateral branches, and both appear to the eye whiter and fainter than the rest of the solar surface. The spots, *pl.* 9, *figs.* 10, 11, 12, 13, were discovered by Pastorff, in Frankfurt on the Oder, May 24th, 1828. The largest (*fig.* 10), *abcd*, had at *ab* a diameter of 100 seconds, and at *cd* one of 60. It now appears, however, as a straight line of 392 geographical miles on the surface of the sun, under an angle of almost one second, seen from the earth; consequently, the true diameter of this spot amounted to 39,200 geographical miles at *ab*, and to 23,520 at *cd*. The greatest diameter amounted to more than five diameters of the earth (= to 6880 geographical miles), consequently the surface of this spot contained nearly 928,000,000 square geographical miles. Furthermore, *ef* (*fig.* 10) had an apparent diameter = 110 seconds; $gh = 60$; $no = 68$; $pq = 30$; ik (*fig.* 11) $= 38$; lm (*fig.* 12) $= 66$; $rs = 24$; tu (*fig.* 13) $= 46$; and $wx = 12$. All these numbers multiplied by 392 give the dimensions in geographical miles, altogether equal to an area of 2,496 millions of square geographical miles, or about 17 times as great as the whole surface of the earth.

The honor of the first discovery of the sun's spots appears due to the English astronomer, Harriot, who saw them Dec. 8, 1610. The eminent physician Averrhoes (in the twelfth century) was perhaps the first who saw a spot with the naked eye, erroneously supposed by him to be the planet Mercury. Phrystus of Wittemberg published the first treatise on these spots in 1611; the Jesuit, Father Scheiner, however, sought to appropriate the

discovery. He seems to have exhibited, in May, 1611, the first spot to his pupils in Ingolstadt, where he was professor. Galileo had observed the spots of the sun as early as the beginning of 1611, nearly contemporaneously with Fabricius, and very soon presented correct views of their nature.

Topography of the Moon.

52. The most interesting object visible by a good telescope in the heavens, is certainly the surface of the moon, whose peculiarities, as being of all heavenly bodies the nearest to us, are known best of all. At an early period Galileo, Scheiner, and Hevelius, and afterwards Grimaldi, Riccioli, Cassini, and Lahire, attempted to construct a chart of the moon; the maps of the moon by Hevelius, Mayer, and Schröter, are well known. Lohrmann, however, uniting an intimate knowledge of facts with sound judgment, first published accurate and beautiful maps of the moon, four in number, whose continuation was unfortunately arrested by his death. Beer and Mädler, finally, in 1836, published four large sheets with a general map of the moon, on Lohrmann's plan, indeed, but founded on original observations. Of this general map *pl.* 11, *fig.* 1 is an accurately reduced copy. Our chart represents the moon inverted, or as it would be exhibited to the observer in an astronomical telescope of from 60 to 80 magnifying power. North is consequently below, South above, East to the right, and West to the left. From want of room, and to avoid crowding, the single mountains and craters are indicated by numbers, whose import will presently be explained. On the moon there can be directly distinguished nothing but differences of level and illumination; consequently only *mountains, craters,* and *colors.* The two first are exhibited best in the growing and waning moon, when the part to be observed lies near the illuminated border; the colors we see to the most advantage at the full moon. Many of the *ring mountains* have, following Riccioli's example, been named after eminent philosophers; while for the rest, with Hevelius, the names of mountains, rivers, &c., have been borrowed from the earth.

53. The numbers annexed to the names in the following list, indicate the depth or the inner descent of the wall in Paris feet; where a second number, inclosed in brackets, occurs, it indicates the height of the wall above the outer inclosure. All these numbers are derived from original measurements by Beer and Mädler.

I.—Northwestern Quadrant of the Moon.

The Map of the Moon (Fig. 1) to the Left Below.

1	Schubert		47	Römer	10,864
2	Reper		48	Littrow	
3	Firmicus	4638	49	Lemonnier	8475
4	Apollonius	5100	50	Bessel	3606 (1464)
5	Taruntius	3272	51	Linnæus	
6	Maskelyne	4362	52	Conon	
7	Sabine	2485	53	Hadley	14,208
8	Ritter	3718	54	Bradley	12,639
9	Dionysius		55	Gauss	
10	Arago	5022	56	Burckhardt	13,672
11	Sosigenes		57	Geminus	11,577
12	Cæsar	5082	58	Bernouilli	11,868
13	Ariadäus		59	Messala	3360
14	Godin	6780	60	Berzelius	
15	Agrippa	6342	61	Franklin	7436
16	Boscowich		62	Posidonius	5346 (2976)
17	Hyginus		63	Kalippus	7230
18	Rhäticus		64	Theätetus	7004 (3312)
19	Triesnecker	5086	65	Aristillus	10,464 (4750)
20	Uckert		66	Autolycus	8457 (4485)
21	Condorcet	8410	67	Cassini	4098 (3870)
22	Hansen		68	Struve	
23	Alhazen		69	Schumacher	
24	Azout	5300	70	Mercurius	
25	Picard	4982 (2867)	71	Hook	
26	Proclus	7790	72	Cepheus	8588
27	Jansen		73	Oersted	
28	Vitruvius	4227	74	Atlas	10,261 (3462)
29	Maraldi		75	Hercules	10,209
30	Plinius	5904	76	Mason	5705 (3336)
31	Ross		77	Plana	
32	Acherusia Cape	4536	78	Bürg	6372 (4297)
33	Taquet		79	Eudoxus	13,980
34	Menelaus	6164	80	Aristotle	10,031 (4266)
35	Sulpic. Gallus		81	Egede	320
36	Manilius	7223	82	Endymion	15,690
37	Agarum Cape	9666	83	Strabo	
38	Eimmart	9683	84	Thales	
39	Oriani		85	Gärtner	
40	Plutarch		86	Democritus	5302
41	Seneca		87	Christ. Mayer	3906
42	Hahn	9095	88	Meton	
43	Berosus	10,722	89	Euktemon	
44	Cleomedes	8194	90	Scoresby	9216
45	Tralles	8202	91	Barrow	8832
46	Macrobius	14,409	92	Archytas	3704

II.—Northeastern Quadrant of the Moon.

The Map (Fig. 1) to the Right Below.

1	Pallas	4193	38	Carlini	
2	Bode		39	Delisle	5586
3	Schröter	2340	40	Wollaston	
4	Gambart	2500 (2154)	41	Lichtenberg	
5	Stadius	630	42	Lavoisier	
6	Copernicus	10,584 (2478)	43	Kirch	
7	Reinhold	8819	44	Pico	6624
8	Hortensius		45	Laplace	9022
9	Encke	1698	46	Heraclides	3681
10	Kepler	9400	47	Maupertuis	
11	Reiner	9306	48	Bianchini	7936
12	Hevelius		49	Sharp	6949
13	Cavalerius	8990	50	Mairan	7520 (4890)
14	Olbers		51	Louville	
15	Marco Polo		52	Gerard	
16	Eratosthenes	14,678 (6204)	53	Plato	6810
17	Gay-Lussac		54	Lacondamine	3996 (2490)
18	Mayer		55	Bouguer	
19	Milichius		56	Harpalus	14,872 (2576)
20	Marius	4270	57	Œnopides	
21	Bessarion		58	Repsold	
22	Cardanus		59	Harding	
23	Kraft		60	Xenophanes	
24	Huyghens	16,934	61	Cleostratus	
25	Wolff	11,316	62	Timäus	
26	Archimedes	5084 (3636)	63	Epigenes	
27	Timocharis	6786 (3426)	64	Fontenelle	6372
28	Pytheas	(5500) (2241)	65	Horrebow	
29	Lambert	5580 (2259)	66	Anaximander	
30	Euler	5585 (1062)	67	Pythagoras	
31	Diophantus	4000 (2392)	68	Gioja	
32	Lahire	4788	69	Anaxagoras	8943
33	Herodotus	4032	70	Philolaus	11,604
34	Seleucus		71	Anaximenes	7487
35	Briggs		72	Sömmerring	4282
36	Aristarchus	7058	73	Vasco de Gama	
37	Helicon	2482			

III.—Southeastern Quadrant of the Moon.

The Map of the Moon (Fig. 1) to the Right Above.

1	Malapert		6	Casatus	19,514
2	Cabeus		7	Klaproth	9365
3	Short	17,532	8	Wilson	12,954
4	Moretus	14,994	9	Gruemberger	13,024
5	Newton	22,362	10	Cysatus	11,900

11	Blancanus	16,892
12	Scheiner	
13	Kircher	16,530
14	Bettinus	11,616
15	Bailly	13,910
16	Hausen	
17	Zuchius	
18	Clavius	15,792
19	Deluc	
20	Maginus	13,553
21	Longomontanus	13,641
22	Rost	7406
23	Weigel	
24	Segner	7617 (6006)
25	Bayer	7572
26	Schiller	11,856
27	Phokylides	8250
28	Wargentin	(400) (1392)
29	Saussure	
30	Pictet	
31	Street	4188
32	Tycho	16,662
33	William I.	10,389
34	Heinsius	8130
35	Hainzel	10,880
36	Drebbel	
37	Schickard	7938
38	Inghirami	11,460
39	Lehmann	
40	Nasireddin	10,308
41	Orontius	
42	Sasserides	7415
43	Lexell	7236
44	Walter	13,479
45	Hell	5034
46	Gauricus	8730
47	Wurzelbauer	5160
48	Pitatus	
49	Hesiodus	
50	Cichus	7938 (4848)
51	Capuanus	8022
52	Ramsden	3000 (1722)
53	Vitello	4000 (4810)
54	Piazzi	
55	Lagrange	
56	Bouvard	
57	Regiomontanus	5964
58	Purbach	7594
59	Thebit	7882
60	Mercator	4358
61	Campanus	6126
62	Hippalus	
63	Kies	2292
64	Bulliald	8410
65	Doppelmayer	
66	Fourier	8877
67	Vieta	13,739
68	Cavendish	7038
69	Byrgius	6516
70	Mersenius	7890
71	Eichstädt	
72	Arzachel	12,750
73	Alphonso	6600 (5196)
74	Alpretagius	11,291 (3540)
75	Davy	
76	Guericke	1926
77	Lubiniezki	924
78	Agatharchides	3456
79	Gassendi	8970
80	Letronne	3015
81	Billy	3183 (2292)
82	Zupus	4399
83	Lontana	
84	Sirsalis	9528
85	Crüger	
86	Rocca	
87	Ptolemäus	7452
88	Herschel	8832
89	Mösting	7062 (1524)
90	Lalande	
91	Parry	1536
92	Bonpland	
93	Fra Mauro	
94	Landsberg	9064 (2802)
95	Euclides	
96	Flamsteed	5479 (1320)
97	Damoiseau	
98	Grimaldi	
99	Lohrmann	
100	Riccioli	
101	Hansteen	3522 (2649)

IV.—Southwestern Quadrant of the Moon.

Map of the Moon (Fig. 1) to the Left Above.

1	Schomberger	
2	Simpelius	9645
3	Boguslawski	10,468
4	Boussingault	

5	Manzinus	9991
6	Mutus	11,525
7	Pentland	
8	Curtius	20,898
9	Pontecoulant	
10	Hanno	
11	Biela	
12	Hagecius	
13	Nearchus	
14	Rosenberger	
15	Blacq	9762
16	Hommel	
17	Pitiscus	9463
18	Baco	11,616
19	Jacobi	9684
20	Zach	11,414
21	Lilius	9286
22	Oken	
23	Vega	
24	Steinheil	11,046
25	Fabricius	7818
26	Nicolai	5874
27	Clairaut	
28	Barocius	11,366
29	Maurolicus	13,314
30	Cuvier	11,272
31	Licetus	12,766
32	Stöfler	10,950
33	Marinus	
34	Fraunhofer	
35	Furnerius	10,100
36	Stevinus	10,782
37	Rheita	13,464
38	Metius	12,372
39	Reichenbach	10,996
40	Neander	7476
41	Stiborius	11,343
42	Riccius	
43	Rabbi Levi	
44	Zagut	
45	Lindenau	11,136
46	Büsching	4218
47	Buch	
48	Gemma Frisius	12,796
49	Poisson	6888
50	Aliacensis	13,624
51	W. von Humboldt	
52	Hekateus	
53	Legendre	7627
54	Petavius	10,158
55	Snellius	6400
56	Borda	10,338
57	Fracastoro	7988
58	Santbech	12,894
59	Piccolomini	14,574
60	Polybius	7278
61	Pons	
62	Fermat	
63	Sacrobosco	11,295
64	Pontanus	
65	Azophi	13,644
66	Abenezra	10,254
67	Appianus	8787
68	Playfair	11,290
69	Werner	14,658
70	Ansgarius	
71	Vendelinus	5046
72	Cook	2959
73	Colombo	7540
74	Magellan	
75	Bohnenberger	
76	Beaumont	5778
77	Theophilus	14,939
78	Cyrillus	
79	Katharina	15,423
80	Kant	
81	Tacitus	9762
82	Almanon	5724
83	Geber	8112
84	Abulfeda	9600
85	Airy	
86	Albategnius	13,943
87	Parrot	
88	Kästner	
89	Maclaurin	
90	Langrenus	11,616
91	Messier	5256
92	Goclenius	4386
93	Gutenberg	
94	Capella	9504
95	Isidor	9005
96	Torricelli	
97	Hypatia	6904
98	Delambre	14,046
99	Hyparchus	
100	Réaumur	
101	Biot	
102	Lacaille	

54. With only a tolerably good telescope, the bright spots of the moon can be determined as mountains, by means of their shadows. Their shadow lies always on the side opposite to the sun, and is the longer the less the altitude of the sun for the mountain. The grey spots were at one time

erroneously supposed to be seas. The distinct brilliant points, so many of which are seen on the full moon, are only rarely elevations; oftener steeply precipitous depressions. The structure of the lunar mountains in general is very different from that which prevails on our earth, in that they present themselves for the most part as circular closed walls, with a hollow sloping cavity. They are called *ring mountains*, as, for instance (*pl.* 11, *fig.* 1), Ptolemaeus (III. 87), Alphonso (III. 73), and Archimedes (II. 26); Plato (*fig.* 4). The greater of them are called *walled plains* when they inclose a plane surface: the smallest ring mountains are called *craters*. The most diminutive of these, on account of their small size, could not be represented in *pl.* 11 by mountain streaks, and therefore small circles have been employed for these. They are seen on a larger scale in *fig.* 3, namely, Pliny and Vitruvius. The walled plains occurring most frequently in the south-western part of the moon, appear to belong to the earlier formations of the moon's surface, since they are unmistakably inferior to later forms of every kind. The light streaks passing frequently through walled plains, as also over the other regions of the moon, are not, as a whole, actual elevations, as they are sought in vain under an oblique illumination, when actual elevations are indicated as such by their shadows.

Next to the walled plains follow, in order of size, the ring mountains proper, which, often truly circular, exist in great numbers. Frequently their wall slopes inwards and outwards in so called *terraces;* and in their interior, they generally present an elevation known as the *central mountain.* The most of these central mountains, however, do not reach as high up as the wall. Where the central mountain stands, the inside is sometimes dark grey; commonly, however, as bright as the outer wall. In the southern hemisphere of the moon, the most of these mountains with their walls and environs are so much alike in color and light, that at full moon nothing more can be distinguished of them. The same thing occurs frequently in the deepest, most conspicuous, and most varied ring mountains and walled plains, sometimes even in the grey spots. These latter, consequently, as for instance in *fig.* I., Mare Crisium, Mare Fœcunditatis, Mare Tranquillitatis, and Mare Serenitatis, cannot possibly be seas. Hence it also follows that the view of the full moon is entirely different from that at the first or last quarter, since here the shadows of the mountains and craters present themselves, while there it is only various colors and their shading that are seen.

Mountain chains occur here and there upon the moon, as upon the earth, but never of so great length. Even those mountains connected with one another, and termed *mountain chains,* have by no means the same valley and hill structures as the mountain chains on our earth, but they approach more to the crater form, and do not run out into various branches. The names of these mountain chains have been derived from those on the earth, as may be seen on the maps of the moon (*pl.* 11, *fig.* 1). The greatest elevations of the following mountain chain are given in Paris feet.

Caucasus	17,138	Altai	12,459
Appenines	16,934	Mountain on Sinus Iridum	14,022

Pyrenees	11,178	Hemus	6,222
Alps	11,136	Carpathians	5,970
Taurus	8,472	Riphæans	2,580

Again, perfectly insulated mountains, called *cone mountains*, exist in great numbers on the moon. With respect to the grey level regions of the moon, at one time called seas, they are universally intersected by ranges of heights, either long and straight, or running in great free curves. The latter are with some inaccuracy termed *mountain spurs*, since they are not ramifications and extensions of greater mountains, and vanish entirely at full moon, casting a shadow only at sunrise and sunset, by which they are then recognised. The *streaks of light* must be considered as very remarkable and difficult of explanation, many of which run along singly, but most form radiating systems. These streaks of light stretch indifferently over mountains, valleys, and plains, without altering their shape, direction, or color. They are almost always four to twelve miles broad, and vanish under an oblique illumination. Equally singular are the *channels*, those extremely narrow but deep furrows, which run generally in rectilineal directions through plains, and more rarely through mountainous regions. These channels cannot be streams or their beds, nor can they remind us of canals and highways. These channels are indicated by narrow parallel lines on the map of the moon.

As regards the colors of the moon, we can only indicate them on our chart in the most general outlines. At least ten different shades, from dark grey to the most brilliant white, can be distinguished. In general, the elevation is the brighter, and the depression the darker; this relation is, however, sometimes inverted. The brightest spots very rarely belong to the higher mountains; it is rather great depths that shine with uncommon brilliancy.

55. For the sake of giving a clear idea of the appearance of particular regions of the moon through a good refractor of 200–300 magnifying power, we have furnished the figures 2, 3, 4, and 5, which surround *fig.* 1, *pl.* 11. The region *fig.* 2 contains the mountains Caucasus, Calippus, Eudoxus, and Aristotle, found in quadrant I. (*fig.* 1); *fig.* 3, the mountains Jansen, Plinius, Vitruvius, and Littrow, with a part of Mare Serenitatis, also occurring in quadrant I. *Fig.* 4 represents the mountains found in quadrant II. (*fig.* 1), namely, Kirch, Pico, Alps, Plato; as also a part of Mare Imbrium and of Palus Nebulum. Finally, *fig.* 5 represents the mountains Saussure, Pictet, Tycho, Sasserides, and Gauricus, of quadrant III. (*fig.* 1.)

A degree of the equator, or $\frac{1}{360}$ part, is equal to 60 geographical miles; a degree of the moon's equator is equal to 16 $\frac{36}{100}$ geographical miles; the whole surface of the moon is therefore about equal to the area of America.

The visible hemisphere of the moon is represented in *fig.* 1, *pl.* 11, as it appears at the time of mean libration. Consequently only the central parts appear in their true proportions; for nearer the borders all circular crater groups must appear oblong.

It is erroneous to suppose that with better instruments and higher magni-

fying powers, it will be hereafter possible to perceive more and minuter details of the moon's surface (as, for instance, artificial structures) than at present: for with these improvements the difficulties and hindrances will increase in like proportion. We need only refer to the atmosphere of the earth and the borrowed light of the moon. These difficulties are even now experienced in the application of the best telescopes, as the moon, of all the heavenly bodies, is that for which the highest powers are unsuited. It is by means of very accurate and long continued observations that we are to have our knowledge of the moon increased. It is only then that better comparisons with the earlier observations, and more accurate conclusions may be drawn than now, when only since the time of Lohrmann and Mädler the moon has been attentively examined with the more improved refractors. Posterity will be able to verify changes which appear to be taking place on the surface of the moon, and our successors will probably ridicule many of our opinions, and reject them as untenable. One fact is certain, however, that *Selenography* (description of the moon) must commence with generalities and progress to particulars, while *Geography* (description of the earth) pursues the opposite method. Selenography has the advantage of Geography, as we do not possess so good a general view of the earth as of the half of the moon which is visible to us.

To become most readily acquainted with the mountains, craters, &c., of the moon, it will be necessary to examine attentively the moon at the time of the first or last quarter, through a telescope of about 40–60 magnifying power, and to make constant reference to the lunar maps. During the full moon, this, at least to a beginner, is not very satisfactory, as at this time the sun stands directly over the centre of the visible moon's disk, and the shadows of the mountains are not seen. In the first and last quarters, however, the sun moves above and below the centre of the visible disk, and at this time, accordingly, the shadows of the mountains are greatest and most evident.

The Planets Mars, Jupiter, Saturn, and Uranus.

56. *Pl.* 8, *fig.* 18, represents the planet Mars in his not entirely illuminated condition, as seen August 16, 1830, by Sir John Herschel, at Slough, with a 20-foot reflector. We see plainly enough presumptive continents and seas; the first distinguished by their reddish color, characterizing the light of this ever red and fiery planet. In contrast with this color, the seas, if we may so term them, appear of a greenish hue. These spots cannot always be seen with equal distinctness, which is probably owing to the fact of Mars not being entirely free from an atmosphere. This supposition is confirmed by the exhibition of brilliant white spots at the poles of Mars. These spots are probably snow, as they vanish when they have been long exposed to the sun, and on the other hand are largest on emerging from the long night of their polar winter. By observations on the spots, Mars has been found to have a period of rotation in 24 hours, 39 minutes, 21 seconds; and the inclination of the axis of rotation to the ecliptic, amounts to about 30° 18′.

Since the time of Herschel and Schröter, Mädler in Dorpat has first, in our day, carefully and attentively examined the surface of Mars. It is moreover known that Mars alone of all the superior planets exhibits phases to the earth, or some slight deviation from the perfect roundness of his disk.

57. The disk of the great and beautiful planet Jupiter, always appears with dark streaks drawn across in a determinate direction. *Fig.* 19 gives a view of these streaks as observed at Slough with a 20-foot reflector on Sept. 23, 1832 ; these are, however, by no means the same at all times. Remarkable dark spots also, resembling masses of clouds, are not rare ; and from careful and continued observations of these spots, the conclusion has been derived, that Jupiter rotates in 9 hours, 55 minutes, 50 seconds (sidereal time), upon an axis perpendicular to the direction of the streaks. On account of the parallelism of these streaks with Jupiter's equator, their oft-occurring changes, and, finally, from appearances of the spots, it may be maintained that these streaks belong to Jupiter's atmosphere, forming tracts in a tolerably serene sky, and are produced by currents similar to our trade winds. They have, nevertheless, a much more permanent and decided character than the clouds in our atmosphere, which may result from the enormous velocity of rotation of the mighty planet. Moreover, that we perceive in the streaks the proportionally darker body of Jupiter is clear, from the well known circumstance that these streaks do not reach to the very edge of the disk, but fade gradually away before they arrive there.

58. *Saturn* (*fig.* 20) is surrounded by an attendant of entirely unique and wonderful character; for accompanied as he is by seven moons, he is also surrounded by two broad and flat, though thin rings, concentric with each other and the planet. Both rings lie in the same plane, and are entirely separated from each other by a narrow, and from Saturn by a much broader interspace. The interval between the planet and the inner edge of the inner ring amounts to 16,572 geographical miles, the breadth of the inner ring to 29,820, the interval between the inner and the outer rings to 1556, and the breadth of the outer ring to 18,356, and finally the thickness of each ring to 88 geographical miles. *Pl.* 8, *fig.* 20, gives a view of Saturn surrounded by his rings, and with dark streaks on his surface, tolerably similar to those of Jupiter; they are however broader and not so evident, although probably originating in the same cause. The supposition that the double ring of Saturn is a solid and opake mass, is confirmed by the fact that it casts a shadow upon the planet, and is shadowed by it in certain positions with relation to the sun. The parallelism of the streaks with the plane of the ring, makes it probable that the axis of rotation of Saturn is perpendicular to this plane, this supposition also being confirmed by the extended dark spots on the planet. From accurate observations of these spots, the period of rotation has been established at about 10 hours, 29 minutes, 17 seconds.

59. Uranus is too distant for the observation of spots on his surface which might assist in determining his period of rotation. Nevertheless it must be supposed that like Jupiter and Saturn, Uranus possesses a very short term of rotation, since Mädler has plainly discovered a flattening of

the planet's disk, which by measurements instituted has been found to be quite considerable. Consequently by reason of this considerable flattening, the velocity of rotation of Uranus must be very great.

The Comets, Nebulæ, Groups of Stars.

60. *Fig.* 15, *pl.* 8, represents the comet of 1819, which, suddenly emerging from the beams of the sun, appeared to Europe, in the beginning of July, of remarkable size. Arago maintained that the light of this comet exhibited traces of polarization, which can only be exhibited by reflected light. This fact speaks strongly for the theory of Olbers, who maintained that the comets are non-luminous bodies, only rendered visible by the reflected light of the sun. Comets consist of a usually spherical nebulous envelope, with a somewhat brighter nucleus, although occasionally without the latter. Sometimes the nucleus is of great size; thus, for instance, that of the great comet of 1811 had a diameter of at least 560,000 geographical miles. In general the nebulosity does not entirely surround the nucleus, but exists as a spherical hull, elongated on the side of the tail, so that the tail appears as a continuation of the nebulosity. It seems besides, that this nebulous envelope constitutes the chief peculiarity of comets, as many of these wandering stars are seen, some without tails, some without nucleus, none however without the nebulosity. The tail is generally found on the side opposite to the sun; at times, however, it deviates from this direction, which may be a consequence of the resistance of the ether in which the comet moves. The length of the tail is very various. Thus for instance the length of the tail of the great comet of 1811 (*pl.* 8, *fig.* 17) amounted on Oct. 12 to above 88 millions of geographical miles. The tails also become broader towards their extremity, and are often divided longitudinally in their middle by a dark line, so that it seems as if the tail were double. This was plainly perceived on the 10th of Sept. 1811 in the great comet, as shown in *fig.* 16. The sun undoubtedly produces the tail, as this is always first visible when the comet approaches the sun, becomes larger as the approximation increases, and again diminishes with the gradual increase of distance between the two bodies. Consequently the tails appear, for the most part, to consist of very thin vapor developed by the heat of the sun from the nucleus of the comet. The alterations arising from this cause, which, according to numerous observations, must often be enormous, and may take place even within a few days, doubtless produce the changes observed in the size, shape, and brilliancy of comets. When a comet becomes visible to the naked eye, it is generally seen but a short time, and has a very different course in the heavens from the planets, though it follows the usual daily motion of the heavens. Formerly, on account of the rarity of comets, their remarkable appearance, and their course, it was supposed that they were not true heavenly bodies. Newton, however, first showed that they, like the planets, are heavenly bodies belonging to our solar system. Like the planets, they describe, according to the same laws,

orbits around the sun. The eccentricity of their orbits is very great. It is now also known that telescopic comets, or those only visible to the eye when assisted by a good telescope, occur in far greater numbers than others, and that multitudes of them are probably always present in the space belonging to our solar system.

There are thus far, only three comets whose return has been calculated several times, and whose orbits are accurately known. The first of these is Halley's comet. Halley found that the comets of 1456, 1531, 1607, and 1682 were one and the same, which, he predicted, would return in the beginning of 1759. His prediction was nearly fulfilled. It again appeared in 1835, and is next expected in the end of 1911. The period of revolution of this comet, which can approach within 48 millions of miles of the sun, and recede from him 2920 millions of miles, embraces 76 years. The inclination of its orbit is 17° 44′, and its eccentricity is $\frac{97}{100}$ of the semi-major axis. The motion of this comet is retrograde.

The second comet, discovered by Pons in Nov. 26, 1818, is the direct comet of Encke, with the very short period of $3\frac{1}{2}$ years. Encke found that it had been already observed in 1786, 1795, and 1805, that its perihelion distance was 26 millions of miles, its aphelion 340 millions, the inclination of its orbit 13°, and the eccentricity $\frac{85}{100}$ of the semi-major axis. The predictions of later reappearances of this comet, whose course is affected by the resistance of the ether (sec. 50), were fulfilled in 1825, 1828, 1832, &c.

The third, also a direct comet, was discovered by Biela, 28th Feb. 1826, and named after him. This comet revolves in $6\frac{3}{4}$ years about the sun. Its aphelion distance amounts to 508 millions of geographical miles, while its perihelion comes very near the earth's orbit. This, as is well known, is the comet which in 1832 passed near the earth, and on that account excited universal apprehension. The inclination of its orbit amounts to 13°, and the eccentricity to $\frac{74}{100}$ of the semi-major axis.

Besides these three comets, there are some others recently discovered, whose periods have been found with great accuracy. Their return in the calculated time must, however, be determined by experience. Of these, the best known is the one discovered in the constellation of the Fly, by Olbers, March 6, 1815, and named after him. The motion of this comet is direct. It approaches to within 100 millions of geographical miles of the sun, and recedes to a distance of 2840 millions. The inclination of its orbit amounts to 44°, its eccentricity to $\frac{93}{100}$ of the semi-major axis, and its period to a little more than 74 years. Its re-appearance may consequently be looked for about the year 1889.

While in earlier times comets were looked upon as harbingers of misfortune, or as indicative of Divine wrath, more recently the fear has been excited, lest, on account of their great number, and the various positions of their orbits with respect to that of the earth, a comet may at some time or other come in contact with the earth. This fear, however, Olbers, more than 40 years ago, by his copious investigations of the numbers and orbits of comets, sought to remove as far as possible.

In conclusion, it remains to say that very recently, two more great and

beautiful comets, namely, those of February, 1843, and May, 1845, have appeared and been carefully examined.

61. We turn now to several interesting objects of the starry heavens, which are represented on the left and right sides of *pl.* 8. *Fig.* 1 is a crowded group of stars of irregular outline, seen in the constellation of Hercules, under a right ascension 248° 45′, and north declination of 36° 48′. Stars of the 10th to the 15th magnitude stand very close together, the diameter of the whole amounting to about 8 minutes.

We have already referred in general terms, in sections 15 and 16, to the groups or clusters of stars, as also to nebulous stars and nebulæ. *Fig.* 2 gives a representation of a beautiful circular group of stars in Aquarius, resolvable near the centre by a good telescope, and seen under 321° 15′ right ascension, and 1° 34′ south declination. Towards the centre it is very clear and uniformly brilliant, although the stars do not stand thicker here than towards the border; the central brightest part amounts to six seconds of diameter.

Many groups of stars are fan-shaped, as in *fig.* 3 or *fig.* 4, which latter occurs in the constellation Cancer. The round nebula (*fig.* 5) is found in Ursa Major. In Gemini, under right ascension 109½°, and north declination 26° 26′, there is a nebula (*fig.* 6) whose central star has a great, irregularly oval atmosphere. In Leo Major, 167¾° right ascension, and 41½° north declination, may be perceived a star (*fig.* 7) in the middle of an elliptical nebula, very much pointed at the ends. Two other nebulæ (*figs.* 8 and 9) occur in Monoceros; one of them (*fig.* 9) of right ascension 97½°, and south declination 8° 53′, is a star of the 12th magnitude, with a luminous nebulous train of about one minute in length, not unlike the tail of a comet.

Two and even more stars are often seen enveloped in one nebulous mass, evidently belonging to them both, standing in the two foci or the two vertices of the elliptical nebula. One of this kind is to be met in Canes Venatici, under right ascension 192¾°, and north declination 35° 47′, where the two stars (*fig.* 10) are of the 10th magnitude; in Sagittarius there is a bright elliptical nebula (*fig.* 11), with a star in each of the foci. In the constellation Auriga, a nebula with three stars is observable, which is round (*fig.* 12) according to some, but triangular (*fig.* 13) according to others. *Fig.* 14 represents the great nebula in Andromeda, figured also from another view on *pl.* 13, *fig.* 10.

The Aurora. Mock Suns and Mock Moons.

62. Although the phenomena now to be referred to, belong more properly to the department of meteorology, to which a special section will be devoted, yet they cannot remain entirely unnoticed under the present head. The Aurora, improperly called northern light, as it appears at the south pole as well as the north, is the name of a luminous, often circular meteor, which sometimes appears in the vicinity of the magnetic pole of the earth, and

shines with an indescribable hue. It is only recently that astronomers have included the Aurora or polar light within the circle of their observations, and have found in almost every case, that simultaneously with the appearance of a northern light, there are certain phenomena about the south pole, sometimes consisting only of unusual disturbance of the magnetic needle. The eighteenth century was very prolific of northern lights, particularly the middle part of it; since 1820 they have also become much more frequent. *Pl.* 14, *fig.* 57, is a view of the remarkable Aurora Borealis which was seen at Christiania in Norway, at 6 o'clock in the evening of January 7, 1831. *Fig.* 58 represents an Aurora Australis or south polar light.

The *mock suns* and *mock moons,* or *parhelia* and *parselenia,* are a result of a peculiar reflection of the light of the sun, of the moon, and even of the brighter planets and fixed stars, upon particles of condensed vapor or ice crystals. When these halos are very much complicated about the sun and moon, they appear as if composed of many circles intersecting each other, of which one is generally horizontal, encircling the whole heavens. It is in this case that they form mock suns and mock moons. Generally, only two or four are seen; Hevelius, however, in 1660, saw six at once; sometimes they have comet-like trains. *Pl.* 14, *fig.* 59, exhibits a view of two such mock suns.

The Shooting Stars.

63. The *shooting stars* have been long and universally known, but it is only in recent times that their true character, their peculiarities, and the various circumstances under which they are perceived, have been matters of observation to astronomers and meteorologists. The periodical and abundant return of the shooting stars towards the middle of August and November, has in many places been diligently observed and investigated. We need only mention the efforts of Benzenberg, Brandes, Olbers, Bessel, Erman, Boguslawski, Quetelet, Feldt, Herrick, and Olmstead, to ascertain their direction, their height above the earth, and their velocity. The reasons for the now generally received hypothesis (of Alexander von Humboldt) are well known; his theory being that these luminous appearances are caused by innumerable small bodies revolving about the sun, which become visible by their combustion when entering the atmosphere of our earth. There are yet many difficulties in the way of the establishment of this theory, as also of the supposition of Biot, that these falling stars are the same bodies which, seen at a distance, form the zodiacal light.

Benzenberg and Brandes divide all shooting stars into three classes: 1, of the first and second magnitudes, similar to balls of fire, in which may be distinguished a ball with a luminous train; 2, of the first and second magnitudes, without the ball, and with a luminous path; 3, from the third to the sixth magnitudes, the last being telescopic, and only visible through a comet seeker.

The number of shooting stars is incredibly great. Humboldt and Bon-

pland observed a magnificent fall of these bodies in 1799, and Brandes counted during one night, in a fifth part of the horizon, 480 shooting stars, from which he estimated the whole number, during the same time, at some thousands. Every one is familiar with the extraordinary showers which have appeared in America at various times. Benzenberg estimates the mean number appearing every night to be 30, 50, and even more. The train appearing behind the greater shooting stars deserves particular attention. Brandes has accurately observed and described the remarkable appearances presented by the last.

The height of these bodies is very various; some of them seem tolerably near the earth, while others are beyond the outer layers of the atmosphere, assuming this at 80 to 120 geographical miles from the surface of the earth.

The velocity of these meteors amounts, according to calculation based on observation, to between 16 and 32 miles in a second, thus reaching sometimes twice the velocity of the motion of our earth in space. Olbers has first shown how the shape of their orbit can be determined.

With regard to the substance of which the shooting stars are composed, nothing satisfactory is known.

The Antipodes of our Earth; the Habitableness of the Worlds of the Solar System.

64. The inhabitants of our earth may be considered, astronomically, under two points of view, namely, in respect to the degrees of latitude and longitude under which they live, and also in respect to the direction in which their shadow falls. By *Antipodes* is meant the inhabitants of that place on the earth's surface lying the distance of a complete diameter of the globe from some other place, as, for instance, Leipzig. The antipodes have an opposite geographical latitude, and a geographical longitude differing by 180 degrees; consequently, with the exception of those near the equator, an opposite time of year, and a time of day differing by 12 hours. Leipzig, for instance, has a north latitude of 51°, 20′, 20″, and a longitude 30°, 1′, 52″ east of Ferro; the antipodes of Leipzig, therefore, must dwell on a part of the earth's surface having a south latitude of 51°, 20′, 20″, and a longitude of 210°, 1′, 52″ east of Ferro. This point, however, lies in the great southern ocean, and, consequently, there are no proper antipodes to Leipzig.

Antæci (αντιοικοι) are the inhabitants of two places on the earth's surface, lying under the same meridian and at the same distance from the equator, but on opposite hemispheres. These have noon at the same time, but of course opposite seasons. At the two poles the antæci are also antipodes.

Periæci (περιοικοι) are those persons, who, living under the same latitude, differ in longitude by 180°. They have consequently the same seasons, but their days differ by twelve hours. At the equator, antipodes and periæci are synonymous.

65 Heteroscii (ἑτερο-σκιοι), or one-shadowed, are those inhabitants of the earth whose shadows among themselves always fall in the same direction; in an opposite direction, however, to the inhabitants of the opposite hemisphere. Consequently, the inhabitants of the temperate zones are one-shadowed; the shadows in the north temperate zone falling north, those in the south temperate zone, south.

The inhabitants of the frigid zones acquire the name of Periscii (περι-σκιοι), at the time during which the sun remaining above their horizon causes their shadows to move in a circle around them in the space of 24 hours.

Amphiscii (αμφι-σκιοι), or double-shadowed, are those inhabitants of the tropics who for one half of the year have their shadows directed towards the south, and for the other half towards the north. The cause of this lies in the fact of the latitude being less than the obliquity of the ecliptic. At any place whose latitude is, for instance, 17° north or south, a body will cast no shadow at all at noon of the day, when the declination of the sun is likewise 17° north or south. For this reason the Amphiscii are then termed Ascii (α-σκιοι), or shadowless. The inhabitants of the equatorial line itself will be shadowless on the 21st of March and 23d of September, since on these days the sun stands in the celestial equator. It has, consequently, then no declination, and the inhabitants of the line have no latitude.

66. It is not absolutely impossible that the sun may be habitable, as the black places which are seen in the middle of the sun's spots represent, perhaps, not the true nucleus of the sun, but only an obscure atmosphere similar to our own. The inhabitants of the sun, therefore, undazzled by the piercing light of the external solar surface or photosphere, and protected from its excessive heat, may live, ignorant, however, of the alternation of day and night. Neither can they know anything of seasons, since they are surrounded on all sides, and at all times, by the sources of heat and light. They can thus know nothing of the existence of the planets, moons, and comets; and can perceive the starry heavens in their beauty only through the openings which those violent agitations in the sun's photosphere sometimes produce.

67. Although *Mercury* receives an illumination from the sun almost seven times greater than that of our earth, the heat thus produced may be very greatly tempered by the numerous high mountains with their long shadows, the rapid alternation of the seasons, and the probably very rare atmosphere. In this manner an alleviation of temperature may result, which will without difficulty admit of this planet's being habitable.

Certain phenomena must take place in the climate and seasons of *Venus* which hardly admit of a comparison with ours. A day of Venus is very nearly equal to one of the earth, but it is very different with respect to the seasons, if we assume, as established, that the axis about which Venus rotates daily is inclined to its orbit at an angle of nearly 72°, and that thus the obliquity of her ecliptic is nearly three times that of the earth. It is well known that the seasons are determined by the obliquity of the ecliptic, and that, consequently, a much greater obliquity than that of our earth must necessarily involve a corresponding influence upon the temperature, and

especially upon the alternation of the seasons; it may, therefore, be assumed for Venus, that when the given circumstances take place, the torrid zone, whose inhabitants have the sun in their zenith, must cover a space of 144 degrees in breadth, while the breadth of this zone on our earth is but 47°. The frigid zone will lie at the two poles, extending 18 degrees towards the equator. The inhabitants of this zone will never see the sun in their zenith. In the temperate zone, occupying a space of 54 degrees from the frigid zone towards the equator, the inhabitants will not see the sun at all during one part of the year, and during another will have him in their zenith, thus resembling the inhabitants of both frigid and torrid zones on our earth. Hence it happens that the dwellers on Venus must experience such sudden alternations of season and climate as our imagination can hardly comprehend.

68. With respect to our moon, the curious arrangement of day and night, the want of an atmosphere similar to ours, of great bodies of water, and of seasons, as also the universal sterility and drought which seem to reign upon its surface, must undoubtedly exert an influence upon vegetable as well as animal life, of which we can have no adequate idea. The inhabitants of the moon, therefore, should such exist, must be very different from those of the earth. Even the view of the starry firmament from the moon must be very peculiar, since to its inhabitants the earth never sets. They who live in the centre of her visible disk always have the earth in the zenith, while those living on the border see us in their horizon. Sun, planets, and all other celestial objects, complete their paths in 29½ days, and thus rise and set, to the moon, every 14–15 days. The difference of seasons vanishes almost entirely on the moon, on account of the slight inclination of the equator to its orbit; and while the inhabitants of her equator have the sun always in the zenith, those of the poles see him always in the horizon. In the former part, therefore, there reigns an eternal summer; in the latter, an eternal winter; while intermediate between the two, a perpetual spring prevails. Days and nights are ever almost equal in length.

69. The physical condition of Mars most nearly resembles that of the earth; it is impossible, however, to offer any plausible hypothesis with regard to the asteroids, Vesta, Iris, Astræa, Juno, Ceres, and Pallas, as their distance and diminutive size conceal from us the peculiarities of their surface.

70. Since the various alternations of Jupiter follow each other with great rapidity, his hypothetical inhabitants must possess great quickness of mind and body. If their size bears any proportion to that of the planet, their height must be about 70 feet.

The inhabitants of *Saturn*, if any, must be entirely different from ourselves, and we have no cause to envy them. The nights of Saturn, as also the winters, last 15 years; there are total eclipses of the sun which last whole years; the sun appears ninety times less to them than to us; and, for years, total darkness and general torpidity, in all probability, reign supreme.

Whether the ring and the seven moons of Saturn, and whether Uranus

and Neptune with their moons, are habitable, are questions which must be left unanswered, as these planets, although little smaller than Saturn, are so remote as to render it impossible for us to ascertain the peculiarities of their physical condition by means of our telescopes.

As it is hardly supposable that all the planets and moons revolve about the sun as uninhabited worlds, and that only the earth has the prerogative of being peopled with any kind of created beings, so it seems not impossible, yea, rather entirely suited to the omnipotence of the Deity, to assume a certain habitableness of the comets. Shall these heavenly bodies, which are by far the most numerous of all the worlds belonging to our solar system, and in proportion to which the numbers of primary and secondary planets vanish away—shall these be entirely uninhabited, simply because man cannot comprehend of what sort the beings dwelling on the comets must be? In fact, it is a sentiment completely recognising the all-wise beneficence of the Creator, to presuppose that, upon those worlds wandering through such an immense extent of the heavens, certain beings may exist, as comfortable and happy as man, who so complacently considers himself as lord of the earth.

The Calendar in General; the Greek, Julian, Gregorian, Russo-Grecian, Jewish, Turkish, and French Republican Calendars in particular.

71. The Calendar belongs, from its nature and particular application, to the department of Chronology. Chronology, however, forms a part of astronomy, and, indeed, is a part of the greatest importance and most material value to the sciences, particularly the historical and political. For this reason it will not be superfluous to present here the most important features of the different calendars.

72. *Calendar* means partly the division of time employed by any people into definite years, months, &c., and partly the register of single days answering to a certain year of such a division. The word calendar is derived from the Latin calendæ (καλεω, *Lat.* calo, to call), by which name the Romans indicated the first day of every month, whose name and new moon were proclaimed by the priests, the calendar of any given year containing not only the religious and political festivals, but also the most important celestial events of that year. Among the latter are to be reckoned especially, the rise and setting of the sun and moon, the length of days and nights, the quarters of the moon, the eclipses of the sun and moon, the appearance of the planets, &c. We distinguish also, not only the calendars of different nations, but also special calendars, according to purport or object. Thus, for instance, we have astronomical, civic, centennial, economical, people, and states' calendars. We shall now hurry over the calendars of the most important nations in order.

Calendar of the Ancient Greeks.

73. The ancient Greeks assumed a lunar year of 354, later of 360 days or 12 months of 30 days each, which they then sought to accommodate to the true solar year by intercalations. In honor of the Olympic games, the beginning of the year was placed at the first new moon after the summer solstice. Nevertheless, this did not always fall in July, as the Olympiads themselves consisted sometimes of 49, sometimes of 50 months. This Greek calendar was as complicated as the Macedono-Grecian calendar, introduced at a later period by Philip of Macedon, which commenced its year at the autumnal equinox, and which was employed for the names of the months, but not in their order, by the Greeks, Phœnicians, Babylonians, Medes, &c. The months of 30 days were called *full;* those of 29 days *deficient;* and each of these fell into three decades. By the gradual introduction of the Roman calendar among the nations subjected to that power, the Grecian fell into disuse.

Calendar of the Romans; the Julian Calendar.

74. The Roman calendar, improved by Numa Pompilius, was based on a lunar year, having one more day than a solar year. The first day of every month (the new moon) was called the calend, and besides this, two others were distinguished, the 18th before every new moon the *Ide,* and the 9th before the Ide the *None.* According to the reckoning of Numa, four months (Martius, Maius, Quintilis, and October) contained 31 days, and the rest 29 days, except February with 28, and the Macedonian month which was intercalated every two years with 22–23. There were therefore nones of seven and of nine days; with later arrangements the Ides also happened differently. The intermediate days were counted backwards from the Ides of the same, or the calends of the next month, as also from the nones of the same month. The Romans had besides periods of nine days, called *nundinæ,* which were indicated by the letters A to I. The *dies fasti* and *nefasti* were days of good and bad omen. Through the irregularity of distribution, and the then low state of astronomical science, it came to pass that about 46 years B. C. the Roman calendar varied about 59 days from the true day (that is, from the true place of the sun in the ecliptic). This induced Julius Cæsar, by the advice of the Alexandrian mathematician, Sosigenes, to adopt the *solar year* of 365 days, 6 hours, and so arrange the calendar that every year divisible without remainder by 4, should consist of 366 days, the rest consisting of 365. With respect to the subdivisions of the calendar, Julius Cæsar retained the old terms of calends, nones, and ides. This calendar is called the Julian.

Gregorian Calendar.

75. The Christians retained the astronomical arrangement of the Julian calendar in general, but established weeks of several days; January was the first month, December the last: January, March, May, July, August, October, and December, had 31 days, while April, June, September, and November, had 30. February had generally 28 days, except during the intercalary or leap year, when it had 29. At a later period, however, the length of the year was altered. Julius Cæsar had originally assumed too great a length for the year. This required, even in the time of Augustus, a correction of three days, and in 1581 the error had again amounted to ten days. The then reigning pope decreed on the 24th of February, 1581, an improvement of the calendar, by which ten days should be omitted from October of that year (counting 4, 10, 15 October), and in four centuries three leap years should be omitted, by which the years 1700, 1800, 1900, 2100, &c., were not leap years. This Gregorian calendar was soon introduced into all Catholic countries, while the Protestants retained the old system until 1699. In the year 1700, in February of the new improved calendar of the German Protestants, ten days were omitted, so that March 1st came immediately after February 18th. With respect to Easter, which in the Gregorian calendar of the Catholics was counted after the Epact, the Protestants assumed a purely astronomical mode of calculation, so that this always came on the Sunday after the first full moon succeeding the vernal equinox (thus between March 22 and April 25). This improved calendar agreed for the most part with the Gregorian, but with respect to the different determinations of Easter, cases soon occurred in which one calculation made Saturday, and the other Sunday, the day which determined the celebration of Easter on the Sunday following. According to the latter calculation, therefore, this festival would happen eight days later than as determined by the former, which produced great confusion among those Catholics and Protestants living near each other. When, in 1778, this was about occurring, Frederick II. of Prussia succeeded in persuading the Protestants of Germany to adopt the determination of Easter by the Epact. To this arrangement the other Protestant states of Europe have also conformed.

It will be necessary to add a few explanations of certain expressions which are connected with the determination of single days and festivals in our calendar. The determination of Sundays and week days depends upon the *Sunday* or *Dominical Letter*, and the *Solar Cycle*. The Sunday letter is that which falls upon the first Sunday, when we call the first day of the year A, and count from A to G. Should the year be leap year, count the Sundays after February 24th one further. It is necessary to know these letters to understand the construction of the perpetual calendar.

The number of years after which the same week days fall upon the same date again, is determined by the solar cycle. Were there no leap years, then, as the date annually advances one on the days of the week, the same

date would fall every seven years upon the same day of the week; since, however, every fourth year is leap year, this coincidence takes place every 28 years, which form one solar cycle, the series beginning the ninth year B.C.; thus, in 1847, the solar cycle is VIII., that is, it is the eighth year of a sun cycle in the nineteenth century.

The *Lunar Cycle*, the *golden number*, and the *Epacts*, serve to determine the phases of the moon, and occurrence of Easter. The lunar cycle is the series of years after which the new and full moons again fall upon the same day of the year. This cycle amounts to 19 years of $365\frac{1}{4}$ days, and the number of any year indicating its place in the cycle, is called the *golden number*. This cycle was discovered by Meton, 430 B.C., and the golden number is the remainder after adding one to the number of years, and dividing by 19. The quotient indicates the number of lunar cycles which have elapsed since A.D. up to the given time. Should the golden number be one, then the new moon falls on Jan. 1. For the determination of the new moon of any other years, the *Epacts* are employed, or the number which, for every year, determines the age of the moon on New Year's day, or gives the number of days by which the last new moon of the previous year preceded New Year's day. Should the new moon fall on Jan. 1, then the Epact = 0.

As the lunar year is 10 days, 15 hours, 11 minutes, 25 seconds, or, in round numbers, 11 days shorter than the solar, it follows that the Epacts increase annually by 11; so that if, in one year, the Epact was 23, in the following it would be 3, as the series commences with 1 after 30. The passage from the last year of a lunar cycle to the first of the following, amounts, not to 11, but to 12 days, and is called the *leap of the Epacts*. This intercalary day is necessary, from the fact that the lunar year is not quite 11 days shorter than the solar year. For the year 1847 the golden number is 5, and the Epact 14. Knowing the Epacts, it is possible in a perpetual calendar to determine immediately every day upon which new moon falls. Thus, for the Epact 14, of 1847, the first new moon falls on the 16th of January; nevertheless, small errors cannot always be avoided, as, for instance, is the case in the year mentioned, when the first new moon actually occurs early on the 17th January, at 1 hour, 34 minutes. After the occurrence of the new moon, we can readily determine the remaining phases of the moon.

RUSSO-GRECIAN CALENDAR.

76. The calendar of the Russians and Greeks has, in respect to the months, weeks, main festivals, &c., the same arrangement as the Gregorian calendar; nevertheless, these nations have retained the Julian in its essential features, that is, with respect to Easter and the festivals dependent upon it. They are, therefore, about 13 days behind the Gregorian date now, and will, in 1900, be 14 days; so that in the present century, for example, when they have March 1st according to their calendar, it will be March 13th

according to the Gregorian. The Greeks and Russians have, thus, the *old style* reckoning; the other Christians, the *new style*.

JEWISH CALENDAR.

77. The Jewish years are lunar years, counted from Oct. 7th of 3761 B.C. The Jews have, according to the improvement of Rabbi Hillel Hanaffi, a cycle of 19 years, among which 12 are common, and 7 intercalary years. One of the former has 354 days, 21 hours, 48 minutes; one of the latter, 383 days, 21 hours, 48 minutes. To satisfy the priestly arrangement there are six different years, namely, three *common years* of 12 months each, the short year having 353, the mean year 354, and the long year 355 days; and three *intercalary* or *leap years*, of 13 months each, the short year having 383, the mean 384, and the long 385 days. The beginning of the year can fall neither on Sunday, Wednesday, nor Friday. The months of the civil year are in order as follows: TISCHRI, MACHESVAN, KISLEV, TEBETH, SCHEWAT, ADAR, W'ADAR (intercalary month), NISAN, JJAR, SIVAN, THAMUZ, AB, and ELUL. The religious year begins with the month NISAN, in which the principal festival, the *Passover* (Easter), falls, and, indeed, always on the 15th NISAN. This Passover, which can never fall on a Monday, Wednesday, or Friday, is of the greatest importance in the arrangement of the Jewish calendar, and generally occurs in our Passion week. In conclusion, it may be stated, that our Saturday is kept under the name of Sabbath (day of rest) just as Sunday is with us.

TURKISH CALENDAR.

78. The Turks and almost all adherents of Mohammedanism count their years from *Hedschra*, or *Hegira* (July 15, 622 A.D.). They have a cycle of 30 years, each consisting of 354 days, except 11, which are leap years of 355 days. Their year, whose mean length amounts to 354 days, 8 hours, 48 minutes, is divided into 12 months of 30 and 29 days: MOHAREM, SEPHER, RABI EL AUWAL, RABI EL ACHAR, DSJOMMADA EL AUWAL, DSJOMMADA EL ACHAR, RADSJEB, SCHABAN, RAMADAN, SCHAUWAL, DSULKADE, and SULHADSJE. In the leap year, the last month, SULHADSJE, instead of 29, has 30 days, the latter being the intercalary day. The festivals of the Turkish calendar occur unchangeably on the same day of the month.

CALENDAR OF THE FRENCH REPUBLIC.

79. By a decree of the French National Convention of Oct. 5th, 1793, a new reckoning of time was adopted, dating from September 22d, 1792, on which day, previously fixed upon, the establishment of the Republic was decreed. As this was also the date of the equinox (at $9h\ 18'\ 13''$ of the morning) there was an allusion given to equality both of days and rights.

The following years were also to begin with the midnight preceding the true autumnal equinox. This new French year was divided into 12 months of 30 days, and to complete the number of days, five, or in leap year six supplementary days (*jours complémentaires*), were added. Instead of weeks, the months were divided into three decades of 10 days each. These ten days were called *Primidi, Duodi, Tridi, Quartidi, Quintidi, Sextidi, Septidi, Octidi, Nonidi, and Decadi.* The nomenclature of the months was derived from the characteristics of the seasons, as follows :—

For Autumn: Vendemiaire, or vintage month (October) ; *Brumaire,* or cloud month (November) ; *Frimaire,* or frost month (December).

For Winter : Nivôse, or snow month (January) ; *Ventôse,* or wind month (February) ; *Pluviôse,* or rain month (March).

For Spring: Germinal, or growth month (April) ; *Floréal,* or bloom month (May) ; *Prairéal,* or meadow month (June).

For Summer: Messidor, or harvest month (July) ; *Thermidor,* or heat month (August) ; *Fructidor,* or fruit month (September).

The additional days are attached to this last month, and have the following appellations:—1*st, Fête du génie;* 2*d, Fête du travail;* 3*d, Fête des actions;* 4*th, Fête des recompenses;* 5*th, Fête de l'opinion.*

In addition to what has already been said, every day of the year had its especial name, which, instead of being taken from some saint, was derived from objects of agriculture appropriate to the time on which the days fell. This calendar, however, lasted only 12 years, for Napoleon abolished it by a Senate decree of September 9, 1805.

ASTRONOMICAL INSTRUMENTS ; OBSERVATORIES.

80. *Astronomical instruments* embrace all the apparatus which the practical astronomer needs in his observations of celestial phenomena, to impart to them that accuracy and certainty so necessary as the basis of delicate calculations, as also to find objects in the heavens invisible to the naked eye. Astronomical instruments have two purposes: the one is to afford a clear understanding of those objects and phenomena of the heavens, which, on account of their distance or minuteness, are either imperfectly or not at all visible to the naked eye ; the other purpose is the accurate measurement of various angles and spaces. The following may therefore be considered as the most important astronomical instruments :—Transit Instrument, Equatorial, Refractor, Meridian Circle, Universal Instrument, Comet Seeker, Heliometer, Simple Circle, Theodolite, Multiplication Circle, Reflecting Sextant, Barometer, Thermometer, Pendulum Clock, Chronometer, &c. Of the more ancient instruments, only the Zenith Sector and Mural Quadrant are retained at the present day ; all the rest, as, for example, the Octants, Quadrants, Telescopes without tubes, &c., are consigned to deserved oblivion.

The following are the names of some artists distinguished for the excellence of the astronomical instruments constructed by them. Those

marked + are deceased. Baumann (in Stuttgart); Breithaupt (in Cassel); Cauchoix (in Paris); Dollond+ (in London); Douve+ (in Berlin); Emery+, Ertel (in Munich); Fraunhofer+ (in Munich); W. Herschel+, Jürgensen (in Copenhagen); Kessels (in Altona); Lerebours (in Paris); Merz and Mahler (in Munich); Oertling and Pistor (in Berlin); Plössl (in Vienna); Ramsden+ (in London); Repsold, sen.,+ and Repsold Bro's (in Hamburg); Reichenbach+ (in Munich); Lord Rosse (in Ireland); Troughton+ (in London); Voigtländer (in Vienna); and others.

THE TELESCOPE; ITS DIFFERENT CONSTRUCTIONS.

The Dorpat Refractor.

81. The Telescope is that optical instrument which represents distant objects more distinctly and of larger size than they appear to the naked eye. There are two principal kinds of telescopes, dioptric or *refractors,* and catoptric (or more properly catadioptric) or *reflectors.* In refractors, observations are conducted by means of glasses alone, as, for example, the eye glass and object glass; in reflectors a concave mirror is used instead of the object glass. Both were invented in the sixteenth and seventeenth centuries; the refractor in 1590, by Jansen of Middleburg, and the reflector in 1644, by Mersenne.

Refractors consist of a cylindrical tube at whose two extremities are placed the lenses employed. The lens receiving the rays of light from the object, is called the *object glass* or *objective.* The one through which the image produced is seen by the eye is called the *eye-glass* or *ocular.* Eye-glasses are *simple* when but one lens is used, *compound* when several lenses are combined together. Besides the *magnifying power* of the telescope, reference must be had to the *diameter of the field of vision,* the *illumination* or *amount of light,* and the degree of distinctness of vision.

There are three kinds of refracting telescopes; the *astronomical,* invented by Kepler; the *terrestrial,* invented by De Rheita, 1665; and the *Galilean.* The astronomical telescope is formed by the combination of two convex lenses; the one, the eye-glass, can be made to approach to or recede from the object glass. This form of telescope, although it represents objects inverted, exhibits them very clearly and much magnified; having also a large field, it is on this account principally used by astronomers.

The second kind, or the terrestrial telescope, is a combination of four convex lenses; three of them, fixed immovably in one tube, can be made to vary their position with respect to the fourth—the object glass. This telescope, which exhibits objects erect, may be considered as a combination of two astronomical telescopes, of which the one represents the inverted image of the other again inverted, consequently actually erect. Nevertheless, it is more advantageous to furnish the terrestrial telescope with four eye-glasses, as is generally done. The eye-glass nearest the eye thus

obtains (as also happens in astronomical telescopes) a second eye-glass viz. the so called *field glass*, for the purpose of enlarging the field of vision.

The Galilean telescope is the simplest of all, consisting of but two lenses—a convex object glass, and a concave eye-glass; this represents objects erect. It has, however, the inconvenience of possessing a very small field of vision, especially when the eye is not exceedingly near the eye-glass.

The common telescopes, however, do not exhibit any very great degree of clearness of vision, as all images produced by them are surrounded by a colored border, and a considerable magnifying power can only be obtained with a length of 5 or 6 feet. Dollond remedied this defect in 1757 by his *achromatic* (colorless) object glasses. An achromatic object glass consists of two highly polished lenses, combined so as to be everywhere in absolute contact, the one convex, made of crown glass, the other concave, of flint glass. The two kinds of glass have different refractive powers, so that chromatic aberration is reduced almost to a minimum. Telescopes with such object glasses, and of a moderate length, afford with a considerable magnifying power, and great brightness and distinctness, images almost entirely without colored borders. Very large instruments of this kind are known pre-eminently as refractors. The preparation of the glass required, so as to be everywhere uniform and without bubbles and streaks, presented great difficulties, until they were surmounted by Fraunhofer and Uzschneider.

On account of the differences in eyes, and the varying distances of objects, a special adjustment is required for the eye-piece. In small telescopes this is done by moving the tube in or out with the hand alone; in larger instruments a screw adjustment is necessary, by which the eye-piece (ocular) can be moved almost insensibly. The larger telescopes are set up either on a three-legged frame or upon a pyramidal support, and their motions are either horizontal and vertical, or for astronomical purposes parallactic, i. e. applicable to any direction.

As an illustration of the mode of constructing and setting up a great telescope, we have selected the giant achromatic refractor constructed by Fraunhofer for the observatory of Dorpat. The instrument arrived there November 10th, 1824, and the first glance was directed by Struve, six days afterwards, towards the moon and some double stars. The principal parts of the instrument (*pl.* 15, *fig.* 2) are—1, the stand A, A, A; 2, the axes F and I, with their circles *d* and *k;* 3, the telescope B, B; 4, the counterpoises E, E′, K, M, H; 5, the clock-work, *e, f, g,* with the weights. The stand is parallactic; upon it rests the hour axis F, parallel to the polar axis, with its hour circle, *d,* at the lower end. A second axis, I, stands perpendicular to the first, and consequently in a rotation of the instrument about the latter, describes the plane of the equator. At one end of this second axis is the declination circle, *k;* at the other, the bed of the wooden tube, B, B, of the telescope, with brass caps at the two ends for the lenses, and the finder, D, D, at the upper side. The counterpoises are five; two, E and E′

on the telescope, to balance the greater weight of the anterior half of the instrument; two, M and K, in the direction of the declination axis; and one, H, to diminish the friction of the hour axis. The clockwork, *e*, *f*, *g*, lies on the left side of the hour circle, *d*, and acts upon an endless screw which catches in it, and even without the clock-work, may be employed to produce a gentle motion about the hour circle. A second micrometer screw, *i*, is attached to the declination circle, *k*, to produce the adjustment in the declination. In a perpendicular position of the telescope, the height of the whole instrument amounts to 16 Paris feet, and the weight to about 4000 Russian pounds. The steel hour axis, 39 inches in length, carries the hour circle of 13 inches diameter, with a graduation on silver on its lower face; each of the two verniers reads to within 4 seconds of time. The declination circle is 20 inches in diameter, and its motion is regulated by a clamp and micrometer screw. The circle itself has a graduation on silver, and the vernier reads to 10″. The length of the telescope is $13\frac{7}{12}$ Paris feet, with a diameter of 10 inches at the upper end, and $7\frac{3}{4}$ at the lower. The object glass has a free aperture of 9 inches, and a focal length of 160. The four free oculars of 1420, 210, 320, and 480 magnifying power, as also the various micrometer apparatus with 14 oculars, can be screwed into the end of the draw tube. The aperture of the objective of 30 inch focus, belonging to the brass finder, amounts to 29 Paris lines. This finder has two oculars of 18 and 26 magnifying power. The clock-work intended to give to the telescope a motion on its hour axis, uniform with that of the fixed stars, and represented on a larger scale in *fig.* 20, consists of two principal parts; the clock proper and the wheelwork connecting this with the hour circle. Both are fastened to the bearing piece at the lower end of the hour axis. This wheel consists of an axis to which are attached the two wheels *d* and *e*; on *d* the disk *f* (the latter is omitted on the figure) is screwed; a smaller disk, *g*, is attached anteriorly by the screw *h*. An endless thread with the weights runs over *f* and *g*. At *i* is seen the spring and trigger catching in the teeth of the disk *g*; *d* is in connexion with the little wheel *k*; *l* and *m* are apertures. The clock drives an axle, *n*, with a double-motioned endless screw, working in the wheel, *e*. The motion of the clock is regulated by a centrifugal balance wheel which is connected with the weights by means of three wheels and pinions; *p* and *q* are cog-wheels, *r* a crown wheel acting on the pinion of the perpendicular axis *t*. The centrifugal balance wheel works within the box *u*; the parts, *w*, *x*, *y*, *z*, serve to regulate the motion of the clock, which, as well as that of the friction weight, continues to move more than an hour. Both the clock and the friction weight may be removed without disturbing the motion of the telescope. The micrometer arrangements inside the telescope are adjusted with great accuracy, but their enumeration and explanation here would carry us too far out of our way.

The value of this great refractor consists in its optical completeness, in the great accuracy of its adjustments, in the regularity of rotation about the hour axis by means of the clock, as well as in the perfectly unalterable micrometrical apparatus. In defining power and intensity of illumination,

this refractor leaves all known reflecting telescopes far behind. Struve saw the multiple star τ Orion certainly 16-fold, while Schröter with his 25 foot refractor only saw it 12-fold; the little companion of the star β Orion (Rigel) was seen through the Dorpat refractor very distinctly just before sunset; and even ω^2 Leonis, one of the most difficult double stars, in this instrument, was recognised without any difficulty as a double star. In conclusion, it remains to be mentioned, that this telescope is placed in a building specially arranged for rotation. The cost of the instrument amounted to 10,500 florins ($5000).

More recently the observatory at Pulkowa has received a somewhat larger refractor from Munich, whose object glass has an aperture of $14\frac{9}{10}$ Paris inches. The new Berlin observatory also possesses a large refractor of remarkable excellence from this same celebrated manufactory at Munich. The observatories of Cambridge, Washington, and Cincinnati, likewise possess refractors of great power.

When in use, the telescope is directed to the object to be investigated, whose motion is followed by changing the position of the instrument. If the telescope be arranged as in the Dorpat refractor, all the clamps are to be loosened, the object found with the seeker, and the instrument then fixed; upon which, by means of the clock-work, it is moved in such a manner as always to have the object in the field of vision.

The Reflecting Telescope; Herschel's Giant Telescope.

82. Catoptric (catadioptric) telescopes, commonly called reflectors, form the second grand division of these instruments.

The reflector is a telescope which, instead of an object glass, has two mirrors, an objective and a reflecting mirror. Newton constructed the first reflecting telescope of the form now bearing his name. This consists of a hollow cylinder, so placed upon a frame as to be readily directed to any point of the heavens. The one end of this cylinder is closed by a spherically concave metallic mirror or speculum, whose focus lies in the common axis of the cylinder and mirror. At a little distance from the focus of the speculum is placed a plane mirror inclined to the axis of the cylinder at an angle of 45°. A beam of light impinging upon the concave mirror, is reflected in a cone upon the small plane mirror, and thence into the ocular placed in the side of the instrument and at right angles to it. This arrangement, therefore, represents objects inverted, unless, as in the terrestrial telescope, erecting lenses be placed in the eye-piece. After Newton's death another form, the *Gregorian*, with the smaller reflector concave, came into use; this represented objects erect. A third kind, the Cassegrainian, was also introduced. This last has the small anterior reflector convex instead of plane, as in the Newtonian, or of concave, as in the Gregorian; objects are represented in it inverted.

Herschel improved the Newtonian telescope by omitting the small reflector, fixing the large concave speculum at a slight angle to the axis.

The focus consequently was formed at the anterior edge of the instrument, where it was received by the ocular. The observer sat directly in front of the open tube looking through the ocular.

In this manner Herschel constructed his 20 and 30 foot telescopes; the 20 foot with a speculum of 10 inches diameter. A seven foot reflector finished by Herschel in 1780 was of great excellence; with it he discovered Uranus on the 18th of March, 1781. The magnifying powers were 230, 460, and 930; but to his greater reflector Herschel could apply powers of 500–2000, without overtasking them for strongly illuminated objects. The giant 40 foot telescope completed in 1789, is represented in *pl.* 15, *fig.* 1. The tube, DD, constructed of sheet-iron, was 40 feet long, 4 feet 10 inches in diameter, and the whole telescope weighed about 5100 pounds; the great speculum alone weighed 2148 pounds. The magnifying powers of the instrument were, for the planets, 250 and 500; for the fixed stars, 1000–6400. The distinctness of the objects seen is said to have been astonishingly great. The cost of the whole apparatus amounted to about £2000 sterling. During observations with this colossal instrument, Herschel sat at the side of the tube at its upper end, in a frame H, fixed to the ladders, G, G, which accompanied the tube in its movements. He thus looked into the instrument, with his back to the star, and examined this latter directly with the eye-glass. Unfortunately the mirror, during a single damp night, lost its polish, and the whole instrument in a few years after its construction was entirely useless. The figure gives, without the necessity of further explanation, an idea of the strong scaffolding between which the telescope could be moved in a perpendicular direction by means of several ropes, AE, FE; the horizontal motion of the whole apparatus, scaffolding and tube, was produced by a rotation by means of rollers running upon the periphery of a horizontal circular railway, ABAB. Around and above the whole was built a round tower with a revolving roof, whose opening could be brought towards the part of the heavens to be observed. More recently Lord Rosse has constructed a gigantic telescope, more than 12 feet longer than that of Herschel, having a speculum of 6 feet in diameter.

The Mural Quadrant.

83. As long as astronomical observations did not possess that degree of accuracy now exhibited, the *mural quadrant* was, of all instruments used in measuring altitudes, the most useful. *Pl.* 15, *fig.* 19, represents the celebrated mural quadrant of Tycho. This had a radius, DC, of eight feet, by means of which, aided by a vernier, E, very small arcs could be read off on the limb CC. The iron grating, DCC, which formed the body of the quadrant, was fastened to a wall, GAA, placed in the meridian, and the rule DD, with the telescope, moved up and down on this grating. In this manner it was possible to observe not only the passage of the meridian by a star, but also its altitude or zenith distance. The depending plumb-line, DA, served to fix the quadrant in its proper position in the vertical plane.

Movable quadrants were also used; and the constructions of Dollond and Troughton (*fig.* 18) were the most convenient. The principal part consisted of the quarter circle, EF, and two attached radii, IE and IF, perpendicular to each other, all of metal. Through the centre of gravity of the movable part of the whole instrument, passed a cylindrical tube, fastened to the quadrant IFKE, and including the axis of rotation; this gave off a vertical post resting on a solid base, AAD, adjustable in a horizontal plane by the screws B, B, B. This post was received in such a manner into a tube, C, fastened to the base, as always, in rotation, to preserve a vertical position. The body of the quadrant was united in such a manner to this part containing the axis of rotation, as to have its plane constantly vertical, and parallel to the axis of rotation. Upon the part containing the axis of rotation was attached the azimuth circle, DD′, graduated to ten minutes, readable to ten seconds by means of a vernier. The quadrant, divided to five minutes, was readable to single seconds by the micrometer, G. The quadrant, with the movable telescope, KL, was so placed that one of the above mentioned metal radii was rendered perfectly horizontal by means of an attached level. H was a lens for reading off the graduation. Finally, as another means of determining the vertical position, a plumb-line was suspended in the above mentioned tube of the axis of rotation, whose proper position was given by four microscopes. K was the position occupied by the observer when looking at the stars through the telescope KL.

The Transit Instrument, or Meridian Telescope.

84. The transit instrument, one of the most important instruments of practical astronomy, was invented by Roemer in 1706. It is intended to obtain with greater accuracy the right ascension of a star, and consequently the solar time. It consists (*pl.* 15, *fig.* 13) of an astronomical telescope, FD, fastened at right angles to a horizontal axis, B, and movable up and down in such a manner that the plane described always lies in the plane of the meridian of the place of observation. For the sake of the greatest possible firmness, the pillars, AA, upon which the two pivots of the horizontal axes rest, must be fixed separately, each one consisting of a single block of granite, and going deep into the earth, without any communication with the masonry of the building.

Portable transit instruments have also been constructed, differing, however, from the fixed only in their smaller size and their being adjustable to any point. *Pl.* 15, *fig.* 22, represents such a portable transit instrument. Although the transit instrument is not usually employed to obtain the meridian altitude of a star, yet for the approximate attainment of this end, a circle graduated to numbers is fastened to one side of the instrument in a vertical plane, as at I in *fig.* 13, and D, *fig.* 22.

In the fixed, heavy transit instruments, it is absolutely necessary, for the purpose of lessening the friction, and the wearing of the pivots, to diminish

as much as possible the weight resting on the beds of the horizontal axis: this is done by counterpoises, as seen at H, H (*fig.* 13), L, L (*fig.* 11), and G, G (*fig.* 3). These act on one arm of levers whose fulcra are supported by the solid parts of the instrument, the other arms carrying stirrups through which the axis is received, so that this latter just touches the socket in which its pivots turn.

In the interior of the telescope, at the focus of the eye-glass, is the wire plate of fine threads of wire or other material, of which two are horizontal and an indefinite number vertical. The object here is to give greater precision to observation, by dividing the field of view into a certain number of subdivisions. This wire plate can be so moved as to be brought accurately into focus, and there regulated; to render it visible, however, it must be illuminated from without. This is done by making one half of the axis hollow, and reflecting the light of a lamp through this cavity into a hole in the side of the tube of the telescope. By this means, the cross lines are dark on a light ground. The illumination employed by Fraunhofer is, however, much more convenient; this is applied between the eye of the observer and the focus. Here the whole field of the telescope remains dark, the cross lines alone being illuminated, so that object and cross lines can be distinctly seen at the same time. As much depends upon the perfectly horizontal position of the axis of the instrument, it becomes necessary to apply frequently a test of this, which is done by a tubular level placed above or below, as in *pl.* 15, *fig.* 13, L. In the portable transit instrument (*pl.* 15, *fig.* 22), the stand, AABBCC, consists of a cast iron crown, upon which the two parts for the axis are immovably fastened. The horizontal position of the instrument is controlled in one direction by a level placed upon the axis; in the other, by a level, F, placed upon the declination circle, D.

The rectification of an astronomical instrument, or the determination of its faults, must precede its use. Three principal errors may attach to the transit instrument: in the first place, it will almost always deviate from the meridian, that is, it will have a small eastern or western azimuth; in the second place, it will almost always be inclined at a small angle to the plane of the horizon, which is determined by the dependent level, L (*fig.* 13); thirdly, the optical axis of the telescope will deviate at a slight angle from the line drawn perpendicular to the axis of rotation: this last error is called the *error of collimation.* These three errors must be ascertained and rectified, partly mechanically, and partly by calculation; as also the four errors to which the diaphragm or wire plate is generally subject.

Fig. 25 is a side, and *fig.* 31 a back view of a small transit instrument constructed by Repsold for the observatory of St. Petersburgh. A are the pillars of the axis G, fastened to a granite block: the pieces in which the pivots of the axis turn, are shown more in detail by *figs.* 26 and 27. E is the declination circle with its vernier, F, and the level, I, situated above the axis. The telescope, BD, is only partly given. At D is the adjusting screw for the ocular, given more in detail by *figs.* 28 and 29. Its micrometer arrangement is shown in *fig.* 30, where the wire plate is moved in the box, *b*, by the screw *a*. *Fig.* 32 is one of two supports which stand in excava-

tions of the two pillars, A, carrying friction rollers above, upon which the axis G rests. *Fig.* 34 is a handle for directing the telescope.

Circular Instruments.

85. As the irregularities produced by changes of temperature, eccentricity, specific gravity, &c., are greater in a part of a circle than in an entire one, it follows that even the most perfect quadrants do not afford the greatest possible degree of accuracy; for this reason full circles were introduced, now used almost exclusively in the determination of altitudes, and to which the remarkable precision of the astronomical observations of the present day is owing. To the circular instruments belong: 1, the *repeating circle;* 2, the *simple circle;* 3, the *meridian circle;* and 4, the *theodolite.*

86. The *repeating circle* of Dollond (*fig.* 14), intended for observations out of the meridian, rests upon a tripod stand, AA, which, by means of the adjusting screws, and the level F, can be made perfectly level. Upon this rests the horizontal circle, B, on which the direction of the alidade, H, can be accurately read off by means of four verniers provided with lenses: of these, only D and E are visible. The alidade H, which supports the four posts, I, I, I, I, of the full circle, is adjusted by the telescope T. The posts carry the beds for the horizontal axis of rotation of the two circles, O and L, and the principal telescope, M, whose horizontal position is regulated by the level attached to the strips K, K. The main telescope, M, has an adjustable ocular, provided with a micrometer arrangement. This telescope is fastened to the vernier carrier, PQ, which then determines altitudes in the fixed circle, O. The manner in which the repetition or multiplication is effected will be understood by referring to what has been said in the part of the work relating to measuring instruments (*p.* 65). We will here only remark, that for this purpose the vernier carrier, PQ, is fastened to the axis by clamps, which can be loosened in repeating. Reichenbach has very much improved the repeating circle; nevertheless, as there are always defects in the instrument, attention is now turned almost exclusively to *simple fixed* circles.

The Meridian Circles.

87. The most prominent and costly instrument of modern practica. astronomy, is incontestably the *meridian circle,* another kind of full circle, used to determine the altitudes of stars. This instrument serves not only to observe in the most accurate manner the culmination of the stars (as in the transit instrument), but also their zenith or polar distance. The entire instrument must therefore be set up in such a manner that its horizontal axis of rotation shall lie accurately in an east and west direction. The planes also of the two circles perpendicular to this axis, as well as the optical axis of the telescope, must be in the plane of the meridian. The meridian circle has much the same construction of the individual parts that is found

in the transit instrument, as also the same errors and corrections. To the latter is to be added the verification by the observed altitude or polar distance of the star. The best meridian circles of an earlier date are those of Ramsden (particularly the one at Palermo) and of Troughton (at Leipzig). Perhaps the most perfect of more modern construction are those made by Reichenbach, and especially by the brothers Repsold in Hamburg.

In the meridian circle erected at the Hamburg observatory in 1836, its constructors, A. and G. Repsold, sought to solve the problem of avoiding every error arising from flexion, by the greatest equality and counterbalancing of the individual parts. For this reason the instrument is symmetrical in all its parts, the axis, BB (*pl.* 15, *fig.* 11), being bored within as well as without; two circles, F, F, of equal weight, with the accompanying microscope carriers, burden equally the axis, BB, and require equally heavy counterpoises, L, L, on both sides. To avoid a possible alteration of the axis, the attachment is very near the telescope, and a corresponding counterpoise on the other side restores the equilibrium of the whole. The circles are of cast brass, 3 feet 2 inches (French) in diameter, graduated on silver for every 2 minutes. These circles have four verniers, by which the angles can be read off to single seconds by lenses, R, R, fastened to the microscope carrier, FF. The massive centre, B (*pl.* 15, *fig.* 12), of this microscope carrier, itself composed of hollow tubes for the purpose of measuring absolute heights, is fitted to the axis in such a manner as to move freely without great friction in the boxes. The telescope (*fig.* 11), CE, with a Fraunhofer object glass, C, of 5 feet focus, consists of two equally heavy conical tubes, CB, BE, of hammered brass, which, firmly united to the axis BB, admit of no bending. The illumination of the cross lines is effected through the hollow axis by a mirror in the tube, a lamp being placed in one of the tubes running out in the prolongation of the axis. The obscuration, as also the regulation of the illumination, is quickly effected by a wedge of colored glass worked by a rack. The beds of the axis, entirely independent of the other parts of the instrument, are screwed fast to blocks of brass in the pillars, A, A′, behind which are the brass plates which support the posts, M, K, for the counterpoises L, L.

88. The meridian circle at the central observatory of Pulkowa (St. Petersburg), also erected by the brothers Repsold, is very similar to the preceding, though on a larger scale. *Fig.* 3, *pl.* 15, represents it in perspective. The two pillars, A and A′, are of grey granite, $7\frac{3}{5}$ feet high, and 18 inches broad each way at the upper end. The telescope, CB, has a focal length of $83\frac{1}{5}$ inches, $5\frac{4}{5}$ inch aperture of objective, and possesses magnifying powers of 170, 238, and 245. The wire plate in the focus at C, consists of two horizontal and nine vertical wires. Each of the two circles, BKEK and BK′E′, has a diameter of 48 inches, and is divided on silver to two minutes. For the counterbalancing, the counterpoises, G and G′, are attached to special metal posts, I, H and I′, H′. The whole instrument can be raised at F and F′. The level N rests on the cross piece M. *Fig.* 4 exhibits the microscope carrier on a larger scale. This consists of the hoop EE; the four microscopes themselves are at K, K, K, K; LL and

L′L′ are the two levels, whose end views are given immediately to the left hand, with their mode of attachment; *d, d, d, d, e, e, e, e,* are the spokes; *c, c* and T, T are hollow tubes. *Fig.* 5^a is a front view of the eye-piece of the telescope, CB (*fig.* 3); the letters *a, b, c, f, g, h,* in *figs.* 5^a, 5^b, indicate the separate parts of the wire micrometer, and *d* the pin for its adjustment; *c* (*fig.* 5^b) is the separate tube of the ocular. *Fig.* 6 shows the construction of one of the four microscopes, K, attached to the microscope carrier; *figs.* 7, 8, 9, and 10, represent particular parts of the micrometer arrangement. *Fig.* 7 is the inner, *fig.* 8 the outer plate; *fig.* 9 the spindle of the micrometer screw *h;* and *fig.* 10 the external view of the whole micrometer arrangement from above.

The Equatorial.

89. The *Equatorial* is an instrument by means of which, not only the declination, but also the difference of right ascension of a star and the zenith can be ascertained. Some idea of the instrument may be obtained by supposing an altitude circle so arranged, that the axis of rotation, previously vertical or perpendicular to the plane of the horizon, shall be perpendicular to the equator; in other words, that this axis placed in the meridian shall form an angle with the horizon equal to the height of the pole at the place of observation. Thus, in the altitude and azimuth instrument, the axis of rotation moves towards the zenith; in the equatorial, towards the pole of the equator. Its axis thus becomes parallel to that of the earth, and the azimuth circle of the simple circle becomes an hour circle, and the altitude circle a circle of declination. This instrument, when very accurately constructed and adjusted, possesses the exceedingly important advantage of giving, out of the meridian, the same determinations which the meridian circle affords at the moment of culmination alone.

The equatorial, as at present constructed, rests upon a prismatic stand. In the smaller portable instruments, however, the middle of the polar axis, I (*fig.* 15, which represents the one constructed by Repsold for the Hamburg observatory), rests upon a vertical brass pillar, A, with three feet; LMN is the hour circle, figured more intelligibly in *fig.* 17, with its micrometer arrangement, N, and the counterpoise, K. GHF (*fig.* 15) is the declination circle; C, D, K, the counterpoises for diminishing the friction; and OP the movable telescope. A very minute division of the hour circle is, strictly speaking, not necessary, as the exact determination of the right ascension is obtained in another manner. *Fig.* 16 shows the external and internal construction of the axis, EH, of the declination circle, GHF. In a well constructed equatorial—1, the axis of rotation must lie in the plane of the meridian, and 2, must form an angle with the horizon equal to the altitude of the pole at the place; 3, the plane of the declination circle must be parallel both to the axis of rotation and to the optical axis of the telescope.

The stands of the greater equatorials consist of a solid pyramidical base

similar to that of the Dorpat refractor. One of this character, for example, is to be found in Munich, with a telescope of 8 feet focus and 6 inches aperture; the hour circle has a diameter of 9 inches, graduated to 4 seconds of time; and the declination circle, a diameter of 12 inches, divided to arcs of 10 seconds. The telescope, admitting a magnifying power of 400, follows, by means of a clock with a centrifugal pendulum, the diurnal motion of the stars.

The Theodolite.

90. Another instrument to be noticed in this place is the Theodolite. *Fig.* 35 represents a lateral, *fig.* 36 an edge, and *fig.* 37 a superior view of a Theodolite constructed by Ertel of Munich. In the three views the same letters refer to the same parts. It rests upon a tripod stand, AA, with three adjusting screws, of which only two, B, B, are represented. On this tripod is a short column, C, and upon this column is placed the horizontal circle E, graduated to degrees, &c. Upon this, and turning on its centre, the standards, H, H, rotate. These carry at their extremities, pivot holes for the horizontal axis of the telescope, N, whose optical axis moves in a vertical plane. The whole arrangement is similar to that of a transit instrument, with this difference, that the Theodolite has a very finely graduated vertical circle for measuring altitudes, while the horizontal circle, E, is intended to measure horizontal angles, which are read off by the lenses, G. This horizontal circle, E, can be fastened or loosened at pleasure by the clamp arrangement, F*abb*. K, K, are lenses or microscopes for reading off the vertical angles measured on the circle L, and M is the level required for rectifying the station of the instrument.

Theodolites are divided into two principal kinds—*Compensating* and *Repeating Theodolites;* they are also provided sometimes with a so-called rectifying telescope. As regards the use of the instrument, we would refer to what has been said of it in the mathematical portion of the work.

The errors and rectifications of the Theodolite are much the same as those of the meridian and transit instruments (sections 84 and 87). In conclusion, it may be remarked, that for the Theodolite may be substituted a repeating circle, a simple circle, or an universal instrument, as constructed by Ertel of Munich, and A. and G. Repsold in Hamburg. These instruments fulfil the aim of the Theodolite just as well, and even more completely; at least this is the case as far as regards astronomical observations.

The Reflecting Sextant; the Reflecting Sector; the Triquetrum.

91. All the instruments already mentioned, as used for measuring angles, require an immovably vertical or horizontal position. This, however, cannot always be attained, in which case reflecting instruments are

employed, which, by their constructions, compensate for the want of a fixed station. Among these belong first the *reflecting sextant.* This consists of a circular sector, amounting to from 60–65 degrees, and is an instrument of great value on land, but absolutely indispensable at sea. As, however, from the theory of the instrument, the angle indicated on the sextant is exactly half the true angular distance, every degree of the sector indicates two degrees of angular distance. The instrument, therefore, measures angles of 120° to 130°, on which account every half degree of graduation is marked as a whole degree. About the centre of the sector rotates an alidade, which carries a large plane mirror, passing through the centre of the sector; another somewhat smaller plane mirror is fixed perpendicularly to the plane of the sextant, and so adjusted, that when the alidade is brought to the zero point of the graduation, the planes of the two mirrors are parallel. This plane mirror is uncovered in its upper half, and in practice, the signal of one leg of the angle to be measured is seen by direct light, that of the other immediately under it by reflection. At the back part of the sextant, a handle of wood is attached, by which it is held during observation. Between the two mirrors are hinged variously colored glasses for the protection of the eyes when observing in a bright light. The astronomical telescope is screwed in such a manner into the frame that the objective end lies next to the mirror. The alidade carries a vernier with a lens for reading off the degrees. The sextant must not be too heavy, as it is to be held in the hand when in use. Sextants of greater dimensions, as of 8 inches radius and more, have stands specially adapted to them.

The errors of such an instrument must be ascertained before using it. The first of these is the error of collimation; the second, a want of parallelism of the axis of the telescope with the plane of the instrument; the third consists in an unequal distinctness of the direct and reflected image of the same object. The fourth error is when the sides of the mirrors are not accurately plane and parallel to each other; and the fifth has reference to the same circumstance in the colored glasses. All these errors must therefore be rectified before the instrument can be used.

The first application of the sextant is in measuring the angle between two objects at any direction with respect to the horizon. Here the least illuminated object is selected as the one to be seen by direct light. The second application is to the measurement of altitudes. To determine the altitude of an object by means of the sextant, look directly through the telescope at the image of the object in the horizon, which may either be a natural or an artificial one; bring the plane of the sextant into the vertical position, and move the alidade until the reflected image of the object covers its direct image: the angular position of the alidade will indicate double the altitude desired.

Pl. 15, *fig.* 23, gives a perspective view of another form of sextant with a glass prism, of simpler form and less expensive construction. ABB is the body, BB the graduated limb, C the movable alidade with the vernier, D the lens for reading off the graduation, GF the telescope, E the box containing

the prism, in which the two images must be brought in contact; the index will then give the angular distance of the two objects at the station of the observer. In this instrument the colored glasses are wanting.

92. Another kind of reflecting instrument formerly used in measuring angles of moderate value, is the *Reflecting Sector* (*fig.* 24), whose limb, DD, only contains somewhere from 10–15 degrees. The alidade carries the vernier E, with the double tangent screw, FF, for fine adjustment; I and K are the mirrors, GH the telescope with the bent ocular, H, so that the observer at H looks downwards into the telescope. At the present time the instrument is no longer used, owing to the difficulty of rectifying it. Even the reflecting sextant is but rarely employed on land, theodolites having taken its place, being equally convenient to carry when of small size, and giving angles with much greater precision. At sea, however, the sextant retains full sway, as there no other observing instrument can supplant it.

93. This is the appropriate place to refer to the *triquetrum* (*fig.* 21), an ancient instrument, supposed to have been invented by Ptolemy, for determining altitudes and amplitudes of the heavenly bodies. It consisted of a staff, A, placed vertically by the assistance of a plummet, D. Attached to this staff were two others, B and C, movable on hinges, and thus capable of forming various triangles with the first. On one of them, namely on B, were placed the sight vanes, *a* and *b*. The construction and use of the triquetrum (so called from its triangular shape) depended upon correct geometrical principles, although, as is very evident, observations made with it could be of but very superficial character

The Sun-Dial; the Gnomon.

94. Sun-dials are instruments by means of which the true solar time can be determined, when the sun is above the horizon and not obscured by clouds. Before the invention of wheel clocks, they formed the only means for an accurate determination of time. *Gnomonics,* a special department of applied mathematics, teaches the mode of constructing sun-dials on any plane or curved surface. Even the Egyptians were acquainted with the sun-dial; at least, Josephus expressly asserts that the obelisks served for astronomical observations; and Augustus caused an Egyptian obelisk to be erected in Rome for the same purpose. The Jews had them 732 B.C.; and as to their existence among the Greeks, they are to be found in the choragic monument of Andronicus Cyrrhestes in Athens. Papirius Cursor constructed the first sun-dial at Rome 290 B.C. Portable sun-dials were invented by Pope Sylvester in the tenth century.

Sun-dials consist generally of a face of proper form—the *dial surface*—upon which is an *hour ring;* on this latter, the shadow of a *style* or gnomon indicates the hours.

There are various constructions of dials, depending upon the position and character of the dial face. The simplest form, and the one most usually employed, is the *equinoctial* or *equatorial* dial, whose plane is parallel to the

plane of the equator. In this form the hour ring forms a circle divided into 24 hours. The shadow of a style erected perpendicularly to the centre of the dial face, indicates the hours whenever the twelve o'clock line of the dial is fixed in the meridian of the place, and the style rendered parallel to the axis of the earth. This equatorial dial can of course be employed to determine the 24 hours in those countries only where the sphere is parallel, or which have the pole in their zenith. In our latitude only the half circle can be used, and that only from vernal to autumnal equinox.

When the plane of the dial is parallel to the plane of the horizon, it becomes a *horizontal* dial. The meridian line of this dial must be in the meridian of the place, and the index in the direction of the pole. To construct a horizontal dial, draw the line of six o'clock, and, perpendicular to this and bisecting it, the meridian, or twelve o'clock line. At the point of intersection draw a line, forming, with the six o'clock line, an angle equal to the altitude of the pole, or the latitude of the place. Taking a moderate length on this line as hypothenuse, complete the right-angled triangle, by letting fall from its extremity a perpendicular on the six o'clock line. From the intersections of the first mentioned lines as centres, and with the hypothenuse, and that part of the six o'clock line belonging to this right-angled triangle, as radii, describe two semicircles on the six o'clock line. Divide each into twelve equal parts, and from each point of division of the inner semicircle, draw lines parallel to the meridian; and from each point of the outer semicircle, lines parallel to the six o'clock line. Through the intersection of these two sets of parallels, prolong radii of the semicircles. These latter radii, twelve in number, will be the lines of shadow cast by the edge of the style for the 12 hours intervening between 6 A.M. and 6 P.M.: when produced on the other side of the centre, they will indicate the hours from 6 P.M. to 6 A.M. Thus, 7 A.M. produced, will indicate 7 P.M., &c. The outline of the dial plane may be square or circular. The style of the dial must form, with its plane, an angle equal to the altitude of the pole at the place of erection. If the style have an appreciable thickness of material, it will be necessary in the construction to suppose the semicircle divided into quadrants, and these separated by a parallel-sided space, equal in breadth to the thickness of the style. Since a simple index post is easily bent and moved from the required angle, it is preferable to employ a right-angled triangle, whose hypothenuse forms with the base an angle equal to the altitude of the pole.

95. When the surface of the dial is in a vertical plane, it becomes a *vertical dial,* of which there are four forms, named after and corresponding to the four principal regions of the heavens: *morning* (oriental), *noon* (azimuthal), *evening* (occidental), and *midnight dials,* as the vertical planes are turned towards the east, south, west, or north. These dials may be constructed mechanically by means of an equatorial dial and the rays of the sun.

The surface of the dial need not necessarily be turned to any particular part of the heavens, nor be exactly horizontal or perpendicular, although the construction of these *declining dials* becomes more difficult, and requires a greater knowledge of mathematics. *Polar dials* are those traced on a plane

perpendicular to the meridian, and passing through the poles; the index is here parallel to the equator. There are also *cylindrical* dials where the surface is a cylinder; and *annular*, where the hour circle is marked on the inside of a ring. The rays of the sun falling through a hole in a hoop upon this circle, determine the hours. The portable dials are principally horizontal, and must be set up by means of a compass.

It remains to remark, in conclusion, that as the sun-dials indicate only the true, and watches the mean time alone, the two can only agree exactly twice in the year.

96. The *Gnomon* was a contrivance of the ancients, to determine the altitude of a luminous body above the horizon, by the shadow cast by a vertical style upon a horizontal plane. Anaximander made use of the gnomon to determine approximately the obliquity of the ecliptic at 24°; and after that, Pythias and Hipparchus calculated the solstices and altitudes of the sun. In all probability the obelisks of the Egyptians were nothing else than such gnomons, by means of which they obtained the culmination of the sun, and consequently the true noon, for the purpose of regulating their water clocks. An improvement of this apparatus is presented by the *Thread Gnomon,* in which the solar rays are received on a vertical wall perpendicular to the plane of the meridian, and the precise position of the meridian plane, passing through the centre of a circular aperture in the wall, indicated by a depending thread. It will be readily understood that the meridian lines required for each gnomon must be previously determined with the greatest possible degree of accuracy.

The Wheel Clock.

97. By the word *clock,* without further qualification, is meant every machine which, by means of the perfectly uniform motion of wheelwork, is intended to divide mean, solar, or sidereal time, into a certain number of equal parts; the minuteness and accuracy of these latter depending on the more or less complete elaboration of the component parts of the apparatus.

It is an ascertained fact that the first clocks were moved by weights. Galileo and Huyghens first applied the pendulum to regulate the motion of the clock by its regular oscillations. In the sixteenth century a spiral spring was used as a motive power instead of the weights, and the pendulum was replaced by the *balance wheel.* By means of these two substitutions, it became possible to reduce the mechanism of the clock within so small a compass, as to render it sufficiently portable for pocket use; and although the honor may be contested against him, Peter Hele of Nürnberg is to be considered as the inventor of watches.

98. This is not the place to go into a minute description of the mechanism of a clock; it must be remarked, however, that the motive power, whether bent spring or weight, acts upon a wheel with a certain number of teeth, and that by a proper arrangement of wheels and pinions, the indices are moved in such a manner that the one (the minute hand) makes twelve

rotations while the other (the hour hand) makes but one. There is often a third index (the second hand) which makes one rotation in a minute. In the better clocks the second hand springs from one second to another, thus showing each one separately; and as in astronomical clocks minute divisions of time are desirable, in these the division of seconds has sometimes been brought to thirds.

As the motive power can never act uniformly, every clock requires a regulator, which may compensate for the irregularities of the power. The whole wheelwork is consequently in such connexion with a single wheel—the escapement, that when the latter is checked the motion of the whole stops. This escapement, in the pendulum clock, is connected with the pendulum, which vibrates either whole or half seconds; the motion ceases, therefore, between every swing of the pendulum. It is thus seen that the proper motion of the clock depends upon the accurate length of the pendulum, and that as pendulums swing in proportion to their lengths, a clock may be regulated by lengthening or shortening the pendulum. The pendulum itself, however, needs regulating; for, being lengthened by heat and shortened by cold, the correctness of the clock's motion is impaired. As it is not possible always to determine this variation of length, and the variation is often very sudden, it cannot be provided for by any manual regulation. To meet this difficulty Harrison invented a compensation pendulum which regulates itself. In this pendulum, rods of brass and steel alternate in such a manner, that the elongation of the steel rods, and consequently of the pendulum, is counteracted by that of the brass rods, which in this manner shorten the pendulum as much as it is lengthened by the steel rods. Its length thus remains unchanged in all temperatures. Graham's mercurial pendulum is intended to accomplish the same end. (For further details see the article, Compensation Pendulum, under the head of Physics.)

In the second kind of regulators, all the wheels are connected with the balance wheel by means of the escapement, so that this produces the necessary check to the motion. Of escapements there are various forms, all, however, being in connexion with a balance, which is a flat wheel, on whose pallets the scape wheel catches, endeavoring to move it forwards. The pallets are so fixed on the verge of the balance wheel, that the scape wheel must leave them free after a certain time, and then the spiral spring acts upon the balance, bringing it back to the former position to meet a new tooth of the escapement. It will be readily seen that the quicker or slower motion of a clock will depend upon the time in which the balance makes its movement, and that this time depends upon the length or shortness of the spiral spring. There is for this reason a regulator on the watch whose motion alters the length of the spring.

As in the pendulum isochronism of oscillation is effected by the principle of compensation, so in the balance there must also be a compensation, since both it and the spiral spring change their dimensions, and consequently their times of vibration, with change of temperature. Compensation is brought about in the balance by the bending of thermometric metal springs, steel

and brass, or platina plates, being so combined that their changes through temperature counterbalance those which, for the same reason, take place in the balance.

99. Clocks for astronomical purposes must be very carefully constructed, and every tendency to inaccuracy must be specially counteracted. They may be pendulum clocks, as used in observatories, or balance clocks—*chronometers*—as employed at sea to assist in the astronomical determinations there necessary. The English, to whom the perfection of chronometers is due, set great value upon the best of them, and Harrison furnished instruments which, in a voyage round the world, did not vary three seconds. Such chronometers are little different in construction from the best watches, except in having a peculiar escapement; all their parts are, however, very carefully constructed, many precautions taken against accidental injury, and throughout, compensations for the effects of temperature and other physical agents (as magnetism) introduced. Longitude or marine time keepers (box chronometers) for nautical and astronomical purposes, are constructed just like the pocket chronometers; they are, however, larger, and inclosed in a special box.

The Planetarium.

100. Among the numerous helps to the study of astronomy and mathematical geography, must be mentioned those artificial models and contrivances known under the names of *Lunarium, Tellurium,* and *Planetarium.* The lunarium is an apparatus by which the motions of the moon about the earth, her phases, &c., can be readily illustrated and explained. It is usually combined with the tellurium. The tellurium, called sometimes *geocyclic machine,* is a particular form of planetarium, which exhibits the motion of the earth round the sun, the course of the moon about the earth, and with her about the sun, as also all attendant phenomena, such as the seasons, quarters of the moon, &c. Finally, the planetarium is a model, intended to render perceptible to the senses the motions of the planets and all resulting phenomena, on which account it has received various constructions. Common planetaria are moved by hand; the better and more complicated have a wheelwork, which, like a watch, is set in motion by a spiral spring, and causes the planets to revolve with their respective velocities around a globe or lamp placed in the centre. As Lord Orrery was the first to construct a planetarium of this character, they are sometimes known as *Orreries.*

Henderson, formerly director of the Edinburgh observatory, has published the description of a simplified planetarium (*pl.* 15, *fig.* 38), whose construction will be here briefly mentioned. In a circular box standing on four feet, the clockwork is set in motion by the handle D. Upon the upper surface of the box are marked the ecliptic, the perpetual calendar, and other items relating to the planets. In the centre is a large sphere representing the sun, about which the planets revolve on vertical posts fastened to horizontal

rods, with their proportional velocities, and at their proportional distances—Mercury, H; Venus, G; Earth, F, with the moon, *c;* Mars, I; Jupiter, M, with four moons, *e, e, e, e;* Saturn, N, with the ring, and the seven satellites, *f, f, f, f, f, f, f;* as also Uranus, K, with the six moons, *d, d, d, d, d, d.* *Fig.* 39 represents a contrivance, which, attached to this planetarium, serves to give the earth's axis a parallel motion, and to exhibit the cause of the seasons and their succession. It consists of the small globe, F, to whose equator the hollow brass tube, E, is fastened, which carries the weight, C, and the carrier, D. If now the planetarium be hung to the wall by the ring E (*fig.* 38), and set in motion, C being held fast, the earth will revolve, her axis remaining constantly parallel, and thus representing the courses of the seasons, which are indicated on the fixed disk, A.

The best Planetaria and Telluria are those of Riedig and Schulze in Leipzig, and Seifert of Hohenstein near Chemnitz in Saxony.

101. It remains to mention, in conclusion, that there are still other pieces of apparatus, some of them ancient, and others more modern, which are used with excellent results both by the practical astronomer and the teacher. Among these may be mentioned the *circle micrometer*, as also the *differential micrometer* (invented by Boguslawski) for determining the difference of right ascension and declination of two stars: the *dipleidoscope* (invented by Dent of London), an apparatus which replaces the transit instrument; the beautiful model by Möbius of Leipzig, for representing the orbits of the asteroids, Ceres, Pallas, Juno, and Vesta, with respect to their magnitudes, inclinations, and eccentricities; finally, the *mercurial* clock of Kater, and the *astrograph* of Steinheil.

Observatories.

102. The place where astronomical observations are conducted and the necessary apparatus erected, is called an *observatory*. The choice of such a place is sometimes very much restricted; where this is not the case, it should be established in a dry locality, where, remote from all motion which might produce vibrations, the foundations and lower stones of the buildings and the instruments may be protected from the influences of weather and temperature. The building itself must be constructed in the most solid manner, and, if possible, facing the four quarters of the heavens. It is desirable to have an elevated station from which the horizon can be surveyed in every direction; where this is impossible, the parts of the building in which are placed the principal instruments must be much elevated. The foundations of the edifice must be very solid, and each principal instrument must have an isolated base, or must be connected with the ground by special foundations, not touching any part of the edifice, in order that all shaking of the instrument may be avoided. A free view in all directions must be had, and for the meridian there must be a vertical slit passing through the whole height of the building. The place also where the refractor, or other instrument supplying its place, stands, must be so arranged that the whole of a vertical plane can be seen

in any direction, for which reason there is generally a roof attached which turns on a railroad, or can be entirely removed. In the first case, the roof is divided by a vertical section of four feet in breadth, into two halves; the aperture, however, can be closed by trap-doors. Such an arrangement, for example, is to be found in the building for the great refractor at Dorpat, and in the turning cupola of the observatory at Washington.

The best observatories are those at Altona, Berlin, Dorpat, Göttingen, Greenwich, Königsburg, Mailand, Munich, Ofen, Pulkowa near St. Petersburg, Seeberg near Gotha, Vienna, &c. In the other parts of the world, those at the Cape of Good Hope, Paramatta in New South Wales, and in the United States, are the best known. In the United States, the principal observatories and instruments are at Cambridge, Philadelphia, Washington, Cincinnati, Hudson (O.), &c.

Practical Astrognosy.

103. The finding of particular stars and constellations is effected by means of the celestial globe and star maps; as also by the method of *alignments* already mentioned (sec. 26). This latter method will now be detailed a little more at length. It was there seen that from the position of the Great Wain, or the Great Bear, the polar star could be determined; and in the same manner other stars are identified. For this the star maps (*pl.* 12) are employed, on which the alignments of the principal fixed stars are given. Produce the direction of the stars ζ and η towards the wain, it will strike a star of the first magnitude, Arcturus in Bootes, which, with the polar star and Vega in Lyra, forms an isosceles triangle, Arcturus being at the vertex. The polar star, which, by its almost unchangeable position, is very well calculated for the purpose, serves as a point of departure for the rest of the heavens; the altitude of the polar star above the horizon being nearly equal to the geographical latitude of the place. Twice the length of a straight line from Vega to Arcturus strikes Spica, a star of the first magnitude in Virgo. Spica forms, with Denebola (in Leo) and Arcturus, and also with Arcturus and the star α Libræ, triangles nearly isosceles. Spica forms the vertex of the first of these; Arcturus of the second. The star α Libræ lies almost in the continuation of the connecting line between the polar star and Arcturus. Furthermore, the alignments of Vega, the polar star, Capella, and Aldebaran, form a large flat arc. Aldebaran, a star of reddish light, is one of five stars lying near to each other, which form a V, and are called Hyades. Aldebaran and Capella form an almost right-angled triangle with Castor, a star in Gemini. A line drawn from Denebola to the polar star, and produced some distance beyond, strikes a bright star, which, with three others, forms a large almost regular quadrilateral, the greater part of Pegasus. A line from Perseus to Aldebaran, and sufficiently prolonged, strikes three bright stars, the belt of Orion. Produce the line indicated by this belt to the left, and it will meet the brightest star in the sky, Sirius (the Dog star). In this manner, straight lines may in succession be drawn from two known stars to others, and the triangles thus formed, constructed in the heavens.

104. With the help of a celestial globe, the same end may be attained more readily, by setting up the sphere for the place of observation, as also for the day and hour of observation. It is then only necessary to look in what direction and at what elevation above the horizon any star is found on the globe, and then direct the eye towards the corresponding part of the heavens, to be able to identify them on both spheres. In this way, for instance, it might be observed by means of the celestial globe, that at 7½ o'clock of Jan. 18, a star of the first magnitude, Capella, stands a little to the south-east of zenith, outside of the milky way. It is then only necessary at that time to look a little to the south-east, out of the milky way, actually to see Capella.

105. Knowing the twelve constellations of the zodiac, it will not be difficult to find the visible planets, Mercury, Venus, Mars, Jupiter, Saturn, and even Uranus, in these constellations, distinguishing them with certainty from the fixed stars by their peculiar appearance and their varying position with respect to the neighboring stars.

PHYSICS.

PLATES 16–22.

In the general introduction to the preceding portion of the work, a concise summary of the entire system of the natural sciences was given, in which *Physics*, in a restricted sense, or *Natural Philosophy*, occupied a very important place. The following sections will be devoted to this science. In the above-mentioned introduction the system was traced out in its broadest features. Taking the general divisions there indicated, it will be now necessary to subdivide them, and to examine each subdivision with special attention.

Natural Philosophy, or Physics, may be divided into *pure* and *applied*. *Pure Physics* will then form the theoretical portion of the science, teaching the laws of nature, as far as they may be inferred from careful and long continued observations of natural phenomena, afterwards verified and established by actual application to practice. Hypothesis can only be verified by its enabling us to develope the phenomena belonging to a certain class, and to predict the manner of their occurrence under certain circumstances, and at certain times. In this way Newton deduced the flattening of the poles of the earth from the law of gravitation; Laplace calculated the two different diameters of the earth, and actual measurement has proved the truth of his results. The predictions of astronomy are founded on such theories; and the actual occurrence of solar and lunar eclipses, and other similar phenomena, years after they had been foretold, shows the firm and sure ground on which these theories are based. This discovery of natural laws is then the object of *pure physics*, while the application of the laws thus found to surrounding nature, belongs to the department of *applied physics*. The various sections of the latter are referred to in their appropriate places in this work: attention will be directed for a moment to pure natural philosophy.

The single branches of science with which pure natural philosophy is occupied, are, 1, the theory of equilibrium of forces, or *statics;* 2, the theory of motion, or *dynamics*. These two parts taken together form what is generally termed *mechanics*, properly a part of applied mathematics. 3, the philosophy of sound, *acoustics;* 4, of light, *optics;* 5, of heat, *pyronomics;* and 6, of *electricity* and *magnetism*, which latter have in more recent times made astonishing progress.

A few general observations on the peculiarities of bodies must precede the minute investigation of particular parts of the subject. We refer to

those peculiarities which form the essence of what is known as *body, matter, material,* which thus apply to all bodies without any exception. Among these peculiarities may be first mentioned *extension* and *impenetrability.* A body must have a certain extension, that is, must occupy a certain space; it must nevertheless be impenetrable, or must fill this space in such a manner, that no second body can also occupy it at the same instant of time. We must not fall into the error of supposing that one body can penetrate another, as a nail can a board, in the physical sense of the word. As the nail is driven through the board by mechanical force, it pushes aside the fibres of the wood, and occupies their place; the particles of the wood and iron are therefore contiguous, but not in the same place. *Penetration,* in the physical sense of the word, is the destruction of one substance by another, not a mere displacement. In the latter case, there is not necessarily an increase in bulk, as the board with the nail occupies no more space than without it; and a measure of water mixed with a measure of sulphuric acid will not fill two measures: penetration, nevertheless, has not taken place, no atom having been annihilated, as may be proved by weighing. *Divisibility* is another general property of bodies, by means of which they are supposed to be capable of division into smaller and smaller portions—atoms. The pulverization of solid bodies, the small globules of fluids, as the blood globules, whose diameter is only $\frac{1}{450}$ of a line, and the great space which gaseous bodies can occupy, show this property on a large scale, while the atomic theory follows it to the smallest molecules. Nearly allied to divisibility, are two other properties of bodies, *extensibility* and *compressibility,* which are opposed to each other. By these terms is meant an increase or diminution of the space which a body, under certain circumstances, occupies, without the connexion of its molecules or atoms being thereby affected. As these atoms are supposed to be unchangeable, this change of space must necessarily be referred to an expansion or contraction of the interspaces which exist between these atoms, in the natural state of the body. This extension is the result of a stretching or heating; the contraction takes place under the influence of cold or pressure.

The mention of interspaces between the individual atoms of a body, leads us to the consideration of another property of bodies, called *porosity,* possessed, as far as we know, by all. In ordinary language, however, the term *pore,* which may be considered, scientifically, as referring to an interspace infinitely small, is applied to those only which are large enough to allow the passage of fluids or gases. It is by means of these pores that the parts of one body penetrate between those of another, as water a sponge. In other bodies the pores are so small as not even to admit the entrance of gases, as, for instance, glass.

The atoms of which a body is composed are not always homogeneous, and hence the different kind of bodies; thus cinnabar is composed of atoms of sulphur and mercury; water, of oxygen and hydrogen atoms, &c.; such bodies being called *compound,* as distinguished from *simple* (elementary or elements), in which the atoms are homogeneous. These investigations, however, belong to the department of chemistry, and as such, do not belong

to this subject. The manner in which atoms are combined, or their *aggregation*, is also deserving of mention, as the same atoms may be considered as combined under different forms and conditions; thus, ice, water, and steam, are all composed of oxygen and hydrogen, in the same proportions, yet all possess very different properties. Three conditions of aggregation are known, according to which bodies are divided into *solid, liquid,* and *gaseous.*

By *solid* bodies are to be understood those which, apart from the changes produced by heat and mechanical agency, have an unchangeable volume, and an independent definite form. In these the single atoms are brought in the closest possible connexion. The connexion of atoms in *liquid* bodies is less intimate, possessing an almost unchangeable volume, even when a small quantity is exposed to great pressure; they have, however, no definite form. In *aeriform* or *gaseous* bodies, the connexion of the atoms is exceedingly slight, there being neither an unchangeable volume nor a determinate form, both depending upon surrounding influences. All bodies, under certain circumstances, may be transformed from one condition of aggregation to another, although the means to be employed, namely, change of temperature and pressure, may not be applicable to a sufficient degree to effect this in certain cases. Thus, for example, mercury at a temperature of, and below —39° F., is a solid; at the ordinary temperature, it is a liquid; and by an increase of heat, it becomes converted into vapor. Inversely, watery vapor, by cooling, becomes a liquid: water—and a still further reduction of temperature turns this into a solid: ice. Mercury also can be converted from a vapor into a solid in the same way. Faraday, within a recent period, has succeeded in converting many gases into liquids and solids, for which great cold and pressure were both necessary.

There must be a certain force which maintains the single atoms of a body in their mutual situations, giving to these bodies their structure and external form; another force again must cause the tendency to separation exhibited by these atoms, as among the gases. These two molecular forces are the force of *cohesion* or *attraction,* and the force of *expansion* or *repulsion;* and as heat converts solids into liquids, and liquids into gases, it has been customary to consider heat and expansiveness as identical. The predominance of one or the other force determines the conditions of aggregation in a body. In solids, the former predominates; in gases, the latter; in liquids, the two are in equilibrium.

Bodies may be considered under two conditions, namely, in a state of rest, and of motion; and this consideration brings us to another general property—that of *inertia.* Neither a part nor the whole of a body has in itself any tendency to change its present condition, that is, to pass from a state of rest to one of motion, or the contrary. The first case is illustrated daily; the second, however, although true, is not so evident, as we see everything come to rest, after a time, from a state of motion. The cause of this cessation of motion, however, is not in the body itself, but in external influences operating upon it: if these latter be neutralized, the motion continues. The principal obstacles to a continuation of motion are—friction,

and the resistance of the atmosphere. The motions of a body will continue in proportion as these influences are counteracted. Thus, a top will spin on the smooth plate of an air pump, under an exhausted receiver, for hours after being set in motion. A body opposes a certain resistance to the force attempting to overcome its inertia, so that every motion is conditioned, on the one hand by the intensity of the influencing force, and on the other by the force of resistance of the body: its *mass*. The mass of a body is the amount of matter of which it is composed.

A body let fall from a height will descend till it meets some obstacle. This is produced by *gravitation*, another general property of bodies. The *falling* of a body is, however, not the only result of gravitation. But more of this hereafter. The direction of gravitation coincides completely with the direction of a body suspended freely from a thread, as, for instance, a plumb-line; this direction, therefore, is called perpendicular, plumb, or vertical: the surface of standing water, as will be learned hereafter, is perpendicular to this elevation. From this mutual relation has been deduced the proposition, that the direction of gravity is always perpendicular to the earth's surface. As, however, the earth's surface, or the water surface, is that of a spheroid, the perpendiculars to it must be in the direction of the radii produced; whence it follows, that the direction of gravitation always tends towards the centre of the earth. Hence vertical lines are not parallel to each other, a fact which becomes inappreciable at short distances. At a distance of 600 feet, for example, the angle at the centre of the earth, between two perpendiculars, amounts only to about $6\frac{1}{3}$ seconds.

The force of gravity is exhibited by *pressure* when opposed to a resistance. The magnitude of this pressure is termed *weight*, this increasing with the number of material particles of which the body is composed, so that as the mass of a body is always proportional to its weight, the latter serves as an expression of the former.

There remains to mention, in conclusion, among the general properties of bodies, their *density;* in other words, the proportion of their weight to their volume. All bodies have a certain density, which depends upon the mode of aggregation, and the material of their single atoms. This density is termed *specific gravity.* As it is necessary to have a standard to which all densities may be referred, the weight of pure water, in its greatest density, has been taken as the unit of reference. By the density, then, or specific gravity of a body, is to be understood the ratio which its weight bears to an equal volume of pure water. If a certain mass of iron weigh 7.8 lbs., while an equal volume of water weighs 1 lb., the specific gravity of the iron is said to be 7.8. More will be said hereafter as to the proper mode of determining specific gravities.

MECHANICS.

A. The Statics of Solid Bodies.

a. General Ideas.

When two or more forces, acting in different directions upon the same body, are so adjusted as completely to neutralize each other, no change being produced in the body, the body is said to be in *equilibrium*, or the forces are said to hold each other in equilibrium. Statics investigates the conditions of equilibrium in bodies, being divisible into three sections, according to the three different states of aggregations: statics of solids—*Geostatics;* statics of liquids—*Hydrostatics;* and statics of gases—*Aerostatics.* The laws of the motions produced, when, among the different forces, the laws of equilibrium are not satisfied, are investigated by *Dynamics.* This, also, is divisible into dynamics of solids—*Geodynamics;* dynamics of liquids—*Hydrodynamics,* or *Hydraulics;* and dynamics of gases—*Aerodynamics,* or *Pneumatics.*

A point acted upon by a single force must move in the direction of the force and likewise, in a straight line. Equal forces are those which, when acting in diametrically opposite directions, neutralize each other completely. Two equal forces acting in the same direction are equal to twice the amount of one of them acting in this direction: several forces, even though unequal, act, in the same direction, as a single one equal to their sum. This is called the *resultant.* Resultants acting in precisely opposite directions, neutralize each other either entirely, when equal, or partially, when unequal: in the first case there is equilibrium, in the second there is motion, in the direction of the greater resultant. If the forces act at an angle with each other, motion is in a direction between them, obeying a mean force, the resultant of the different lateral forces. The magnitude and direction of this mean force is known from a law called the *parallelogram* of forces, explained by *pl.* 16, *fig.* 1. Let the lines AB, AC represent the direction and intensity of two forces, acting at the same instant on the body A. Completing a parallelogram from the angle BAC, and its sides, AB and AC; DA, the diagonal of the parallelogram, ABDC, will represent the direction and intensity of the force, which, if acting alone upon the point A, would produce the same effect upon it as the two simultaneous forces BA and CA. If a lateral force be supposed capable of urging the point A as far as B in a certain time, and another lateral force be capable of carrying it to C in the same time, the two together will carry it from A to D.

In a manner similar to the preceding, by which two forces may be considered as one, one force may be separated into two, of which it may be considered the resultant. The problem then becomes, to determine the intensity and direction of two forces, which, acting upon a body at a given angle, shall produce the same effect as the single given force. Suppose, for instance, that in *pl.* 16, *fig.* 2, the force AC act upon the body A, and it be

desired to divide this into two others, of which one, AD, shall be given in intensity and direction; then the other force will be found in intensity and direction by the third side, CD, of the triangle ACD. Draw, for instance, AB parallel and equal to CD, then AB and AD will form two sides of the parallelogram of forces, whose diagonal is the given mean force, AC, this being the resultant of the two forces AB and AD, determined in intensity and direction. If neither of the lateral forces be given in intensity and direction, then the first might be assumed at pleasure.

When three forces, AB, AC, AD (*fig.* 4), act upon a body, the resultant of the first two may be found, then that of this resultant and the remaining force. The diagonal, AG, proceeding from A, will be that of a parallelopipedon, which may be constructed from the edges, AB, AC, AD. This parallelopipedon is called the *parallelopipedon of forces*, by means of which it becomes possible to determine the direction and intensity of the mean force, when the three forces, AB, AC, AD, do not lie in the same plane. In this case, supposing AB, AC, AD, to be projections of these forces, then the line AG will be the projection of the diagonal of the parallelopipedon formed on these three lines—in other words, the projection of the resultant of the three forces; and in the theory of projection we have already learned how from the projection of a line to obtain its true size and direction.

The mean force of three or more forces acting together on a body, is found by the simple construction in *fig.* 3. From the extremity, B, of the line AB, representing one of these forces (any one being taken indifferently), draw a line, BC‴, parallel and equal to the second force, AC; from C‴, a line, C‴D‴, parallel and equal to the third force, AD; from D‴ the line D‴E‴, parallel and equal to the fourth force, AE. The line AE‴, drawn to the extremity of the last of these parallels, will be the mean force required. That the line A*c* is, in magnitude and direction, the general resultant, is a consequence of the fact that, when the parallelograms of forces, ABB′B″, ACC′C″, ADD′D″, AEE′E″, are constructed on this mean force, the single forces, AB″+AC″+AD″+AE″=AE‴, and that all the parallelograms have a common side in the line B′E′.

An equilibrium between three forces must occur whenever any two of the forces are equal and opposite to the third. The proposition of the parallelogram of forces can be exhibited practically. Let, in *fig.* 15, the points A and B be fixed pulleys, in the same vertical plane, over which is passed a string. Let now the weight, W, act on one end of the string, W″ on the other, and W′ between the two, then all will be in equilibrium in any one position of the string. Three forces are now acting upon the three points, A,B,C, in the directions CA, CB, and CW′. It can be readily shown whether the law of the parallelogram has its application here. Suppose, now, that W= 2 lbs., W″= 3 lbs., the question becomes, what must be the magnitude of W′ when the angle ACB is, for example, = 120°. Construct a parallelogram of which one side = 2, the other = 3, and the angle included between the two = 120°, and find the diagonal about = $2\frac{3}{4}$, making the weight of W′= $2\frac{3}{4}$ lbs.; then the angle ACB, made by the string, will be = 120°. DB represents the amount of the force W″, AE that of W, and

CE that of W′. *Pl.* 16, *fig.* 16, extends this construction to the case of several weights, and forms the basis of the Funicular Machine of Varignon, of which more hereafter.

It is known that every body is subject to the influence of gravitation, and that this gravitation acts upon every molecule of the body. All these single influences of gravitation may be considered as united into a mean force of gravitation, which then is called the *weight* of the body. This union can and must take place in a single point, the *centre of gravity;* and a force acting on this centre of gravity, and equal to the weight of the body, will hold it in equilibrium. Gravity and weight, therefore, differ as cause and effect. Gravity is that natural force which causes the weight of bodies, and the centre of gravity the point in which the entire weight of the body may be supposed to reside. It is a fixed point, whose situation does not change, whatever be the position of the body. Whenever this point is supported in any way, the body rests in equilibrium.

The centre of gravity of homogeneous bodies of regular shape, is easily obtained by geometrical constructions. The centre of gravity of a straight line is evidently at its middle point (*fig.* 5). That of a triangle, ABC (*fig.* 6), lies where lines drawn from the angles to the centres of the opposite sides intersect each other. It may also be found by drawing a line from one angle to the middle of its opposite side, and trisecting this line; the first point of division, S, starting from D, will then be the centre of gravity. That DS must equal $\frac{1}{3}$DB, is shown by drawing DE; DE will evidently $=\frac{1}{2}$AB. The triangles DSE and ASB are, however, similar, whence SD : SB : : DE : AB; as, however, DE $=\frac{1}{2}$AB, SD must $=\frac{1}{2}$SB $=\frac{1}{3}$DB.

The centre of gravity, S, of a parallelogram, ABCD (*fig.* 8), is the intersection of its diagonals; that of a regular polygon, ABCDEF (*fig.* 7), as also of a circle, is the centre. If a rectilineal figure of an even number of sides, as, for instance, the six-sided one, ABCDEF (*fig.* 7), be so constituted as to be divisible by a diagonal, CF, into two symmetrical halves, the centre of gravity will lie in the middle of this diagonal. If, moreover, as in the figure, all diagonals have a common point of intersection, this point itself will be the centre of gravity.

In those bodies which have a regular shape, and whose mass is distributed with perfect uniformity, the centre of gravity may be likewise determined geometrically. Thus, the centre of gravity of a cube or parallelopipedon is also in its geometrical centre: it is obtained either by passing a plane through two opposite edges, AB, DE (*pl.* 16, *fig.* 10), and finding the centre of this plane, or by finding the centres of gravity, S, S′ (*fig.* 11), of two opposite planes, and bisecting the connecting line at S″. From the first method it follows that the centre of gravity of a parallelopipedon lies in the point of intersection of two of its diagonals.

The centre of gravity of a pyramid (*fig.* 12) is obtained by connecting the apex, G, with the centre of gravity of the base, S, and on this line cutting off the fourth part from the base, so that SS′$=\frac{1}{4}$GS. The centre of gravity of the cone is found in a similar manner. To obtain the common centre of gravity of two different bodies, as of the cubes AG and *ag* (*fig.*

13), obtain first the centres of gravity, S and *s*, of the two, by means of diagonals, and unite the two points by the straight line, S*s*; upon this latter determine the centre of gravity, S′, as will be explained more fully under the head of the lever. The same method is to be pursued in determining the centre of gravity of irregular surfaces, as for instance, ABCD (*fig.* 9).

b. Of Simple Machines.

Simple machines, or *mechanical powers,* are those simple arrangements of which all machinery is compounded. Of these, six are generally distinguished: the *lever,* the *wheel and axle,* the *pulley,* the *inclined plane,* the *wedge,* and the *screw.* All these, however, may strictly be reduced to two—the lever and the inclined plane; on which account these two are looked upon as the elementary machines. The ancient Greek mathematician, Pappus, enumerates the above-mentioned simple machines, with the exception of the inclined plane, which is of more recent introduction. Instead of the latter power, Varignon added the funicular machine to the five others, which, however, consisting simply of ropes on which the forces act in different directions, and being intended to elucidate the proposition of the composition of forces, cannot properly be called a simple machine. See *fig.* 14, where the forces act in the same plane and in different directions upon the combined ropes at A, E, P, P′, P‴. These will hold each other in equilibrium when BC is equal and opposite to the mean force of BA and BP′, CD equal and opposite to the mean force of DE and DP‴, and CP equal and opposite to the mean force CB and CD.

The mathematical lever, in its simplest form, is an inflexible line supported in one point (*fulcrum, hypomochlium*) on which two or more forces operate, endeavoring to move it about this fulcrum. The distances from the fulcrum to the points of attachment of the forces are the *arms of the lever.* There are two kinds of levers: *levers of the first class,* or double-armed levers, in which the forces operate on different sides of the fulcrum; and *levers of the second class,* or one-armed levers, in which these act on the same side. The same conditions of equilibrium, however, apply to both, viz. that the forces must be inversely as the arms of the levers. Thus, when the arms of the lever are equal, the forces must be equal, and when the arms are unequal, the forces must be unequal, the greater force acting on the shorter arm, and the lesser force on the longer arm, these forces being in the same proportion as the arms of the lever. *Pl.* 16, *fig.* 23, represents a lever of the first class, in which the acting forces are the weights, P and W. F is the fulcrum, and for equilibrium, the proportions P : W :: BF : AF must exist. *Fig.* 25 represents a lever of the second class, which is supported at F, and operated upon in opposite directions by the weight W and the weight P, passing over the pulley and attached to A, the former weight drawing the lever downwards, the latter raising it up. Equilibrium can only subsist when P : W :: BF : AF. *Fig.* 26 is properly a lever of the second class, although in it the fulcrum is

above, and the force, P, draws upwards, while the weight, W, draws downwards. This form by some has, for this reason, been called a lever of the *third class*. In this lever, the above-named conditions still hold good, and the same is the case in the bent lever (*fig.* 30). Here, however, the bend of the arms, A′F and FB′, of the lever, is not to be considered, but only the direct distances from the fulcrum, B′*b* and A′*a*, or the levers, AF and FB, equal and parallel to them. Here also in a state of equilibrium we have P : W : : BF : AF.

Hitherto we have had reference to the mathematical lever, that is, to a line without weight; if the actual material lever be the one in question, where the weight of the arms of the lever comes into account, then the same proportions of the arms of the lever being retained, but with greater curvature of one or other arm, and consequently greater weight, the proportion, P and W, might change greatly without any disturbance of equilibrium.

Considering closely the proportion P : W : : BF : AF, we have P . AF = W . BF, this product of the two extremes and the two means being called the momentum of the forces. The *momentum* therefore of a force, is the product of the force by its leverage, and the preceding laws can be expressed in shorter phrase, by saying, a lever is in equilibrium when the momenta of the forces acting upon it are equal.

The case is somewhat different when the forces acting on the lever are not parallel to each other, as in *fig.* 29, where the two forces, P and W, are carried over pulleys. In this case each of the two forces must be decomposed into two others, of which one is perpendicular, and the other parallel to the lever. Expressing P by DA, and W by BG, calling also the angle, DAC, α, and the angle, GBE, β, then the force, DA, may be divided into the two forces, AC = P cos. α, and DC = P sin. α; the force, BG, likewise into BE = W cos. β, and EG = W sin. β. The proportion then becomes P cos. α : W cos. β : : BF : FA. This proportion only holds good, however, when the lever can only turn on the fulcrum without shifting. Should it lie but loosely upon the fulcrum, there must be equilibrium of the horizontal part of the forces, and the proportion P : W : : sin. β : sin. α.

Among the numerous applications of levers of the first class is to be reckoned the *balance*, that arrangement by which the weight of a body is determined. The common balance consists of an equal-armed lever, in which the two forces—the body, P, to be weighed, and the weight W—must act perpendicularly to the two arms of the lever. In the equality of the arms of the lever, the forces must necessarily be equal, that is, the weight to the weighed. When one beam of the balance is longer or heavier than the other, by even a very slight amount, the equality of the weight and the object weighed is destroyed, and the balance is false.

In the *steel yard* (*pl.* 16, *fig.* 24) other conditions of equilibrium exist. In this the beam, AB, is a lever of unequal arms; the arms, BF and AF, are supported at F, where the balance is either suspended as in the figure, or else held in the hand. A definite proportion exists between the lengths of the two arms, as 1 : 4 or 1 : 10, &c., and the forces will, according to the

preceding law, be inversely proportional to the lengths, that is, one pound at the end of the longer arm will balance 4 or 10 pounds at that of the shorter. As the short arm, $BF = C$, is fixed, and the weight, W, subject to great variation, and as the counterpoise, P, is likewise constant, the arm, $AF = D$, must be variable to hold any weight, W, in equilibrium. This is attained by shifting the point of suspension of the weight, P. Thus, let $BF = 1$, $AF = 4$, $P = 2$, then will $P : W :: BF : AF$, or $2 : W :: 1 : 4$; then $W = 8$, and 2lbs. at A will balance 8 at B. If, however, W weigh less than 8lbs., then A hanging at P, the arm AF will preponderate, and P will have to be shifted towards the fulcrum. Supposing equilibrium to occur at D, and that $DF = 3$, then we shall have the proportion $2 : W :: 1 : 3$, and W will be equal to 6. This mode of calculation is, however, too tedious in practice, and therefore the long arm, AF, is previously graduated in such a manner, that when the weight and the counterpoise are in equilibrium, a number on the scale opposite the latter indicates the amount of the former. It is evident that the balance is accurate only so long as BF, P, and FA, remain unchanged in length or weight.

The law of the lever finds numerous applications in the determination of the centre of gravity. To obtain the centre of gravity of an irregular figure, as of the quadrilateral, ABCD (*fig.* 9), divide it by a diagonal into two triangles, determine by the preceding methods their centres of gravity, and consider the connecting line, SS′, of these centres, as a lever upon which, at S and S′, forces operate proportional to the surfaces of the two triangles. The centre of gravity or fulcrum, S″, is obtained by dividing the line, SS′, in such a manner, that $SS'' : S'S'' ::$ triangle BCD : triangle ABC. By continuing this process the same end may be attained for figures of more than four sides. The centre of gravity of two combined bodies, BE and *be* (*fig.* 13), is obtained by uniting their separate centres of gravity, and dividing the connecting line, S*s*, into two such parts at S′, that the distances of this point from the centres of gravity shall be inversely proportional to the masses of the two bodies.

If more than two forces act on one lever, striving to move it in two determinate and opposite directions, equilibrium occurs when the sum of the momenta of all the forces acting on one arm, is exactly equal to that of the forces operating upon the other arm. Thus in *fig.* 27 must $P . AF + P' . A'F + P'' . A''F = W . BF + W' . B'F + W'' . B''F$. When the forces on the same arm of the lever operate in different directions, some upwards and others downwards, as in *fig.* 28, then equilibrium takes place when the difference of the momenta of the forces acting on one arm, is equal to the same difference in the momenta of the forces operating upon the other arm; thus, when $W . AF - P . CF = P' . DF + P'' . EF - W' . BF$.

Fig. 31 represents a compound lever, consisting of three simple levers, AB, A′B′, A″B″, acted upon in opposite directions by the weights P, W. Upon the middle lever, whose fulcrum is F′, the force $\frac{P . AF}{BF}$ operates at A′, the force acting on $B' = \frac{W\, B''F''}{A''F''}$: both of these forces press A′B′ upwards,

and to produce equilibrium, $\frac{P \cdot AF}{BF} \cdot A'F'$ must $= \frac{W \cdot B''F''}{A''F''} \cdot B'F'$, or $P \cdot AF$
$A'F' \cdot A''F'' = W \cdot BF \cdot B'F' \cdot B''F''$.

The lever considered thus far has been the mathematical or weightless one; in practice, however, its weight must be taken into account as acting at its centre of gravity. Calling, therefore, the weight of the lever Q, and the distance of the centre of gravity from the fulcrum, q, the conditions of equilibrium in *fig.* 23 will be $P \cdot FA + Qq = W \cdot FB$; for *figs.* 25, 26, and 30, $P \cdot FA = W \cdot FB + Qq$; for *fig.* 29, $P \cdot \cos. \alpha \cdot FA = W \cos. \beta \cdot FB + Qq$; and for *fig.* 27, $P \cdot FA + P' \cdot FA' + P'' \cdot FA'' = W \cdot FB + W' \cdot FB' + W'' \cdot FB'' + Qq$.

The general principles of the rectilineal lever apply to the case of bent levers, or those whose arms form an angle with each other at the fulcrum. Here, however, equilibrium is established when a line drawn from the fulcrum, perpendicular to the straight line connecting the extremities of the lever, divides this line into two parts which are inversely proportional to the forces acting on the ends of the lever. The *bent lever* is much more sensitive than the straight, when its angle is directed upwards, for which reason, in the better scale-balances, the beams are not rectilineal levers, but the fulcrum or point of suspension is generally somewhat lower than the points of attachment of the weights.

To the preceding proportions respecting the lever, it becomes necessary to add, that in every lever, the spaces traversed by the arms of the lever are inversely as the weights or forces, and directly as the lengths of the arms, so that when, for instance, the arms are as 1 : 3, the spaces traversed will be 1 : 3. This proposition is of great importance, as it follows from it that by an elongation of the arm of the lever to which the power is applied, the effect of the lever may be increased in proportion, but that the time required for the production of a particular effect is also increased; so that what is gained in power is lost in time. Archimedes, after developing the law of the lever, was correct in saying, "Give me a fulcrum out of the earth and I will raise her from her foundations." But let us see what effort it would cost him. Supposing him to work for ten hours each day, and to exert a force of 30 pounds in pulling an arm of the lever through 10,000 feet per hour, he would, in the space of 1,473,973,790 centuries, have elevated the earth just one inch! For, let the force exerted = 30 lbs., the weight of the earth =W, and the arc described by the long arm of the lever in moving the short arm one inch = x, then $30 \times x = W \times 1$, and $x = \frac{W}{30}$; that is, to the entire weight of the earth divided by 30.

Now, supposing the earth to be a sphere of a mean radius = 3949 miles, then, since the volume of a sphere $= \frac{R}{3}(4\pi R^2)$, the earth will contain about 256,827,726,120 cubic miles. As a cubic mile of water, at the rate of $62\frac{1}{2}$ lbs. to the cubic foot, will weigh 1,752,400,000 lbs., and as the mean density of the earth, according to Cavendish, is $5\frac{1}{2}$ times that of water, the cubic mile of earth will weigh $5\frac{1}{2}$ times this amount, or 7,638,200,000 lbs. The entire

weight of the earth in lbs. will then be 1,961,701,537,649,784,000,000 Dividing this by 30 = 65,390,051,253,772,800,000 inches=arc described by the long arm while the short arm is moved an inch. Reducing this to feet, and considering that, at ten hours per day, 3,650,000,000 feet would be traversed in a century, we shall have for the final result, 1,473,973,790 centuries as the time required to raise the earth one inch.

The wheel and axle is a simple machine which consists of a cylinder (the axle) and a wheel, both having a common axis, at whose extremity are pins or gudgeons on which the whole can turn. The power operates generally at a tangent to the circumference of the wheel, the resistance being attached to a cord around the axle. *Pl.* 16, *fig.* 33, shows the ordinary construction of the machine, where the gudgeons of the axle are at FE, turning in the parts of the frame HF and AE; the weight, W, is raised by the cord, G, wrapped about the axle, and the power is applied to the wheel, ISB, either by the cord I, or the hand-pins S, S, S. Sometimes, instead of the wheel, arms only, like spokes, are fastened to the axle, or else a winch is employed; the effect, however, is the same. The axle may be vertical, or in any other position, without changing in the least the principle of its operation. The wheel and axle is sometimes called an endless or constant lever, as it is in fact a lever on whose arms power and resistance act always normally, although the lever rotates about its fulcrum, and weights can therefore be raised to any height. In the simple lever, the space traversed by the power is always limited. A catch wheel is attached at D.

The same conditions apply to the wheel and axle as to the common lever. The radius of the wheel is the power arm of the lever, the radius of the axle is the resistance arm, and equilibrium takes place when, in the normal action of the two forces, the power is to the resistance inversely as the radii (arms of the lever) on which they act. It is evident that an increase of power is brought about either by diminishing the radius of the axle, or by increasing that of the wheel, or the winch on which the power acts. This must, however, be within certain limits, as the axle may become too thin and break, and the wheel or winch may become inconveniently large for use. Another obstacle is found in the principle, that the greater the difference between the two arms of the lever, the greater will be the space traversed by the power in proportion to that traversed by the resistance. To obviate the first difficulty, the construction represented by *pl.* 16, *fig.* 35, has been employed. The credit of the invention has been ascribed to the renowned George Eckardt, although its date is more than a hundred years before his time. Here the part A of the axle is stronger than B, and the rope, I, I′, which passes round a pulley and supports the resistance, W, is wrapped about two parts of the axle in opposite directions. When the winch, P, is turned in such a manner that the rope winds up on the stronger cylinder, at each revolution a portion of rope is unwrapped from the smaller cylinder equal to the circumference of the greater. The part of the cord wrapped up, therefore, diminishes by the difference of the circumference of the two cylinders: here the resistance or weight is to the power as the arm of the winch to the half difference of the radii of the cylinder.

A pulley is a circular disk inclosed in a case, turning about an axis passing through its centre, and provided on its circumference with a groove for the reception of a cord. Pulleys are fixed or movable.

In the *fixed pulley* (*fig.* 19), the case is stationary and attached to some object. At one end of the rope which passes over the pulley is the power, at the other end the resistance; the former must be equal to the latter; and the advantage consists only in being able to give the power any desired direction. Thus, a weight may be raised by a power acting horizontally, or vertically downwards. Pulleys of this character (fixed pulleys) occur in *figs.* 15, 16, 25, 26, 28, 29.

Movable Pulleys, as represented in *fig.* 36, are distinguished from fixed in that the case of the pulley is movable. The cord, I, is fastened to a hook, passes under the pulley AB, which carries the weight W, and is then either elevated by the power P, or, as in *fig.* 37, passes over a second pulley to be drawn up from below. In the fixed pulleys, which are properly nothing more than means for changing the direction of motion, the weight must be equal to the power; in the movable, however, another condition occurs. Here the power is to the weight as the radius of the pulley to the chord of the arc of the pulley embraced by the rope. The most advantageous case is exhibited when the two sides of the rope are parallel, and the chord equal to twice the radius of the pulley. The power is here to the weight as 1 : 2, that is, one pound of power will raise two of weight. In the double pulley, the same condition takes place, the second pulley being a fixed one, and only serving to change the direction in which the power is applied.

In a single pulley, the proportion of 1 : 2 is the only one that can be attained, even in the most favorable cases; any desired proportion of weight to power can, however, be effected by a skilful combination of several pulleys, fixed and movable. Of these combinations there are two kinds, those in which but one string is used, and those in which several are employed. *Pl.* 16, *fig.* 38, represents the first kind; *figs.* 39 and 40, the second. In *figs.* 38 and 39, the weight, W, is attached to the movable pulleys, and the power, P, acts upon the last fixed pulley: in *fig.* 40, the relation is just the reverse, without changing the operation. As in one of these combinations all the strings must be stretched equally, and all except that on which the power operates must receive their tension from the weight—this tension, however, equalling that produced by the power—equilibrium will take place when the power is to the weight as 1 to the number of strings stretched by the weight. In *fig.* 38, or the *power pulley*, the pulleys are placed one above the other, and the statical relation of the machine is as 1 : 4; in *fig.* 39, where the pulleys are not immediately one above the other, and are united by several strings, every movable pulley connected with another by a special cord doubles the power of the machine; hence it follows, that in this combination, although the weight is suspended to four pulleys only, A, A′, A″, A‴, the statical relation is as 1 : 16. The combination represented in *fig.* 40 is still more advantageous, in which the weight is fastened to the extremities of all the cords, the axis of the upper pulley alone being attached to a beam, while all the other pulleys are movable. Here, with three movable pulleys,

the weight is to the power required for equilibrium as 15 : 1 ; with n movable pulleys, it will be as $2^{n+1}—1:1$. The combination in *fig.* 43, in which the cords A, A′, A″, work obliquely, is less advantageous and convenient.

White's Pulley is represented in front by *fig.* 42, and laterally in *fig.* 41, consisting of two blocks, Q and R, of which one is fixed and the other movable. Each block has six concentric grooves, which act as so many single pulleys, the weight hanging to twelve cords, *b*, *c*, *d*,—*n*. Hence, with this number of pulleys, the relation between weight and power is 144 : 1. This combination, however, besides the slowness of movement, has the disadvantage that, from the small diameter of the lesser pulleys, the rigidity of the cords is so great as very sensibly to affect the action of the machine.

The inclined plane, as the fourth simple machine, is represented in *figs.* 44–46. AB is the base, BC the height, AC the length of the inclined plane, viewed as a right-angled triangle, up which the weight, M, is to be moved. Divide according to the parallelogram of forces, the weight, W, of M, acting vertically downwards, into two forces, one perpendicular to the direction of the inclined side of the plane, the other parallel to it ; the former will be expressed by $W \cos. BAC = W \cos. \beta = W\frac{AB}{AC}$ the weight sustained by the resistance of the inclined plane, and the latter $W \sin. BAC = W \sin. \beta = W\frac{BC}{AC}$ expressing the amount of the force parallel in its direction to the inclined plane, necessary to produce equilibrium. Hence this force will be smaller as the inclination of the plane is less, or as the length of the plane is greater than its height. Should the force, as in *fig.* 46, act in a horizontal direction, or one parallel to the base of the plane, then the force, P, required to sustain the weight, W, will be $P = W \text{tang.} BAC = W \text{tang.} \beta = W\frac{BC}{AB}$, or the force is to the weight as the height of the plane to its base. The force is thus smaller in comparison with the weight to be sustained, as the height, BC, is smaller with respect to the base, BA ; when, as in *fig.* 45, BC = AB, or BAC = 45°, then P=W, or the power is equal to the weight. Finally, if the height, BC, be greater than the base, AB, or BAC greater than 45°, the force must be greater than the weight.

The *wedge*, the fifth simple machine, is illustrated by means of *figs.* 47 and 48. It has in general the form of a three-sided prism (in the figure appearing as a triangle, ABC) : upon the side AB, and perpendicular to it, a force operates in endeavoring to drive the opposite edge, C, into a body to be split, or between two bodies to be separated ; or, in case this has already been done, to retain it in its place. If, upon the wedge ABC (*fig.* 48), a force operates perpendicularly to its length, DC, endeavoring to drive it out, equilibrium occurs when the power is to the resistance as the sine of half the angle included between the two sides of the wedge, or sin. α, to the sine of the angle included between the direction of resistance and the side of the wedge. The power obtained is as the cosine of the latter angle. *Fig.* 47 represents

the wedge when the force acts abnormally, or not in the direction of the length of the wedge, by which means the wedge is driven in obliquely. In this case, the resistance is to the power as radius to the difference of half the angle included between the sides of the wedge, and the angle made by the direction of resistance with the side of the wedge. In any case, the right-angled wedge may be looked upon as an inclined plane, and the isosceles wedge as the combination of two equal inclined planes. The wedge is the more powerful as the angle included between its sides is greater; it is driven in, however, more easily as this angle is less. The wedge is principally used for splitting, in which the power acts by percussion, so that, practically, no accurate calculations can be made from the principles referred to above.

The screw is merely an inclined plane wound around a cylinder. Construct a rectangle (*pl.* 17, *fig.* 1), divide two opposite sides into any equal number of equal parts, unite the points of division, 1, 2, 3, 4, &c., of the one side, with 2, 3, 4, 5, &c., of the other, by the lines aa', cc', dd', ee', and suppose the rectangle lapped around a cylinder, the circumference of whose base exactly equals an undivided side of the rectangle; then the lines aa', cc', dd', &c., will form on the cylinder a continuous curved line, called a *screw line*, and each single winding is called a *turn of a screw*. The height of a turn of the screw is the distance between two contiguous turns, or between the two points of the screw line lying vertically one above the other (as a, c, or l, m). If, now, a prismatic body be wound around the cylinder on the screw line, it will form the winding or thread of the screw; and the whole taken together will be a *screw spindle*, or *male screw*, when the thread is on the outside of the cylinder: it is a *female* or *mother screw* when the thread is applied to the inside of the cylinder or cylindrical cavity. According as the prismatic body wound around the cylinder is a three or four-sided prism, the thread of the screw is called *sharp* (*pl.* 17, *fig.* 2) or *flat* (*fig.* 3), where A is the spindle, and Q the mother or female screw. This female screw consists of a prismatic body, DE, in whose cylindrical hole a thread, B, is situated. The male and female screws differ in the thread being applied to a cylindrical convexity for the former, and to a cylindrical concavity for the latter. The thread of the screw may have other forms than that of the three or four-sided prisms; these are, however, the most convenient and generally used.

Male and female screws can only be used in combination with each other, and even in cases where one seems to be absent (as the female of a wood screw), it is formed by the one that is present in the material itself. Strictly speaking, the screw, although always included among them, does not belong to simple machines, as it can never be applied without the assistance of a lever to turn the spindle in the nut.

In the movement of a screw three cases may present themselves: either the spindle is fixed and the nut is turned, thus advancing along the former; or the nut is fixed and the spindle moves in it; or, finally, both male and female move, uniformly, but with different velocities, often in different

directions, whence arises a retarded or accelerated differential motion; the theory remains the same, however, in all cases.

With regard to the statical condition of the screw, equilibrium takes place when the power is to the resistance or weight as the height of a turn of the thread, or the distance between two threads, is to the circumference of the circle described by the power. Hence it follows that by prolonging the lever used in producing the rotation, or by diminishing the height of the threads, the greatest resistance can be overcome by a moderate power; here, however, the universal law presents itself, that what is gained in power is lost in time.

An endless screw is a spindle containing only a few turns, which catch either in a half open female screw, cut in the circumference of a disk, or in a wheel whose teeth are placed obliquely to the axis of the wheel, and in the direction of the obliquity of the thread, or in a rack-work with similarly situated teeth. The application of the endless screw to a windlass has been selected as an illustration, and figured in *pl.* 16, *fig.* 34. Upon the axle BC, turned by the winch A, are to be found at D several turns of a screw, which, immovably fastened to the axle, turn with it without advancing. In these turns of the thread, the oblique teeth of the wheel, F, catch, thus moving along the inclined plane of the thread, and causing the wheel to turn. As there are always as many teeth of the wheel caught by the screw as the latter has complete turns, and as for the turns going out at one side, new ones are constantly entering at the other, the motion is endless. This machine, it will readily be perceived, is a combination of the screw with the wheel and axle, and its statical condition will be $P \times AB \times rad.\,F = W \times$ *height of a turn of the thread* $\times$ *rad. of axle.*

The screw, in its various varieties and modifications, finds innumerable applications in machinery; we shall here briefly mention a single one, the differential screw of Hunter, represented in *pl.* 17, *fig.* 4. EF is a plate of metal in which the screw D works, having, for example, ten turns to the inch. The inside of the screw is hollow, and forms at LM a nut, in which works the smaller screw, NO, having perhaps eleven turns to the inch, and forced by the frame, EFGH, to take part in the motion of the screw, D. Suppose now that by means of the handle BC, the screw D is turned round ten times, then A will rise one inch, and will raise the point K to an equal height. Turning the screw NO ten times in the opposite direction, the point K will descend $\frac{10}{11}$ of an inch, and the result of the whole will be an elevation of $\frac{1}{11}$ of an inch. Now, however, while the screw D turns ten times, the turning of NO is hindered by the square shoulder at K, and the result is the same as if NO had been turned ten times in the other direction, and K will consequently ascend only $\frac{1}{11}$ of an inch: for a single revolution of the screw this will amount to $\frac{1}{10}$ of $\frac{1}{11}$, or $\frac{1}{110}$ of an inch, which is the actual ascent or descent of the screw. Suppose the length of the lever, AB, to be only six inches, then to produce equilibrium the power must be to the resistance as 1 to $110 \times 6 \times 2\,\pi = 4146.912$.

With respect to the simple machines, it is to be remarked, that to pro-

duce motion the applied force must be considerably greater than what is necessary for equilibrium, and this increase of power required will be in proportion to the number of obstacles to motion. Of these, the principal is *friction,* which requires a greater or less increase of power, when an actual motion of the machine is demanded. On the other hand, friction admits a diminution of power when equilibrium is to be restored after motion has taken place, or when motion is to be prevented. In the investigation of the action of machines, therefore, reference must be had to friction and similar hindrances, the rigidity of cords, &c., for example.

c. On the Strength and Stress of Materials.

When a solid body is exposed to any stress whatever, whether in the direction of its fibres, or perpendicular or obliquely to them, and this stress be continued until a fracture results, before this last circumstance occurs, there must be a moment in which there is an equilibrium between the resistance of the fibres of the body or its strength, and the stress to which it is exposed; by strength being meant the power resisting fracture, and stress the power tending to produce fracture. By reason of this equilibrium the theory of the strength of bodies comes under the head of statics.

This *strength* of bodies may be considered under three points of view: first, with regard to the *absolute* or *longitudinal strength,* or the resistance presented by a body to a force acting in the direction of its fibres, and tending to tear them apart, as in *pl.* 17, *fig.* 5; secondly, with regard to their *relative, respective,* or *transverse* strength, or the force with which a body supported or fastened at one or both ends, resists a force acting transversely, that is, perpendicularly or obliquely to the direction of its fibres; thirdly, *the strength of resistance,* or the force with which a body resists a pressure tending to crush or crumble it. By *strength of torsion* is meant the resistance of a body to a force striving to twist it about its fixed axis.

The absolute strength of two beams or rods—the form is indifferent—is in direct proportion to the area of their transverse sections. Thus if the body fastened to A (*fig.* 5, *pl.* 17) have at B a transverse section of one square inch, and be just capable of supporting the weight applied to C, then a body three inches square or nine inches in area will sustain nine times that amount. The weight of the body itself, however, must be taken into account, as acting at its centre of gravity. A rod or pole may be made so long as to break or tear asunder with its own weight, as soon as its weight acting at the centre of gravity exceeds the absolute strength of the transverse section. On this account, this centre of gravity should be brought as near as possible to the point of support, and such bodies should always be made stronger above, as in *fig.* 5.

If to a wire or any elastic body weights be suspended, not enough, however, to produce a rupture, and the extension suffered by the operation be measured, it will be found that the relation between the weight, P, and the

extension, E, may be expressed by the following general equation: $\frac{P}{P'}=\frac{E}{E'}\cdot\left(2-\frac{E}{E'}\right)$, where P′ is the weight at which the wire would tear, and E′ the extension produced by it.

However simple the theory of absolute or longitudinal strength may be, that of *relative* or *transverse strength* is exceedingly complicated. Here, not only the area of the transverse section is to be taken into account, but also the shape; and likewise, in addition to the resistance against fracture, that also to every bending of the body which may be produced by the pressure.

If a prism be supported at the two extremities, or fastened at one, and be loaded in the middle, or at the free extremity in the latter case, there will be a bending of the prism. This will take place in such a manner, that while one set of fibres will be stretched, another set will be compressed; in the interior of the transverse section, therefore, a fibre can be imagined about which this bending takes place, without experiencing itself either extension or compression; this fibre is called the *axis of flexion,* or the *neutral axis.*

Supposing the fibres of a beam to be absolutely incompressible, and the beam loaded as in *pl.* 17, *fig.* 8, at Q, then it must turn about its lower point in the line through AC, and every fibre in this direction will be in a state of tension; if all the fibres were entirely unextensible, then the rotation would occur in the same manner, but every fibre in the line would be in a condition of pressure. It is known, however, that all bodies may be both compressed and extended; therefore the rotation will be about neither the upper nor the lower point, but, as in *fig.* 6, about the point B, and the upper fibres will then be stretched, while the lower will be compressed; those in the line AB will be in a condition of neutrality. Now, both above as well as below the neutral axis, a point may be imagined, in one of which the moments of compression, and in the other of extension, are united, these being the means of pressure and tension. In *fig.* 9, let the weights, P and Q′, represent the sum of this tension and compression, then the position of the neutral axis will be determined by the ratio of the moments, and will lie in the middle when the moments are equal. The mean points of compression and extension coincide with the centres of gravity of their respective surfaces.

The mode of finding the neutral axis, and consequently the relative strength, for the case in which the body consists of extensible and compressible fibres, is explained in *fig.* 6. Imagine a body in the form of a parallelopipedon, whose breadth is b, and height h, and which is fastened in such a manner into the wall, CC, as to have in a natural condition the direction BB′. If, by a weight at A, it be bent into the position, BFA, then BFA is the neutral axis. Let EF $=\lambda$ be a smaller part of this axis, so that GK is an element of the body; then, in an uncompressed condition, this will everywhere be equal in length to λ. Draw JK parallel to GG′, and represent the distance, ET, of a fibre, ST, from the axis by $u=$ (FT); also make

g equal to the distance from the axis of the most extended fibre, β, then will $ST = \frac{\beta}{g} . u$, and the force q, producing this extension, will $= \frac{AE}{\lambda} . \frac{\beta}{g} . u$; here A is the absolute strength, and E the modulus of elasticity, or the weight necessary to stretch the body to double its length. GF is, however, composed of an innumerable number of fibres, whose sum, FH, may be represented by h', and the force, P, necessary to extend all these fibres will $= \frac{E}{\lambda} . \frac{\beta}{g} . \frac{bh'^2}{2}$. The compressing force, P′, for the part below the axis, whose modulus of elasticity, or force required to compress it to half its original length, may be represented by E′, will be $= \frac{E'}{\lambda} . \frac{\beta}{g} . b \frac{(h-h')^2}{2}$. The statical moments of the two forces are, $Py = \frac{E}{\lambda} . b . \frac{\beta}{g} . \frac{h'^3}{3}$ and $P'y = \frac{E'}{\lambda} . \frac{\beta}{g} . b . \frac{(h-h')^3}{3}$. The statical moments, however, of the weight Q, whose leverage, $FL = x$, will then necessarily be $Qx = \frac{B}{\lambda g} . \frac{b}{3} (Eh'^3 + E'(h-h')^3)$. Since the fibres at F experience no compression, $P + P'$ will $= O$, or $Eh'^2 = E' (h-h')^2$, Qx then becoming $= \frac{\beta}{\lambda g} . E . \frac{bh . h'^2}{3}$.

Producing GG′ and HH′, until they intersect at U, then UF will be the radius of curvature, ϱ, for the arc element, $EF = \lambda$, and $\frac{\lambda}{\varrho} = \frac{ST}{u} = \frac{\beta}{g}$ and $\frac{1}{\varrho} = \frac{\beta}{g\lambda}$: this value substituted in the formula for Qx, and φh taken for h', where φ is a magnitude dependent upon the situation of the neutral axis, and expressing the ratio of extensibility and compressibility, we will have $Qx . \varrho = E . \varphi^2 . \frac{bh^3}{3}$. The right side of this equation is constant for equal parallelopipeda, and depends upon the elasticity of the body; it is called the moment of elasticity $= W$. Let Q be the mean of several forces, then Qx, the sum of their moments, will $= M$, and $M\varrho = W$; that is, for every transverse section at right angles to a bent parallelopipedon, the product of the radius of curvature by the moment of the force, is a constant quantity.

In most cases, however, the bending of the body is so slight, that the leverage, x, of the weight Q, may be exchanged for the length, $FA = 1$, and $\frac{\beta}{\lambda} = \frac{m}{E}$: we thus obtain, by introducing this quantity into one of the preceding equations, $Q1 = \frac{m}{gE} . W$. Suppose now the body (*pl.* 17, *fig.* 6) to be fixed in the plane HH′, the preceding formulæ will give the moment of the weight, Q, which can break off the body, HDD′H′, at the plane HFH′; Q is also the relative or transverse strength of the parallelopipedon. The co-efficient of fracture, m, must be obtained by trial. Assuming the neutral

axis to pass through the centre of gravity of the surface of fracture, then $\varphi = \frac{1}{2}$, and $g = \frac{1}{2} h$, which gives the relative strength of the parallelopipedon, $Q = \frac{1}{b} m \frac{bh^2}{1}$. The relative strengths, therefore, of parallelopipedal bodies of the same material are as their breadths, as the squares of their depths, and inversely as their lengths. If it be necessary to consider the weight, G, and if the centre of gravity be taken at half the length, we obtain $Q = \frac{\frac{1}{6} m \, . \, bh^2}{b} - \frac{1}{2} G$.

As an illustration of the application of this proportion, let *fig.* 10 represent a rectangular plate, with its longer edge, AF, walled in horizontally: suppose a weight, Q, to be suspended at E, and increased until fracture ensues Required the direction of the line of fracture, BD, and the magnitude of the weight, Q. Representing the height or depth of the plate, BF, by h, then $Q = \frac{1}{6} m \, . \, \frac{BD \, . \, h^2}{GC}$. If the unknown angle, DBC, be represented by α, then $BD = \frac{BC}{\cos. \alpha}$, or if $tg\alpha = x$, $BD = BC \sqrt{1 + x^2}$; also $GC = BC \sin. \alpha = BC \frac{x}{\sqrt{1 + x^2}}$, and these values substituted in the equation for Q, give $Q = \frac{1}{6} m \, . \, \frac{1 + x^2}{x} h^2$. Finding from maxima and minima, the value of x, for which the factor, $\frac{1 + x^2}{x}$, is a minimum, we learn that this is the case when $x = 1$, whence $tg\alpha = 1$, and $\alpha = 45°$: Q is then $\frac{1}{3} m \, . \, h^2$.

The strength of a beam, AB (*fig.* 12), exposed to fracture from a weight, Q, acting in a direction perpendicular to its fibres, is as the product of the transverse section at the place where the weight is applied, and the distance from the centre of gravity of the same cross-section, to the point or line where the fracture terminates. In beams of square sections, the strengths are as the cubes of the sides; in cylindrical beams, as the cubes of the diameters; in two similar beams, as the cubes of the homologous sides.

The strongest rectangular beam which can be cut from a given cylinder, is one in which the squares of the breadth, depth, and diameter of the cylinder are as 1 : 2 : 3. This beam may be found, according to *pl.* 17, *fig.* 7, in the following manner:—Divide the diameter, AE, into three equal parts at G and H; erect GF and DH perpendicularly to these points, and produce them to the circle, BC; A, F, D, and E, will determine the four corners of the beam. Here the breadth of the beam is to its depth as 5 : 7, or more accurately as 12 : 17.

The strain to which beams are exposed, under different circumstances, is determined by very complicated calculation. Let L represent the length of leverage, from the neutral axis to the point of attachment of the weight, W the weight, and α the angle made by the above-mentioned leverage with the horizon at the instant of fracture; then the strain for the case repre-

sented in *fig.* 8, will be $= \text{LW} \cos. \alpha$; for that in *fig.* 11 $= \frac{1}{4} \text{LW} \sec.^2 \alpha$, and for that in *fig.* 12, $= \frac{1}{6} \text{LW} \sec.^2 \alpha$.

The preceding formulæ have had reference to the conditions of equilibrium of beams supported at both ends and loaded in the centre; we will now consider the case where the load is applied elsewhere than in the middle, as in *pl.* 16, *fig.* 50. The weight appended may then be supposed to be divided into two weights, which act on the arms of levers whose lengths are as the parts of the beam. Thus, representing by L the entire length of the beam, m, and n its parts, then the pressure $= \frac{mn}{m+n} . \text{W} = \frac{mn\text{W}}{\text{L}}$. Supposing two equal or different weights applied at different points, as in *fig.* 51, and calling the distance from the left point of support to the left point of suspension of the weight, m; that from the left point of suspension to the right point of support, n; that from the left point of support to the right point of suspension, r; and that from the right point of suspension to the right point of support, o; then for the first weight the pressure will be $\text{F} = \frac{mn\text{W}}{\text{L}}$, and for the second $\text{F} = \frac{or\text{W}'}{\text{L}}$, where W and W′ are the corresponding weights, and L the length between the points of support. To obtain the pressure resulting from this double pressure, upon every other point of the beam, call the distance of this point from the left point of support, s, and that from the right, t, and we will have the following proportion: $n : t :: \frac{mn\text{W}}{\text{L}} : \frac{mt\text{W}}{\text{L}}$, for the pressure exerted by the left weight; and $o : s :: \frac{ro\text{W}'}{\text{L}} : \frac{os\text{W}'}{\text{L}}$, for that of the right; hence the combined pressure at this third point $\text{F} = \frac{mt\text{W} + os\text{W}'}{\text{L}}$.

An application of this proposition is to be found in *fig.* 49, where the weight acts upon the middle of an inflexible bracket. Here the effect of this weight upon the beam is the same as if two weights of half the original one were suspended at the points where the bracket meets the beam. It will be easy, from the preceding, to determine the value of F in the middle of the beam, where, as in *pl.* 16, *fig.* 52, several equal weights are suspended. It also follows, that when the burden is distributed uniformly over the whole beam, its action is the same as if half the amount were attached to the centre of the beam.

The beams hitherto considered have been, for the most part, such as were supported at the ends; and we have found that such a beam is four times as strong as the same beam attached to a wall by one extremity and loaded at the other. Supposing the beam to be walled in at both ends, as in *pl.* 17, *fig.* 12, and loaded by the weight Q, we may assume that it will break at the same instant in A, B, and C, provided Q be of sufficient amount. Represent the forces which produce fracture at these three points by p p', p'', and the two parts of the beam by a, a', the total length of the

beam by L, its breadth by b, and its depth by h. Fracture will then ensue when, according to the preceding formula, $p = \frac{1}{6} m \frac{bh^2}{a}$; $p' = \frac{1}{6} m \frac{bh^2}{a'}$; and $p'' = \frac{1}{6} m \frac{bLh^2}{aa'}$. Q, however, must be sufficient to produce all three fractures; therefore, $Q = \frac{1}{6} mbh^2 \left(\frac{1}{a} + \frac{a}{a'} + \frac{L}{aa'}\right)$; or, as $a' = L - a$, $Q = \frac{2}{6} m \frac{bLh^2}{aa'}$. Calling the distance by which the point, C, lies out of the centre, d, then will $Q = \frac{8}{6} m \frac{bLh^2}{L^2 - 4d'^2}$; if $d = 0$, or if C lie in the middle, then $Q = \frac{8}{6} m \frac{bh^2}{L}$. Hence it follows from this formula that beams loaded in the middle are weakest, but that they can support eight times as much as when attached at one end and loaded at the other.

For the case in which the beam, as in *fig.* 13, is inclined at an angle, as $BAD = \alpha$, to the horizon, the perpendicular lateral force, $CG = Q \cos. \alpha$, can alone tend to produce fracture; the other lateral force, $CF = Q \sin. \alpha$, involving the strength of crushing: Q becomes then $= \frac{1}{3} m \frac{bLh^2}{aa' \cos. \alpha}$.

Those bodies which in all their sections present the same strength are of great importance: *the bodies of equal resistance.* The fracture of bodies of equal section throughout occurs always at the surface of attachment, or where the weight is attached; consequently the transverse sections lying at a distance from these points are too great, and must be diminished. Such a case has been considered (*fig.* 5) under the head of absolute strength; it remains here to mention some others. *Fig.* 14, *pl.* 17, represents a body which, fixed at one end, is loaded at the other with the weight Q, and where transverse sections are throughout, rectangles of equal breadth: representing the height by y, the breadth by z, and the distance from C of the section MN by x, then, according to the preceding nomenclature, $AB = h$, $AC = L$, and $z = b$: we then have $\frac{bh^2}{L} = \frac{by^2}{x}$, hence $y^2 = \frac{h^2}{6}.x$. This, however, is the equation of the parabola; and the outline, BC, must be a parabola, whose vertex lies at C, and whose parameter $= \frac{h^2}{6}$. *Pl.* 17, *fig.* 15, represents a similar body, ABC, upon which the weight, Q, is uniformly distributed. Here the same references are employed, and we have for y in the section MN, the value $y = \frac{h}{L}.x$, whence it follows that the outline, BC, must be a straight line. Finally, suppose *fig.* 16 to represent the body, AB, resting freely at its two extremities, its sections rectangles of equal breadth, and the weight, Q, moving longitudinally above the body; required the conditions according to which the inferior curve line is formed. Let $AC = BC = \frac{1}{2} L = a$, $CD = h$, and for any given section, MN, $CM = x$, and $MN = y$; then $y^2 = \frac{h^2}{a^2}(a^2 - x^2)$, and the curve of outline will be a

semi-ellipse, whose semi-major axis is a, and semi-minor axis $= h$. Should the least height not equal 0, but a quantity, $CC' = c$; then if $M'N$ be taken $= y$, and $MN = y'$, $y' + c$ will equal the height, and the equation becomes $(y' + c)^2 = \frac{h^2}{a^2}(a^2 - x^2)$; and for the points, A and B, beyond which the height remains unchanged, where y' thus $= 0$, we will have $x = \frac{a}{h}\sqrt{h^2 - c^2}$.

It is often desirable to determine the amount of flexion which precedes the fracture of any elastic body; in this case it is necessary to determine the shape of the elastic line formed by the neutral axis. Suppose (*fig.* 17) BZ to be the natural condition of a fibre attached at one end, B, and this fibre loaded at A by Q, and uniformly along its whole length by a weight, which, for a single unit of length, amounts to p; the fibre takes the form of the elastic line, AB. Let AC be the axis of abscissas, A the origin of co-ordinates, and for any given point, E, of the curve, whose radius of curvature is ϱ, take $AF = x$, $FE = y$, the greatest ordinate, $BC = u$, and $AC = a$; let W also be the moment of elasticity, and for the elastic line we will have the co-ordinate equation, $y = \frac{Qx}{2W}(a^2 + \frac{1}{3}x^2) + \frac{px}{6W}(a^3 - \frac{1}{4}x^3)$, and the greatest ordinate, u (where $x = a$), $= \frac{Qa^3}{3W} + \frac{pa^4}{8W}$. If $p = o$, or the fibre be loaded only at the end, then will $u = \frac{Qa^3}{3W}$, and $y = \frac{Qx}{6W}(3a^2 - x^2)$; and if $Q = 0$, or the fibre be loaded only uniformly along its whole length, $u = \frac{pa^4}{8W}$, and $y = \frac{px}{24W}(4a^3 - x^3)$. According to the above formulæ, the co-ordinates are as 8 : 3, thus the depression is much greater when a weight hangs at the extremity of the fibre, than when it is distributed along its whole length.

If the elastic fibre rest, as in *fig.* 18, at both ends, the weight Q being applied in the middle, the equations answering to these conditions result from the preceding. Let Q be the weight applied to the middle, pL that distributed along the whole length, L; then each support receives a pressure $= \frac{1}{2}(Q + pL)$. Suppose, however, the fibre to be fastened at C, and the pressure at A and B to act upwards, then, in the preceding co-ordinate equation, $\frac{1}{2}(Q + pL)$ must be substituted for Q: the second part of that equation must be taken negatively, as it contains p as a factor, and this must necessarily act vertically downwards, or in the opposite direction to $\frac{1}{2}(Q + pL)$. As, moreover, $\frac{1}{2}L = a$, we obtain the new co-ordinate equation $y = \frac{(Q+pL)}{4W}\left(\frac{L^2}{4} - \frac{1}{3}x^2\right)x - \frac{px}{6W}\left(\frac{L^3}{8} - \frac{1}{4}x^3\right)$. The greatest ordinate, also, when $x = \frac{1}{2}L$, becomes $u = \frac{L^3}{384W}(8Q + 5pL)$. If $p = 0$, then $y =$

$\frac{Q}{4W}(\frac{1}{4}L^2 - \frac{1}{3}x^2)\,x$, and $u = \frac{QL^3}{48W}$; if Q again $= 0$, then will $y = \frac{px}{24W}(L^3 - 2Lx^2 + x^3)$ and $u = \frac{5pL^4}{384W}$. Assuming $Q = pL$, then the depression in the two cases will be as $8:5$; consequently, when a weight is distributed uniformly along the whole fibre, the depression will be only $\frac{5}{8}$ of what would result from the application of the same weight to the middle.

In investigating the strength of resistance to a crushing force, we suppose prismatic bodies standing vertically, upon whose upper extremities weights are laid, and then investigate the force necessary for crushing, and that which produces first a bending, and then a cracking. With respect to the force of crushing, it appears, from experiment, to increase in a somewhat greater ratio than the cross section, although it may be properly assumed that if all parts of the cross section experience equal pressure, the force will be proportional to the cross section. Calling, therefore, the strength (obtained by trial) of a certain cross section, m, and the area of the prism to be investigated, A, then $Q = mA$. The capacity for being crushed diminishes as the circumference increases, the area remaining the same; it is, therefore, least in the circle: it is less, also, as the form of the body approaches in height to the cube.

To obtain the law of cracking, let us suppose an elastic rod, AB (*pl.* 17, *fig.* 19), which, fastened at A, assumes naturally the vertical direction AZ; becoming bent, however, into the curve ADB by a weight attached to the upper end, B. To find the co-ordinate equation of this curve, assume the vertical direction, BC, of the weight as the axis of abscissas, and B as their origin For any point, D, of the curve whose radius of curvature is ϱ; let $BQ = x$, $DQ = y$, and $AC = a$, and let the curvature of the rod be so slight that the abscissa may be exchanged for the length of the arc. If, now, y be the leverage of Q, then $M = Qy$, and $Qy = \frac{W}{\varrho}$. By assuming another point of the curve, F, infinitely near to D, and bringing into the calculation the quantities FH, DH, with their trigonometrical proportions, we finally obtain for x the value $\sqrt{\frac{W}{Q}}$ arc. $\left(\sin. = \frac{y\sqrt{Q}}{\sqrt{Qa^2 + W tg\varphi^2}}\right)$, where the one factor is an arc whose sine is equal to the quotient of the two radical quantities, φ indicating the angle at which the geometrical tangent of the point A meets the curve. For y we have the value $\frac{\sqrt{Qa^2 + W tg\varphi^2}}{\sqrt{Q}} \sin. \sqrt{x\sqrt{\frac{Q}{W}}}$. Most generally a is to be taken $= 0$, or the direction of the bending weight passes, as in *pl.* 17, *fig.* 20, through the point of attachment, A. The equation then becomes $x = \sqrt{\frac{W}{Q}}$ arc. $\left(\sin. = \frac{y}{tg\varphi}\sqrt{\frac{Q}{W}}\right)$, and $y = tg\varphi\sqrt{\frac{W}{Q}} \sin. \left(x\sqrt{\frac{Q}{W}}\right)$.

For the points A and B, $y = 0$, thus $x = \sqrt{\frac{W}{Q}}$ arc $(\sin. = 0)$; as, however, arc $(\sin. = 0)$ may be taken $= 0, \pi, 2\pi, 3\pi - - - i\pi$, where i represents

any whole number, it follows, if L represent the length of the rod $= x$, that $L = i\pi\sqrt{\frac{W}{Q}}$, and if $i = L$, $L = \pi\sqrt{\frac{W}{Q}}$, and $Q = \frac{\pi^2 W}{L^2}$. As, however, Q is independent of the amount of the bending, this weight, in any degree of bending, holds the elasticity of the body in equilibrium, or Q is the capacity of cracking of the rod.

Combining these values with those previously obtained by substituting the moment of elasticity for W, we find that in prismatic beams of homogeneous material, the capacities of cracking are as the breadths, as the third power of the thicknesses (least sides), and inversely as the squares of the lengths; in cylinders, as the fourth powers of the radii, and inversely as the squares of the lengths.

With respect to the *strength of torsion,* or twisting, let us suppose a body (*fig.* 21, *pl.* 17) fixed at one of its ends, AA′, and a force, P, acting at the other extremity on the arm of a lever, CD $=$ R, capable of producing a rotation about the axis, CC. If, now, the diameter BB be twisted to B′B, AA′ will be stationary; the homologous diameters, however, of all intermediate sections will be displaced in proportion to their distance from the surface of attachment. The angle BCB′ is then the angle of rotation, and the turning force must be strong in proportion to the amount of this angle, to the strength of the transverse section of the fibres, and to the distance of the fibres from the axis of rotation; the longer the fibres, however, the less need be the force.

An actual twisting apart of the body must ensue when the remote fibres can yield no more without being actually ruptured; and in cylinders of homogeneous material, the statical moments of the forces which produce such a rupture by twisting, are as the cubes of the radii.

B. Dynamics of Solid Bodies.

The theory of motion is much more difficult as well as more comprehensive than that of equilibrium: it calls mathematics into play to a much greater extent, and this in its most abstruse branches.

The motion of a body, which may result from one or several forces, is, in respect to its direction, either rectilineal or curvilineal; in respect to its velocity, either uniform or variable. Motion is said to be *equable* when equal spaces are traversed in equal times: when, for example, the same amount of space is passed over in each successive second. Of this kind is all motion produced by a single force acting instantaneously—in a blow, for instance—provided that the motion meet no obstruction. Motion is *variable* when, instead of remaining the same, it increases or diminishes. If the motion increase or diminish equally in equal times, it is said to be *uniformly accelerated* or *retarded.*

The force itself producing motion may be either momentary or continuous. In the former the force is to be considered as acting for a very

little, or no time at all; in the latter the action takes place incessantly without a conceivable instant in which the force does not exert its influence. Every momentary force imparts to a material point upon which it operates an equable motion; every continuous force operates in producing an accelerated or retarded motion.

The following may be adduced as fundamental propositions in Dynamics, consequently not derived *à priori*, but the results of experience. They are modifications of the well known Newtonian laws of motion.

1. A moving material point continues in a state of rectilineal and equable motion, until affected by some other influencing force.

2. Two forces acting momentarily, are as the velocities which they communicate to the same material point in the same instant of time.

3. A moving body loses just as much motion as it communicates to another body; that is, action and reaction are equal and opposite.

a. Equable Motion.

As a material point or body, in a condition of equable motion, traverses equal spaces in equal times, the spaces traversed in different times are as these times. If, therefore, s be the space traversed in a time, t, and s' that traversed in a time, t', then $s:s'::t:t'$; and if $t'=$ one second, s' is the velocity, c, of the body; thus $s=ct$, $c=\frac{s}{t}$, and $t=\frac{s}{c}$. Thus in equable motion the space described equals the product of the time by the velocity; the velocity equals the space divided by the time; and the time equals the space divided by the velocity.

If a body be acted upon by two momentary forces in different directions, the direction and velocity of the motion will take place as the diagonal of the parallelogram of forces. Representing the velocities of the forces by c and v, and the included angle by α, then the velocity attained, $x=\sqrt{c^2+v^2+2cv\cos.\alpha}$, and the corresponding parallelogram is called the *parallelogram of velocities*. From this it may readily be shown how much a body loses in velocity by moving with a given velocity against a fixed obstruction, and from it, it also follows, that an equably moving body which enters in the direction of the tangent upon a curve, must move in it with undiminished velocity.

b. Varying Motion.

It has been already observed that varying motion may be uniformly so or not. Taking first into consideration the *uniformly accelerated* motion of a body, the velocity after the expiration of any period of time (the final velocity) may easily be determined. In this case the velocity increases equably in equal times. If, therefore, G be the velocity at the expiration

of the first second, the acceleration for the following seconds becomes 2G, 3G - - - tG, and the final velocity is V $= t$G.

To determine the space, s, traversed by the body in the time, t, suppose t to be divided into infinitely small portions, and let the force operate only at the commencement of one of these divisions; if then the number of the divisions observed $= n$, and the velocity at the end of the first division, $\frac{t}{n}$ $=$ W, then the space traversed in the different divisions $= \mathrm{W}\frac{t}{n}$, $2\,\mathrm{W}\frac{t}{n}$, $- - - n\,\mathrm{W}\frac{t}{n}$, and $s = \mathrm{W}\frac{t}{n}(1+2+3 - - - + n)$. If n be infinitely great, then $s = \mathrm{W}\frac{tn^2}{2n} = n\,\mathrm{W}\frac{t}{2}$, and as nW must be the final velocity, v, of the motion, $s = \frac{vt}{2} = \frac{\mathrm{G}t^2}{2}$, and $t = \sqrt{\frac{2s}{\mathrm{G}}}$. From these investigations the following propositions respecting uniformly accelerated motion may be developed:—1, the final velocities attained at the expiration of different times are as these times; 2, the space described during uniformly accelerated motion, is half that which would be described if the motion had been equable and of the final velocity; 3, the spaces traversed are as the squares of the times which have expired during the motion; 4, the spaces traversed in successive equal times increase as the odd numbers, or as 1, 3, 5, 7, &c.

The laws of uniformly varying motion may also be presented geometrically. Suppose the body to begin its motion from a state of rest at A (*pl.* 17, *fig.* 22); draw the straight line, AB, marking off upon it the equal parts, Aa, ab, bc, and erecting the ordinates aa', bb', cc', at the points of division. The abscissas, Aa, Ab, Ac, then represent the time elapsed since the beginning of the motion, and the corresponding ordinates, the final velocities. As these are all proportional to the aforesaid time, it follows that the line, AC, joining the ends of the ordinates, must be a straight line. Assuming the distances Aa, ab, bc, &c., as infinitely small, and drawing to AB the parallels $a'b''$, $b'c''$, $c'd''$, &c., small right-angled triangles result, whose sides, $b'b''$, $c'c''$, give the successive increase of velocity. The surface of the corresponding trapezoid has always an equal numerical value with the length of the path described by the accelerated motion; consequently the sum of all the trapezoids plus the small triangle, Aaa', or the surface, Ahh', represents the entire space traversed from the beginning. This triangle, however, is half the size of the rectangle which serves as the measure of the space traversed in equable motion, hence follows the proposition (No. 2) adduced above.

The laws of the *unequably accelerated* motion of bodies present many difficulties in their development. Suppose, in the first place, that it be desired, from the observed unequably traversed spaces and the corresponding times, to determine the velocity at the different points of the path described. To this end let AB (*pl.* 17, *fig.* 23) represent the axis of abscissas, AC the axis of ordinates of a system of rectangular co-ordinates, and A the starting point of

motion; the times may be taken as abscissas, the spaces traversed as ordinates. Assuming the parts Aa, ab, and c, of the axis of abscissas as infinitely small, then the line Ad connecting the extremities of the ordinates, a', b', cannot be a straight line, but must be curved; the small triangles, $a'b'b''$, $b'c'c''$, must also be unequal; consequently, the velocity of motion must change at every moment. Suppose, furthermore, that at any instant of motion, corresponding to the point c', this motion suddenly becomes equable, then this new motion will be represented by a straight line, c'E, the prolongation of the chord of $c'd'$. As, moreover, the moving point in the instant when the motion is considered, during the elementary time $c'd''$ or cd, would have described the space $d'd''$, it will by reason of the ensuing equable motion describe a space in the unit of time, determined by obtaining the ordinate mn for $c'm$ and $c'n$; the space mn then serves as the measure of velocity for uniform motion, and is thus the final velocity desired.

c. *Freely Falling Bodies and Projectiles.*

The theory of freely falling bodies is a consequence of the preceding propositions respecting uniformly accelerated motion. The force of gravity which here comes into account, must, if the motion be uniformly accelerated, be a constant force. It is known, indeed, that the intensity of this force diminishes as the square of the distance from the centre of the earth; as, however, the greatest space which can be traversed by a body is extremely minute, compared with the earth's radius, it will involve no serious error to consider the action of gravitation within these limits as a constant force. The weight of the body is not taken into account in determining the laws of free falling, as gravitation acts uniformly upon all the atoms of a body, and although practically, weight does seem to be of account, the reason of this lies in the resistance of the atmosphere: all bodies fall with equal velocity in a vacuum.

In the free falling of bodies, the two propositions may be brought into application—that the velocities of a freely falling body are constantly proportional to the time expired, and that the spaces are as the squares of the times. It becomes necessary to determine the acceleration produced by gravitation, that is, the value of the space fallen through at the end of the first second, which can only be done by direct experiment. From carefully conducted experiments, it has been found that at a mean geographical latitude, and a height not too great above the level of the sea, the acceleration amounts to 9.81 metres (31 feet, 11 inches, 11 lines, English; 30′ 2″ 7‴, French; 31′ 3″ 2‴, Rhenish). Calling this acceleration g, the body in the first second traverses $1\frac{g}{2}$; in the second, $2^2\frac{g}{2}$; in the third, $3^2\frac{g}{2}$; and the entire space, s, fallen through in t seconds is $t^2\frac{g}{2}$.

Atwood's machine is best adapted to demonstrate the correctness of results obtained by these investigations. The entire instrument is figured in *pl.* 16, *fig.* 17; *fig.* 18 represents its upper portion on a larger scale.

The machine consists of a post, F, about 7 feet high, with its base, S, capable of being rendered perfectly vertical by the four adjusting screws; on its upper end there is a frame, T, carrying the proper apparatus. This apparatus consists of a wheel, K, united to the axis by the spokes, *a, b, c, d*, and over which runs a string to which hang the weights, A and B. Each end of the axis rests in the angle of two overlapping friction wheels, L, M, and N, O, so that the friction wheels taking part in the motion of the main axis, reduce the friction to its minimum. A divided scale, G, is fastened to the foot by the clamp, R, and upon this scale the two shifting platforms, H and C, may be fastened at pleasure by screws. The clock, D, attached to the post, F, indicates seconds, thus serving as a measure of the times of falling.

As the weights, A and B, are perfectly equal, they will be in equilibrium when attached to the two ends of the string passing over the wheel, K. This equilibrium will, however, be disturbed when an extra weight, n, is laid upon one of them, the heavier weight falling, and the lighter rising with accelerated velocity. As the motion of the two weights is entirely the result of the extra weight laid upon the one, it takes place slower than in a freely falling body, and this retardation of velocity is in the same proportion which the extra weight, or the difference of the two weights, bears to their sum; it takes place, however, as to the rest, according to the laws of freely falling bodies. Thus, if m indicate each one of the originally equal weights, and n the superimposed extra weight, then the velocity and the space fallen through for any given interval of time, is only $\frac{n}{2m+n}$ of the velocity and the interval of time, which takes place in the same time in a free fall. If, for example, $m = 7$oz., and $n = 1$oz., then the space traversed in the first second is only 1 foot, that in the second, 2 feet, in the third, 3, &c.; and by diminishing n in proportion to m, the motion may be rendered as slow as may be desired. To measure the space fallen through, the scale, G, is divided off into fractions of inches; the two platforms may be attached to any part of the scale, and of these the upper has a hole large enough to allow the passage of one weight after the removal of the small bar, I. If the extra weight, n, be so adjusted that $\frac{n}{2m+n} = \frac{1}{180}$, or more precisely, that the space fallen through in one second shall be equal to one inch; if furthermore it be so arranged as to pass through the upper platform, and if the lower one be placed successively at a distance of 1, 4, 9, 16, 25, 36, 49, 64 inches below the 0 of the scale, then the weight will be heard to strike this lower platform after successive intervals of 2, 3, 4, 5, 6, 7, 8 seconds, agreeably to the theory. If again the extra weight be so adjusted as not to pass through the upper shifter, then the descending weight, from the moment of separation from the extra weight, will continue its motion with an equable velocity. Furthermore, as in this case the accelerating force, namely, the extra weight, n, ceases to act, it will be found by placing the upper platform at a distance of 1, 4, 9, 16 inches beneath the zero point, and

adjusting properly the lower one, that the velocity attained amounts to 2, 4, 6, 8 inches in a second; being thus uniform.

The laws already developed serve for the vertical motion of a body; new ones must be obtained when the motion takes place in vacuo, in a direction forming any angle with the horizon. Starting then from the point of view, that all material points of the same body receive an equal progressive motion, it will be possible to restrict our attention to the laws of a single point of a body.

Suppose (*pl.* 17, *fig.* 24) A to be the starting point, and AC the direction in which the body is thrown, this would move with equable velocity in the direction AC, if unacted on by gravitation. This, however, incessantly solicits it in a vertical direction, downwards, so that after one second it would be about 16 feet; after two seconds, 4.16, or 64 feet; after three seconds, 9.16, or 144 feet lower down than if this gravitation did not act.

Calling the initial velocity a, and the angle, CAB, which the original direction forms with the horizon, α, then the projected body under the simple influence of the initial force, would in t seconds traverse the path, $t \,.\, a$. and have reached the height $t \,.\, a \,.\, \sin.\, \alpha$. The force of gravity diminishes this height by gt^2, and the formula becomes $t \,.\, a \,.\, \sin.\, \alpha - gt^2$. It is evident that after a time the ascent of the body will change into a descent, and will finally return to the same horizontal plane from which it started. This takes place when $t \,.\, a \,.\, \sin.\, \alpha - gt^2 = 0$, or $gt^2 = t \,.\, a \,.\, \sin.\, \alpha$, or after $t = \frac{a \sin.\, \alpha}{g}$ seconds. In the middle of this interval of time, or after $\frac{a \sin.\, \alpha}{2g}$ seconds, the body will have reached the highest point of its path, whose height amounts to $\frac{a^2 \sin.\, 2\alpha}{4g}$. The line of projection is therefore a pure parabola. The rectilineal distance of the point where the body again reaches the horizontal plane, from the point where it started, or the distance of projection, is $= \frac{a^2 \sin.\, 2\alpha}{2g}$; it is greatest when $2\alpha = 90°$, or $\alpha = 45°$; that is, when the body is projected at half of a right angle to the horizon.

The theory of projectiles comes most into play in artillery, where it is desirable to determine, not only the path of the projectile in the air, but also the variation of range of the guns with the variation of the angle of elevation. It does not come within the province of this work to adduce to any extent the comprehensive calculations and investigations necessary to determine these paths; a few examples only are given of the modes of ascertaining the lengths and greatest ordinates of the parabola in different cases. Thus, *pl.* 17, *fig.* 25, shows how the parabola is determined when the axis of abscissas of the projectile line, AE, is horizontal, and the direction of discharge deviates from the perpendicular, AB, where then the greatest ordinate passes through the vertex, D, of the parabola. In *fig.* 26 the projection takes place from a height to a depth, the gun standing at A; the greatest ordinate is EB; the line of abscissas, AB, being no longer horizontal,

and there being no angle of elevation, the descending branch of the parabola alone presents itself. *Fig.* 27 represents, by comparison with the projection in the plane, AF, the case where, to attain a greater range, AE, a projection to a lower level takes place with an angle of elevation, BAD; *fig.* 28 shows, in its left hand side, the diminution of range with a greater elevation; the right hand exhibits much the same case as in *fig.* 26.

The preceding remarks are in all strictness to be taken with regard to projection in a vacuum, which, however, never occurs in practice. The resistance of the atmosphere, in which all bodies move which are projected from the earth, changes not only the path but the velocity of projectiles, and is very difficult to calculate accurately; only very dense masses, as balls of lead, iron, &c., approach in their motions to the laws of projection in vacuo, and this indeed in proportion to their size. The range in air is 5 to 10 times less than in vacuo. The greatest range is attained by a much smaller angle than 45° (in cannon even at 20°); the highest point of the path is nearer the end than the beginning; the descending part of the path is therefore much steeper than the ascending.

d. Centrifugal Force.

In the preceding remarks it was assumed that the directions of gravitation, in all points of the path of a projectile, were parallel to each other. This is no longer the case, however, when we come to consider the motion of a body about an attracting point, as, for instance, in the motion of the earth or of any other planet about the sun. In such motion (central motion) two forces are to be imagined as operating: the centripetal force, which incessantly solicits the moving body towards the attracting centre, and the tangential force, which, if the centripetal force were to cease its action, would impel it outwards in a straight line in the direction of the tangent. It depends upon the proportion between these two forces whether the body is to move in an ellipse or in some other curve.

If a ball fastened to the end of a string be whirled around, the string experiences a tension which increases with the velocity of rotation. The cause of this tension is called the centrifugal force. It always acts wherever rotation takes place about an axis, and consequently in the rotation of the earth on its axis; at the equator it is greatest, as here the velocity of rotation is greatest, and opposed to the force of gravitation; at the poles it is zero. In experiments upon the centrifugal force, the apparatus represented in *fig.* 29, *pl.* 17, may be employed, called a *centrifugal machine.* By means of the winch, *d,* the horizontal disk beneath it is rotated, this rotation being communicated by a string, *e,* to a second disk of smaller radius; this latter disk must turn the quicker as its radius is less. With it, and in the continuation of its axis, turns the vertical axis, *c.* If a thin ring of brass be fastened to the lower end of this axis, the upper curve capable of moving freely up and down the axis, this ring, if circular when at rest, will assume an elliptical shape when in motion, and the shape will deviate

more and more from that of the circle, in proportion to the increase of the velocity.

e. Of the Pendulum.

A body which is capable of oscillation about an axis, neither vertical nor passing through the centre of gravity, is called a *pendulum.* Suppose (*pl.* 16, *fig.* 20) a material point, B, to be attached in such a manner to the extremity of a weightless line, AB, that the line can swing freely about the other extremity, A, we shall have a simple or mathematical pendulum; and the combination of a small heavy sphere with a thin thread, to which it is suspended, may be regarded without serious error as a simple pendulum. If such a simple pendulum be brought from its vertical position, AB, which, from the laws of statics, it must assume, into the position, AB′, and left to itself, it will, by reason of the attraction of gravitation, be brought back towards B, and describe the arc, BB′, lying in the same plane with AB. It will arrive at B with a velocity corresponding to the depth of fall, that is, to the segment of the radius, AB, obtained by letting fall a perpendicular from B′ upon this radius. With the velocity thus attained, the resistance of the atmosphere and of friction being now left out of the question, the material point will endeavor to continue its path in the arc, BB″, on the other side of B, until this velocity previously attained has become zero. This point is evidently at B″ when BB″ = B′B. At B″ the same state of things occurs as at B′, and the pendulum must incessantly perform equal oscillations in the arc, B′BB″. In descending the velocity must constantly increase, and in ascending decrease, being greatest at the point of equilibrium or the lowest point of the arc. The motion of the pendulum from B′ to B″, is called the oscillation; that part of it from B to B′ or B″, is the ascending semi-oscillation; and from B′ or B″ to B, the descending semi-oscillation. The *amplitude* is the arc corresponding to the oscillation expressed in degrees, minutes, and seconds: the time necessary to describe this arc is the *duration* of the oscillation. The fact that in the material pendulum, the duration and amplitude of the oscillation continually decrease, results from the friction at the point of suspension and the resistance of the atmosphere. The pendulum being thus retarded, cannot reach the height, B″, and the altitude attained becomes less and less at each successive oscillation.

The laws of oscillation for the pendulum are as follows:—1. The duration of minute oscillations is independent of their amplitudes; they are isochronous; and a pendulum swings through an arc of 5° in neither greater nor less time than through an arc of 1°. 2. The duration of an oscillation is independent of the material and the weight of the ball, one of lead moving no faster than one of cork. 3. The oscillations of two unequal pendulums are to each other as the square roots of their lengths.

When it is said as above that the weight of the pendulum has no influence upon the duration of oscillations, it is to be understood as applying only to

an individual place: if the pendulum be carried to some other place on the earth's surface, where the intensity of gravitation is different, the duration of its oscillations will be changed.

The preceding laws apply only to the mathematical pendulum, and as these cannot actually exist, our investigations must have reference to the compound pendulum. Suppose in some point of the line AB, a molecule, m, and in B the molecule n, then m, being nearer to the point of suspension, will make shorter vibrations than n, and will consequently accelerate its motion, while n will retard the motion of m; oscillations will therefore result, such as would be produced by a simple pendulum shorter than AB and longer than Am. In every material pendulum, therefore, there must be a point whose motion is neither accelerated nor retarded by the rest of the mass, and which will consequently oscillate in the same manner as a simple pendulum whose length is equal to the distance of this point from the point of suspension. This point is called the *centre of oscillation* of the pendulum, and when mention is made of the length of a pendulum, by it is always to be understood the distance from the point of suspension to the centre of oscillation. In very long pendulums composed of very thin threads and very heavy balls, the centre of oscillation lies at an inappreciable distance below the centre of gravity of the ball attached; this centre of gravity, therefore, may without material error be considered as the centre of oscillation.

From the preceding considerations it follows that from observation of the oscillations of one pendulum, it becomes possible to determine the length of another which shall vibrate exact seconds. Borda used a pendulum which was exactly twelve Paris feet in length, and made 1876 oscillations in an hour. Now, as a seconds' pendulum must make 3600 oscillations in the same time, and the lengths of the pendulums must be as the squares of the times of oscillation, it follows that $3600^2 : 1876^2 :: 144 : x$; therefore $x = \frac{144.1876^2}{3600^2}$ $= 39.14$ Paris inches; more accurately, in English inches, 39.12851. The length of the pendulum vibrating seconds at New York is 39.10153 inches.

If a pendulum could be so constructed as to accomplish its oscillations in the arc of a cycloid instead of a circle, the length of the pendulum being equal to the diameter of the generating circle, all its oscillations would be perfectly isochronous; the cycloid possessing the property that great and small arcs are traversed in equal times. Huyghens, who probably first applied the pendulum to the clock, endeavored to make the pendulum vibrate between cycloidal plates or cheeks, so that the thread or spring supplying the place of the rod of the pendulum, would be obliged to bend along these cheeks; the ball moving, therefore, in a cycloidal curve, and describing isochronous oscillations. Nevertheless, the arrangement of these cycloidal plates is attended with great difficulties, and for this reason it is generally the custom to employ circular pendulums of small amplitude, which have the same advantages as the cycloidal, and are of much more easy construction.

Circular or *centrifugal* pendulums are those in which the oscillations, instead of being performed backwards and forwards in the same vertical

arc, take place in a horizontal circle, and always in the same direction. To this end, however, the pendulum rod must be capable of moving about the point of suspension, not in a single plane only, but in any direction at pleasure.

In the material pendulum there is still a circumstance which affects the oscillations, namely, the influence of temperature, which, when elevated, lengthens the pendulum, and when lowered, shortens it. This circumstance is especially injurious not so much in particular experiments, where the length may be regulated each time, as in the application of the pendulum to clocks, where the slightest variation in its length must affect the rate. In this latter case the pendulum, to be an accurate regulator of motion, must first regulate itself; and to this end, many combinations have been devised, of which *Harrison's compensation or gridiron pendulum,* and *Graham's mercurial pendulum,* will alone be mentioned here.

The *gridiron pendulum* (*pl.* 16, *fig.* 21) was invented in 1725 by Harrison, for which, in connexion with his chronometer, he received a premium of £25,000 sterling from the British parliament. It consists of five steel and four brass rods, which alternate with each other, so that the central rod to which the disk of the pendulum is attached, is of steel. These brass and steel rods are so fixed in the heads, *aa, bb,* that while the expansion of the steel rods produces a tendency to elongation in the pendulum, that of the brass rods, which press upwards the head to which the pendulum rod is attached, produce a tendency to contraction. If, now, the lengths of these brass and steel rods are to each other in the proper proportion of their coefficients of expansion, or as 61 : 100, the expansion of one set will elevate the pendulum just as much as it is depressed by the other, and the actual length will be invariable.

This pendulum, philosophical and beautiful as it is in theory, is diminished in practical value by the following considerations: 1. That it is difficult to make the rods sufficiently accurate; 2. It is difficult to give them their proper proportional lengths; 3. That it is more exposed to the resistance of the atmosphere. Other metals may be employed instead of steel and brass.

The *mercurial pendulum* (*pl.* 16, *fig.* 22) invented by Graham in 1715, has a brass rod, *aab,* which carries below a cylindrical glass vessel from 13—14 inches long, and two inches in diameter. This vessel, *o,* filled up to 12 inches with mercury, forms the ball of the pendulum, and lest the expansion of the rod should be too great for that of the ball, the quantity of mercury in the latter may be varied. By the influence of temperature, the rod is expanded; the mercury is expanded at the same time, however, and its centre of gravity is elevated: the pendulum is thus shortened again, and by trial a very accurate compensation may be obtained. The single influence operating against this pendulum is that the mercury sometimes begins to expand before the rod; the variation, however, rarely amounts to more than one eighth of what takes place in good common pendulums. The disk, *d,* serves for the general regulation of the pendulum.

After Galileo had developed the laws of the pendulum, Huyghens determined the centre of oscillation of the material pendulum, and thereby made possible an accurate measurement of time, by applying the pendulum to the

regulation of the clock. Newton, however, first announced the proposition, that the same pendulum, in different places on the earth's surface, must make different oscillations. The astronomer, Richer, who journeyed to Cayenne in 1672, verified this observation, as the difference of the rate of a clock at Paris and Cayenne required a shortening of the pendulum by $1\frac{1}{4}$ line. By means of accurate experiments it was afterwards found, that for the different latitudes of St. Thomas (0° 24′ 41″) and Spitzbergen (79° 49 58″), the length of the pendulum varied from 39.021 and 39.215 Paris inches (more accurately in English inches, and reduced to the level of the sea, 39.02074 and 39.21469).

Even if the highest mountains and the deepest seas produce no change in the general form of the earth, by reason of their small size compared with the earth's radius, yet the rotation of the earth on its axis must theoretically cause a heaping up of its mass at the equator, and a flattening at the poles, so that the earth, instead of being a sphere, must be really an oblate spheroid. Measurements of degrees of the meridian have determined the amount of this oblateness. If, for example, Dunkirk and Formentera lie nearly on the same meridian, and their distance from trigonometrical measurement amounts to 1374438.72 metres (the angular distance being 12° 22′ 14″), it becomes easy to determine the length of one degree of the meridian. If now the earth were a sphere, all degrees of the meridian would be equal. Measurements of degrees in different latitudes, however, have shown that this is not the case, but that the length of a degree of the terrestrial meridian continually decreases from the poles to the equator; the radius of the equator accordingly amounts to 6,376,984 metres, and that of the poles, 6,356,324; a difference of 20,660 metres. The mean radius of the earth corresponds to that of latitude 45°, and amounts to 6,366,745 metres. The length of the pendulum is in strict relation to these measurements, for the seconds' pendulum is shorter, the nearer the place of observation to the equator, so that the seconds' pendulum of Paris would make 126 oscillations less in a day, at the equator. Hence it follows that the intensity of gravity diminishes with the distance from the centre of the earth, and experiments with the pendulum, carried on at different heights above the level of the sea, confirm this statement.

Considering that the centrifugal force increases towards the equator, and that nearer the equator the distance from the centre is greater, it becomes possible, knowing the length of a seconds' pendulum at Paris, to determine that for any other place on the earth's surface; here, however, the greater or less density of the earth's crust comes into account; as it is found that there are always slight discrepancies between the calculated and actual pendulum lengths—differences which may sometimes amount to four or five oscillations in a day. To this belongs the deviation experienced by the plummet in the vicinity of mountains. Bouguer was the first who was struck with the idea of finding in mountains a proof of the universal attraction of matter. His investigations in the slopes of Chimborazo, combined with astronomical measurements, showed a deviation of the plummet of seven to eight seconds. Maskelyne found the deviation at the foot of Shehallien in

Scotland (1772), to amount to 54 seconds, and obtained from this the mean density of the earth at 4.45.

f. Of Impact.

In most cases the forces by which a body is moved, act only on a small part of the molecules of which it is composed, and yet all parts of the body move, those struck as well as those not touched. Thus, for example, a billiard ball rolls along, although, strictly speaking, only a small part is struck by the player. The motion must therefore be uniformly distributed to all the molecules; this takes place, however, in an infinitely short time, and the force has then passed on into the body, and distributed itself in it uniformly. The body thus impelled will continue incessantly to move in the direction of the impulse with uniform velocity, unless hindered by friction or the resistance of the atmosphere. The action of the force is therefore momentary; its effect, however, unlimited.

Under such circumstances the body receives the force, and one and the same force acting upon different bodies must produce very different motions; a force which can impel a small body with tolerable swiftness may hardly move a larger. It is usually said that this difference depends upon the weight, but this is not the case; else, if the body ceased to be heavy, the same force would impel all bodies with equal velocity. This, however, does not follow, as even in vacuo the same force must produce a less velocity, as the matter to be moved is greater; and the theory of mechanics teaches us that the same force operating upon different bodies, communicates to them velocities which are inversely as their masses, that is, as the quantity of their matter. Consequently, the same force that would impel a mass with a velocity = 1, would impel one of ten times the greater mass with one tenth of the velocity. Multiplying each of these masses by their velocity, the products will be equal; this product is called the *quantity of motion*, or the *momentum*. Machines cannot increase the quantity of motion, as they do not generate force, but only change the kind of motion. Thus a laborer can, by means of a rope which passes over a fixed pulley, easily raise 25 pounds to a height of 2½ feet in a second; if, however, the rope were laid over a wheel and axle, where the latter should have a four times smaller diameter, the laborer, with the same exertion of strength as before, would easily raise four times the weight, but would require four fold the time.

If a body in motion meet one that is stationary but movable, it imparts to this latter a part of its motion, without thereby changing the quantity of motion; for if the striking body did not rebound in consequence of its elasticity, and if the blow were a central one, both bodies after the blow would move in the same direction, but always in mutual relation to their masses. The velocity after impact can therefore very readily be obtained by dividing the velocity of the moving body by the sum of the masses of the moving and the stationary body. Suppose a ball moving with a velocity of 1400 feet in a

second, and weighing half an ounce, to strike a ball of 40 lbs. weight suspended to a string, then the common velocity after impact would be to 1400 as $\frac{1}{32} : 40 + \frac{1}{32}$; thus $= \frac{1400}{1281} = 1.09$ feet in a second.

Upon this principle depends the measurement of great velocities by means of the *ballistic* pendulum. This pendulum, represented laterally (*pl.* 17, *fig.* 37), and in front (*fig.* 38), consists of an iron-bound wooden block, B, of considerable weight, which, by means of the iron frame, *r*, *m*, *s*, is attached to the axis, C, in such a manner that it can swing about this axis, which is supported at D. Above is attached a graduated arc, *no*, on which an index shows the amplitude of oscillation; beneath is an arched piece containing a groove filled with soft wax, on which the index, *f*, in the motion of the pendulum, makes a scratch, exhibiting graphically the length of oscillations whenever a ball, A, strikes the pendulum in the direction of the centre of gravity. The pendulum is 10—12 feet in length. To determine the velocity of a cannon ball it is fired against the pendulum, and its motion is thus communicated to the latter. Knowing the arc described by the pendulum, as well as the mass of both pendulum and ball, it is a simple problem to ascertain the velocity of the ball.

C. Statics of Fluids.—Hydrostatics.

a. Pressure of Liquids.

As the statics of solid bodies had reference to the laws of their equilibrium, hydrostatics embraces the theory of equilibrium in liquids, and of the pressure which they exert upon the walls of the containing vessel.

In liquid bodies, two forces are to be considered, namely, weight and molecular attraction; and these two forces may be readily imagined to be separated from each other, that is, a liquid may be supposed to exist without weight. Such a liquid left to itself would not fall: it thus needs no support on any side, and might even sustain a pressure and transmit it according to a certain principle. Hence the following axiom: a liquid transmits pressure acting upon any part of its surface, uniformly in every direction. Suppose a vessel to contain such a liquid, with a suitable piston, also without weight, placed upon its surface. The liquid would not flow out, even if the side of the vessel were pierced by an aperture. If, however, a weight be placed upon the piston, it would sink if not supported by the liquid, whose upper layer would likewise sink unless supported by the one beneath it, and so on to the bottom of the vessel. All these layers of liquid, therefore, receiving successively the same pressure, the result is the same as if the piston with its superincumbent weight pressed directly upon the bottom of the vessel. Hence it follows, that the pressure upon horizontal surfaces is transmitted from above to below without any loss, that is, is equal at every point, and proportional to the surface involved.

The same proposition holds good in reference to the walls of the vessel; for, supposing an aperture made in the side of the vessel by cutting out a piece equal in surface to the piston, the same weight as is placed upon the piston would be required upon this piece to prevent the liquid from escaping, and the resistance would be in proportion to the surface of the piece cut off. If the piston itself were pierced, the liquid would escape through it; liquids, therefore, transmit pressure uniformly in all directions. The laws thus developed for weightless liquids apply equally to those with weight, as it is here the single molecules which receive and transmit the pressure.

Another proposition with regard to liquids is the following: when a liquid is in equilibrium, its surface must be perpendicular to the direction of gravitation. When liquids are in equilibrium, they exert upon each other and all solid bodies with which they are in contact, a greater or less pressure: this pressure upon the bottom of a containing vessel being, without any regard to its shape, equal to the weight of a vertical column of the same liquid, which has the bottom of the vessel for its base, and the perpendicular height of the water for its altitude. Haldat's apparatus (*pl.* 18, *fig.* 1) serves as an illustration of this law. It consists of a bent tube fastened in a box and so adjusted as to admit of attachments of various forms (*figs.* 2—4) being screwed on at one end instead of *dh*. Mercury is now poured into the tube, and the height, *n*, noted to which it rises in the arm *c*. The cylindrical vessel, *d*, is screwed on to the left hand and filled to a given height, *h*, with water, and the increased height, *p*, of the mercury observed in the other arm. The rise of the mercury is evidently the result of pressure exerted upon it by the water in *d*. Let off the water by means of the cock, *r*, and exchange the vessel, *d*, successively for *figs.* 2—4, filling them with water to the same height, the mercury will each time rise to the same height, *p*, although the amount of water in the different cases is very unequal.

The pressure experienced by any portion of the side of a vessel is represented by the weight of a column of liquid, whose horizontal base is equal to the area of the portion in question, and whose altitude is the depth of its centre of gravity below the surface of the liquid. *Fig.* 5 illustrates the pressure upon the different points of the vertical side of a vessel. Erect at any point, *a*, a perpendicular to *rs*, and make this equal to *ar*, or the depth of the liquid at this point below the surface, then *ab* represents the pressure experienced by the point, *a*; suppose similar perpendiculars erected all along *rs*, then the entire isosceles right-angled triangle thus produced, will represent the entire pressure exerted upon the side in question. If *o* be the centre of gravity of the triangle, then a line drawn horizontally from *o* will intersect the wall in a point, *c*, called the centre of pressure: its height above the bottom is one-third of the height of the surface of the liquid.

In vessels communicating with one another in any manner, *figs.* 6 and 7, for instance, the surfaces will stand at the same height, if the same liquid be contained in both vessels. Suppose in *fig.* 6 a horizontal partition to be passed through *m*, then, if F represent the area of this partition, and *h* the height *vv*, the pressure on the partition wall from below will be $= Fh$. In the broader vessel, if the height, *am*, at which the water is supposed to stand

be represented by h', the pressure upon F will be represented by Fh'. Suppose the partition wall now replaced by a layer of water, this will experience a pressure from above of Fh', and a pressure from below of Fh; equilibrium can therefore only exist when $h = h'$, or when the level is equally high in both vessels. If the liquid in the different vessels be different, however, the level will be unequal. If, for example, in *fig.* 8, one vessel contain water and the other mercury, they will meet each other in the plane passing through g. Below the plane gh there is only mercury; above it in the one vessel there is water, in the other mercury, the water pressing upon the mercury so as to force it into the smaller vessel in proportion to its height, never, however, attaining to the same level. The heights of the liquids will naturally be inversely as their specific gravities, and as these are as $1:14$, the column of water must be 14 times the height of that of mercury.

b. Law of Archimedes; Specific Gravity.

Under certain circumstances, heavy bodies may move in a direction opposite to that of gravity. Thus wax and wood rise from the bottom to the top of a vessel filled with water; a piece of brass rises in mercury, &c. All these phenomena depend upon that important law first discovered by Archimedes, and named after him. A body immersed in a fluid loses in weight by an amount equal to the weight of the fluid displaced. This may be explained by means of *fig.* 9, *pl.* 18, where a combination of several vertical prisms is immersed in a fluid. The proposition is readily proved for a single right prism; as in this case the pressures on the different sides of the prism mutually balance each other, it is only necessary to consider that upon the top and bottom. The upper surface experiences a downward pressure equal to that of a column of fluid whose base is this upper surface, and whose altitude is the height of the fluid above the surface of the prism. The lower surface, on the other hand, is pressed upwards by a force equal to the column of fluid whose base is the lower base of the prism, and whose height is that of the fluid above this base, equal, therefore, to the height of the fluid above the prism, plus the height of the prism itself. The heights of these two columns differ, therefore, by the height of the prism, and it is therefore evident that the pressure from below, or the upward pressure, exceeds the pressure from above or the downward pressure, by the weight of a column of fluid equal in volume to the prism immersed. This excess of upward pressure acting contrary to the weight of the body, or to its gravitation, necessarily relieves the latter of an amount of weight equal to that of the fluid displaced. All bodies, of whatever irregularity of shape, may be considered as composed of right prisms, to each of which, and consequently to whose sum, the above reasoning will apply. A convincing proof of the accuracy of this law, which applies to both liquids and gases, may be had by means of the apparatus figured in *fig.* 10. At one end of a common balance is suspended a hollow cube of metal, beneath

which is attached a solid cube, fitting exactly in the first one. Place the one in the other, and bring the balance to a state of equilibrium by loading the opposite scale with weights; suspend the solid cube beneath the hollow one, and allow the former to be immersed in the water, equilibrium will be disturbed, and the weight scale will sink; fill the hollow cube with water, and equilibrium will again be restored.

A perfectly homogeneous body floats in a fluid when its weight is equal to that of the fluid displaced, and it may then assume any position; if, however, its centre of gravity do not coincide with that of the fluid displaced, it only floats when the two centres lie in one and the same vertical line; the position, however, is fixed, only when the centre of gravity of the body is the lower of the two. Thus fishes float in water when they weigh as much as the water displaced; the equilibrium of their position, or the inferior situation of their belly, depends upon the air-bladder, and is so placed that the upper part of the fish is lighter than the lower. By means of the air-bladder, the fish can rise or sink in the water, floating at pleasure at any height, by its simple compression or expansion. As the fish cannot inspire air at pleasure, like an air-breathing animal, the bladder must contain a certain quantity of gas (consisting in most fishes of $\frac{9}{10}$ oxygen and $\frac{1}{10}$ nitrogen), which is compressed more by the muscles than by the surrounding fluid. This muscular compression is, of course, voluntary on the part of the fish, and the compression or expansion of the bladder stands in intimate connexion with it. The apparatus (*pl.* 18, *fig.* 11) known as the *Cartesian Devils,* illustrates this condition of things. The devil is a hollow glass figure, *b*, in which there is a very small opening, generally in the point of the tail. The figure is filled with water just enough to make it float in a vessel filled with water. Cover the vessel with a bladder, and place it inverted upon the stand, in which is placed a strong spring, *e*; then by the pressure of the spring, the air in the vessel is compressed, and the water driven into the inside of the figure, compressing the air already contained therein. The weight thus increased, the figure necessarily sinks to the bottom. Relax the pressure of the spring, and the air in the figure expanding again, forces out part of the water, thus allowing it to rise. Here the figure represents the bladder of the fish, and the pressure of the spring the muscular contractions exerted upon that organ. The gas in the bladders of fish, taken at a depth of about 3000 feet below the surface, sustains a pressure of almost 100 atmospheres. The expansion, when the fish rises to the top, is so great as sometimes to force the viscera out at the mouth.

The determination of the *specific gravity* of bodies is a very important application of the law of Archimedes. Various forms of apparatus have been devised for this purpose; a few only can here be mentioned.

The *hydrostatic balance* (*pl.* 18, *fig.* 12) used for this purpose, is a very accurate balance, such as is employed in chemical manipulation, and as will be described more fully under the head of chemistry. Any chemical balance may be employed for this purpose, by removing one scale-pan and substituting another, which, although of the same weight, is hung much

shorter, and provided with a little hook beneath, from which the body whose specific gravity is to be ascertained, may be suspended. The absolute weight of the body thus suspended, is first to be ascertained by weighing it in the air, the weight being placed in the opposite scale. Place a vessel, D, filled with distilled water under C, and allow the body to be completely immersed in it, taking care to remove all air bubbles from its surface, its weight will of course be diminished, and to restore equilibrium, weights must be placed in C, or removed from D. The amount of these weights indicates the loss experienced by the body in its immersion, and consequently the weight of a mass of water equal in volume to that of the body itself. The specific gravity of the body is the quotient arising from dividing the absolute weight by the weight of an equal volume of water, or the loss of weight experienced when immersed in the water.

A very well adapted and useful hydrostatic balance is represented in *pl.* 18, *fig.* 13, giving a front view, and *fig.* 14, one from the side. To the main pillar, A, an arm is attached above, containing two pulleys, over which strings pass supporting a small beam to which the balance is suspended. The strings are united together into one behind the pulleys, and by means of the screw arrangement, C, may be drawn up or let down, the whole play amounting to 1—2 inches. The shears of the balance beam are pierced above, for the purpose of showing the point of the tongue, and thus determining whether equilibrium be attained or not. To the balance beam, B, are suspended the two scale-pans with small hooks beneath. DD′ is a thin plate attached to a special support beneath the scale-pans, admitting of being raised or depressed at pleasure. This plate, DD′, is pierced to allow passage to the brass wires attached to the hooks beneath the scale-pans. To the wire at D is attached a thin brass cylinder, pierced below, to allow anything to be suspended from it. This cylinder, about five inches long, is covered with paper, upon which an equally divided scale is drawn. In one corner of the plate, DD′, a wire passes with considerable friction through an aperture; to its lower end the index, F, is attached, which, by the friction of the wire in the hole, can be placed at any desired position with reference to the scale. At the lower end of the scale cylinder is attached a weight, G, and to this, by means of a fine wire, the brass ball, P, of about $\frac{1}{4}$-inch in diameter. To D′ is suspended, by a horse-hair, the large hollow glass bulb, P′.

Suppose the weight, G, to be removed, and the wire with P attached directly to the cylinder; suppose P′ also to be replaced with a weight, *z*, heavy enough to produce an equilibrium with the other scale and its appendages, when the middle of the wire, with P attached, is intersected by the surface of the water. The wire to which P is attached must weigh exactly four grains to the inch. As brass is about eight times as heavy as water, the wire will lose half a grain for every inch immersed in the water. If, then, everything be in equilibrium when the centre of this wire lies on the surface of the water, and if the index, F, lie against the middle of the scale cylinder, divided into 100 equal parts, the weight of a body can be ascertained accurately to within $\frac{1}{100}$th of a grain. Thus, lay the body to be

weighed upon the scale at D, and restore equilibrium, so that the difference shall be less than one grain. If the entire balance be raised or depressed, by means of the apparatus, C, until equilibrium is perfect, and if the index, F, point exactly to the middle of the scale cylinder, then the weights laid in D′ exactly represent that of the body in question. If, however, the index, F, point above or below the middle of the scale, as, for instance, to 36, then $\frac{36}{100}$ths of a grain are to be added to or subtracted from the weight already ascertained, as the case may be, to determine the absolute weight of the body in D. To determine the specific gravity, again attach the bulb or cup, P′, restore equilibrium, and then place the body to be examined in P′. The equilibrium again restored by weights placed in D′, and the indications of the index, F, will give the weight of the water displaced.

The specific gravity of solids may also be determined by means of Nicholson's areometer (*pl.* 18, *fig.* 15), which, by an error of the engraver, is represented inverted, and consequently requires an inversion of the plate to bring it right again. A small heavy mass, as a glass ball, filled with mercury, is suspended to a hollow glass body, V, whose upper part on immersion must project above the surface of the liquid. To the upper part is attached a fine rod, *f*, which carries a small pan, *c*. Lay upon this the body to be examined, and cause it, by means of additional weights, if necessary, to sink to a point, *f*. Remove the body from the pan, and substitute as many weights as will bring the point *p* of the areometer back again to the surface of the water: these additional weights give the absolute weight of the body, equal, we will suppose, to *n*. Remove the weights, *n* (not, however, those previously imposed), and place the body in a little basket between V and I. The instrument will not sink to *f*, this requiring the addition of weights in the upper pan. The amount of these latter weights $= m$, will give the weight of the liquid displaced, and the specific gravity $= \frac{n}{m}$.

To determine the specific gravity of fluids, a *scale areometer* (*pl.* 18, *fig.* 16), may be employed. This consists of a cylindrical glass tube, in the lower part of which a ball, *b*, is blown, which is continued into a smaller tube, terminating finally in another ball, *c*. This latter ball is filled with shot or mercury sufficiently to cause the instrument to sink vertically in distilled water to a certain point, the zero. In any other liquid the instrument will sink until its weight is equal to that of the liquid displaced; deeper, therefore, as the liquid is lighter: so that the specific gravity of the liquid can be ascertained by the depth of depression. For this purpose, the areometer of Gay Lussac has the point, *a*, at which it stands in water, indicated by 100, and upon the tube above and below this point, a divided scale attached, so that the volume of the tube included between any two divisions of the scale is $\frac{1}{100}$th of the volume sinking in the water, the numeration being carried from below upwards. An areometer divided in this manner is called a *volumeter*. The specific gravity of a liquid is ascertained by introducing the instrument and dividing 100 by the number on the scale to which it sinks. A volumeter of this character is the more sensitive as the distance

between the divisions is greater in proportion to the thickness of the tube: to avoid making them of inconvenient length, they are not made to be of universal application, but for particular liquids, or for liquids that are lighter or heavier than water. The zero of volumeters intended for liquids lighter than water is placed at the lower end of the scale, that for those heavier than water at the upper part; and the filling of the ball, *c*, is to be adjusted so that the tube *a* may sink to the proper point. The scale, which for every good instrument must be made especially, is generally on a slip of paper placed inside of the tube, which is then hermetically sealed above it. There are other areometers, which, more conveniently, give the specific gravity directly: in these the scale is not equally graduated, but the divisions increase from below upwards. For practical purposes, such areometers are much used for particular liquids, as alcohol, solutions of salt, milk, &c., giving the proportions in which they are mixed with other substances. They receive particular names, according to the fluid for which they are destined: Alcoholmeter, Saccharometer, Lactometer, Hydrometer, Salometer, &c.

c. *Attraction between Solids and Liquids.*

If the extremity of a fine tube be immersed in a liquid, the level of the latter will be higher or lower inside the tube than outside of it, according as the tube is moistened by the liquid or not; thus, in a glass tube immersed in water, it will be higher (*pl.* 18, *fig.* 17), and immersed in mercury it will be lower (*fig.* 18). The force which causes these phenomena of elevation or depression is called *capillarity*, or *capillary attraction*, and comes into play whenever solids and fluids are brought into contact. In such cases, the heights of elevation or depression of the liquid are inversely as the diameters of the tubes; the finer these are, therefore, the higher is the rise or fall of the liquid. For the empirical determination of this law, a very accurate direct measurement of the place of the liquid in the tube becomes necessary; and for this, the apparatus invented by Gay Lussac answers very well. In this, apparatus (*fig.* 19), the height of the liquid in the tube can be ascertained by means of a small telescope, *g*, moved up and down a graduated post, and capable of being fixed at any elevation. Having fixed the post of the telescope in a vertical position by means of the adjusting screws and the plummet *f*, the height of the liquid in the tube is to be noted, the tube then moved aside, and the plate *h*, through which passes with some friction a finely-pointed rod, *k*, laid upon the vessel *a*. The point of this rod is to be brought in exact contact with the surface of the liquid, and the height read off by means of the telescope. The difference of these heights will be the height of the column of liquid in the interior of the tube.

It must not be forgotten that whenever a liquid rises or falls in a narrow tube, the summit of the column is not perfectly flat, but concave in the first case, as in *fig.* 20, and convex in the second (*fig.* 21), the radius of

convexity and concavity being equal to the inner diameter of the tube The regularity of this structure, however, depends entirely upon the cleanness of the inside of the tube.

If a capillary tube which has been employed in any of the abovementioned experiments be raised out of the liquid, the liquid originally contained therein will be retained there by the pressure of the atmosphere, and a drop which may have been suspended to the lower end will even be driven inside; and with sufficiently thin walls, the height of the column of liquid may thereby be raised to nearly double the original amount. Syphon tubes exhibit similar phenomena; and in concentric tubes the phenomena of capillarity take place in the inner tube and the ring between the two, as if each one alone were present. If, therefore, the diameter of the tube be twice as great as the thickness of the tube, the summits of the columns will be equally high in both. Parallel plane plates may be considered as parts of infinitely great concentric tubes, and experiment has shown that the phenomena of capillarity are precisely the same in the two cases. If the plates are inclined at a very acute angle, as in *pl.* 18, *fig.* 22, ADBE and CDBF, the liquid in the narrow part will rise higher than in the wider, and in such proportion, that the areas of the rectangular transverse sections, as *ab* and *cd*, are always equivalent. The shape of the curve, DE, forming the outline of the fluid, is that of an equilateral hyperbola, whose asymptotes, on the one hand, represent the line of intersection of the plates, and on the other, the level of the liquid. If the plates be removed from a vertical position to a horizontal, and a drop of water be interposed, it assumes a circular form, and passes to the line of intersection of the plates, and this with a rapidity greater in proportion to the sine of the included angle. Similar phenomena are exhibited by conical tubes. The small column of liquid, *mm'*, moves towards the point of the tube, as in *fig.* 23, and towards the broad end, as in *fig.* 24, and in the two cases assumes either a convex or a concave outline.

As a general rule, solid bodies cannot come in contact with fluid without the surface of the latter experiencing a greater or less change. Particularly remarkable in this respect are the phenomena of attraction and repulsion presented by bodies swimming in liquid. Two balls swimming in liquid and moistened by it, as balls of cork in water, when within sufficient proximity, attract each other with considerable intensity (*fig.* 25); likewise, two balls not moistened, as of wax (*fig.* 26). On the other hand, two balls repel each other when one is moistened and the other not (*fig.* 27). Similar phenomena are presented by vertical plates (*figs.* 28 to 30).

Another of the phenomena of attraction is the adhesion of plates to the surface of water, so that when they lie horizontally upon this surface, they can only be raised by the exertion of a greater or less force. The amount of this force is dependent upon the density of the fluid, increasing with this density. The material of the plate produces no difference in the result.

We cannot here go into an elucidation of the theory of capillarity, but will only remark that, according to the most recent theory of Mile, capil-

larity is nothing else than a mechanical molecular activity, which produces the drop and the bubble—the negative drop—and which is modified by the influence of the narrow space and of the adhesion.

d. *Endosmosis*

It is well known that a concentrated aqueous solution of any substance may be diluted with perfect uniformity throughout; if, however, there be no immediate contact between the water and the solution, but the two be separated by a porous partition with very fine pores, the liquid must pass through these pores to become mixed together. It may very often happen, however, that this partition admits of a more ready passage to one liquid than to the other, and the levels of the two, in their respective compartments, will then be different. Filling, for instance, a glass cylinder closed at the bottom by a bladder, with a concentrated solution of blue vitriol (sulphate of copper), and placing this in a vessel of water, the water will pass through to mix with the solution; the elevation of the liquid in the inner cylinder consequently rises, that in the outer vessel falling. If the inner cylinder be the one filled with water, the reverse will be the case, a depression here ensuing instead of an elevation. These phenomena investigated by Dutrochet, and by him named *endosmosis* and *exosmosis*, are exhibited sensibly in the apparatus figured in *pl.* 18, *fig.* 31, and by its inventor, Dutrochet, called *endosmometer*. The glass vessel, *b* is closed inferiorly by a piece of membrane or bladder, *cd*, and filled to a certain height with alcohol, the upper end stopped by a cork in which a glass tube, *a*, is fixed air-tight. This apparatus is placed in a larger vessel filled with water, and likewise closed by a cork, through which passes the tube, *a*. If the surface of water in the latter stand, say at *n*, equilibrium soon takes place, the surface of the alcohol standing perhaps at *n'*. Endosmosis now commences, the water penetrates the bladder against the resistance of the alcohol, and the alcohol column rises above *n'*, finally running out of the open end of the tube. If the experiment be reversed, so that the water shall occupy the place of the alcohol in the smaller vessel, the level will fall in the latter, owing to an ensuing exosmosis. Both operations continue until the liquids on each side of the membrane are homogeneous, and the difference of level is simply the result of the pores of the membrane being too minute to permit the action of hydrostatic pressure: for, if this membrane be moistened even on the side opposite to the liquid, no drops are found. Endosmosis and exosmosis play a great part in the organic world, since absorption and the distribution of the nutritious juices are almost entirely results of these operations.

D. Dynamics of Liquids; Hydrodynamics; Hydraulics.

a. Velocity of Efflux.

Hydrodynamics exhibits the laws of motion of liquid bodies; and at the head of this part of natural philosophy stands the law of Torricelli, that when an aperture is made in the side or bottom of a vessel filled with liquid, this liquid escapes with a velocity equal to that which would be attained by a body falling freely from the surface of the liquid to the orifice of discharge. According to this, the velocity of efflux is entirely independent of the nature and specific gravity of the liquid; it is in connexion, however, with the depth of the orifice below the surface, and is as the square root of the height of pressure. A convenient form of apparatus for experiments upon the efflux of liquids is represented in *pl.* 17, *figs.* 32, 33. The main part consists of a cylindrical tin vessel, communicating with a glass tube, in which the liquid stands at the same height as in the vessel itself; this height is measured by a scale attached to the tube. In the side of the vessel are two apertures, *b* and *c*, one above the other; there is a third opening in the bottom of the vessel, on which account the small table supporting it must have a hole pierced through it; a fourth orifice is to be found at *a*, in a short horizontal tube. This latter part is represented on a larger scale in *fig.* 33. Through the wall of the vessel, *aa*, passes a tube, *d*, which ends in a shoulder. In this tube is a second smaller one, capable of rotation about its axis, within the first. In the side of this smaller tube is a thin plate of brass, with the efflux aperture screwed in it, and by turning the tube this aperture may be directed vertically up or down, sideways or obliquely. By means of the valve, *c*, the access of water to the aperture, *b*, can be regulated at pleasure, the other apertures having also valves raised by strings when the water is to flow out through them.

To prove the Torricellian law by experiment, suppose the water to pour out of the point *a*, in *fig.* 32, with the same velocity as if it had fallen from the surface of the water to the depth *a*, then the stream of water must again attain the same height. This, however, is by no means the case, as the water falling from the highest point of the column retards the ascent of that following after it, as is shown by the fact that the stream ascends considerably higher when its direction is so inclined as to prevent this interference. Under favorable circumstances an altitude can be obtained equal to nine tenths of the depth of fall; the remaining tenth is accounted for by the resistance of the atmosphere and the friction of the sides of the tube. Allow the water to pass out from *b* or *c* (*fig.* 32), and the stream will be as represented in *fig.* 31: it will form a parabola whose shape depends upon the velocity of efflux. The theoretical parabola will, however, differ from the actual, in the ordinate being less than that of calculation, the reason lying in the retardations of atmospheric pressure and of friction.

The stream of water, immediately after leaving the orifice, contracts to two thirds of its diameter, this contraction continuing, although in an insensible

degree. In streams directed upwards, the jet expands continually after it has reached its greatest contraction of two thirds of its diameter, at a distance from the orifice equal to its diameter. The stream retains its constant form during only a certain part of its length; then it is separated into greater or smaller currents, which assume very various forms according to the shape of the orifice of efflux.

Should the efflux take place not through a thin plate but through a tube, considerable changes take place if the tube have not the shape of the compressed stream of water. Cylindrical escape pipes do not produce any difference under great pressure; at a less pressure, however, they increase the discharge, this taking place to a still greater extent in conical pipes: in all these cases, however, the velocity of efflux is diminished.

b. Lateral Pressure.

Pl. 17, *fig.* 30, illustrates the laws of the lateral pressure of moving liquids. If water flow from a vessel, A, through tubes, their sides will experience no pressure if there is no friction to overcome, but by this a considerable part of the hydrostatic pressure is lost, and acts upon the walls of the tube. The narrower the tube, the greater is the friction, and so much less the velocity of efflux. The pressure which the walls of a tube, *cf*, have to experience, will be less the nearer to the aperture of efflux, *f;* making then an aperture at *c*, and erecting in it a vertical tube, the water will ascend to a height, *cb*, corresponding to the pressure on the walls of the tube at this point. Midway between *c* and *f*, at *e*, the pressure on the walls is only half as great; the water would therefore rise only half as high as at *c*, namely, to *d;* and placing in any other part, between *c* and *f*, a vertical tube, the level of the water would lie in the straight line, *bf*.

To measure the pressure of falling water, the apparatus represented in *pl.* 18, *fig.* 72, may be employed. Upon the foot, B, stands a cylinder in which the post A may be fixed at different heights. DF is a balance beam, whose horizontal position may be determined by the index on the graduated arc C. At E hangs a common scale-pan, and at F is a plate whose size equals that of the efflux orifice of the vessel G. Letting a stream of water fall upon F, it will press downwards upon this plate, and the horizontal position of the beam is to be restored by weights placed in E. These weights will represent the pressure of the water.

c. Reaction and Impact of Water.

If a vessel be filled with water, without an aperture in any part of it, everything will be in equilibrium; if, however, in any part of the vessel an opening be made and efflux allowed, the pressure ceases at this point, and is consequently less than on the part of the vessel diametrically opposite: the vessel, then, if allowed, would move in a direction diametrically opposite to

that of efflux. Upon this principle depends the efficacy of Segner's water wheel. This consists of a vessel capable of turning about a vertical axis, at whose foot is a horizontal tube, bent in opposite directions at the two extremities, and in the same horizontal plane. The water escaping through these extremities produces a rapid rotation by the reaction of pressure on the sides of the tube opposite the opening; provided, however, that the pressure be sufficient to overcome the friction.

If a stream of water be directed against a movable body, it will cause a change in its position; and the force with which this is done will be in proportion to the amount of pressure. If, during the unit of time, as one second, a stream of water, whose height is M, fall from a height, h, Mh will be the momentum of this column of water; and the force obtained by the impact of the water may be easily calculated.

The most important application of the impact of water is to be found in water wheels used for the propulsion of machinery. The most usual water wheels are vertical, with a horizontal axis. They are divided, according to the point of application of the force, into *overshot*, in which the water falls into the buckets of the wheels, from above and beyond the highest point; *undershot*, in which the water strikes against the lower float boards; and *middleshot*, or a medium between the other two. In the ordinary water wheels a good deal of power is lost; Poncelet has therefore constructed wheels with curved floats, which are much more powerful. Most powerful of all, however, are the so-called *top-wheels*, or *turbines*, invented by Fourneyron. In these the wheel is horizontal and the floats vertical; the water is carried through peculiarly constructed conducting curves against the floats, and turns the wheel around like a top, with such force indeed that 75—80 per cent. of the force of water employed is effective. In the division of the work specially devoted to Technology and Machinery, reference will again be made to the technical application of water power; where also the construction of the water-column machine will be explained—a machine in which the pressure of the water acts upon the piston of a pump, producing a backward and forward motion, which can be transmitted by proper connexions to other machinery. We may mention, in conclusion, another hydraulic machine, which can be employed to great advantage in many cases: this is the *hydraulic ram*, invented by Montgolfier in 1797, and employed in raising water. In *pl.* 17, *fig.* 36, *mm* is a horizontal tube, in which the water flowing from a reservoir moves with a velocity dependent upon the height of pressure. At k is a valve closed by the velocity of the escaping water; by it the aperture at this place may be closed. The water now pressing through the tube i into the cast iron reservoir d, enters, after raising another valve, into a great cast iron receiver (the air-vessel), and in this manner reaches the ascent tube, *ca*. Into this it is driven with a much greater force than would be produced by the height of pressure alone, as by the closing of the first valve, which suddenly obstructs the motion of the water escaping there, a pressure is produced upon the sides of the tube. In the ascending tube, the water rises to the height allowed by the elasticity of the air in the air-vessel, and the pressure of the water already raised; then the

valve leading to the air-vessel again closes; the conical valve first mentioned falls by its own weight; the water commences again to escape through it, and the play of the valves, or the butting of the ram, begins afresh.

E. Statics of Aeriform Bodies, or Gases.—Aerostatics.

Gaseous or aeriform bodies, among which the atmospheric air occupies the most important place, in some respects form a great contrast to the true liquids. At an earlier period, atmospheric air was considered as a simple body —an element; at the present day, however, its component parts are well known, and its place among compound bodies ascertained. It shares with the other gases, as well as with solid and liquid bodies, the same general peculiarities, and is also subject to the influence of gravitation and of molecular forces.

Atmospheric air surrounds the globe on all sides, having a thickness of from 30 to 35 miles; it is the cause of a great number of phenomena, some of which will here be referred to, others belonging to the subject of meteorology.

That the air had weight was known to Aristotle; Galileo, however, and, after him, Torricelli, were the first to prove this by experiment. Exhausting the air from a hollow globe, suspending this to the end of a balance brought into equilibrium by means of weights, and afterwards allowing the air to enter the globe, it will be found that equilibrium is again destroyed, and must be restored by the imposition of more weights: their amount will express the weight of the air contained.

The molecular force acts in gaseous bodies very differently from what it does in the case of liquids and solids, endeavoring to separate the molecules one from another, this influence being called *elasticity* or *tension* of gases. Of the activity of this force we may be convinced by introducing a well-closed bladder under the receiver of an air-pump. When a vacuum is produced, the contained air expands the bladder as exhaustion proceeds. The expansive force of air is unlimited, as in a state of greatest expansion it still exerts a pressure upon the containing walls. For this reason gases can have no free surface like solids and liquids, as they would extend illimitably into space; there is, therefore, for them only one condition of equilibrium, namely, that the elasticity in one and the same layer is equal, For equilibrium, therefore, the lower layers must constantly remain the densest; for which reason the pressure of the atmosphere must be greater at the level of the sea than on the tops of mountains. It must not be understood, however, from what has already been said, that as the air can have no free surface, the assumption of a limit of the atmosphere to some miles is erroneous. This rests upon grounds hereafter to be stated.

The atmospheric pressure may be measured; and to its existence innumerable phenomena testify. Immerse the lower end of an open tube into water, the fluid will rise into it, according to the laws of hydrostatics, to an

equal height with that surrounding it; suck some of the air from the tube, and additional water will enter, because the equilibrium of atmospheric pressure is disturbed. The air within becomes rarer and lighter; the external atmosphere, therefore, pressing upon the external surface of the water, forces it up into the tube until the air therein contained is compressed sufficiently to exert the same pressure with the outer, or, in other words, until the weight of the water raised is equal to the excess of external pressure. Exhaust the air entirely from the inside of the tube, and the water must rise until the weight of the column raised is equal to the weight of a column of air having the same base, and a height equal to that of the atmosphere. It has been found that a column of about **33** feet is the maximum that can be raised in this manner. Torricelli from these facts established the following conclusion: for two different columns of fluids to be in equilibrium, they must be to each other inversely as their densities. Mercury is fourteen times heavier than water; if, now, the pressure of the atmosphere sustain a column of water **33** feet in height, it will sustain one of mercury $\frac{33}{14}$ feet, or about 29 inches. That this is actually the case is shown by a simple apparatus for measuring the pressure of the air, termed the Barometer, consisting essentially of a glass tube about **31** inches long, closed at one end and filled with mercury. After filling this tube, hold the finger on the open end, and inverting it in a basin of mercury, remove the finger. The height of the mercurial column remaining in the tube, which in places at a slight elevation above the sea amounts to a mean height of about 28.6 inches, serves as a measure of the pressure of the air, as this, acting on the external surface of the mercury in the basin, sustains that in the tube. Along the top of the mercurial column, a scale divided into inches and fractions of an inch is attached, sometimes on metal, sometimes on paper, and occasionally upon the tube itself. To ascertain the amount of atmospheric pressure upon any given surface, calculate the weight of a column of mercury whose base is that of the given surface, and whose height is that of the mercury in the barometer.

Many different constructions of the barometer have been made, principally reducible, however, to two kinds, *cistern* and *syphon* barometers. The common barometer (*pl.* 18, *fig.* 32) is one of the first kind. It consists of a long tube, B, curved beneath and dipping into the vessel or cistern, C, upon which the pressure of the external air can act, as it is open. The whole is fastened to a board, A, and a scale, D, with a movable index, E, attached, to mark the variations of pressure by the rise or fall of the mercury. This scale is generally divided into inches, and tenths or twelfths, and a vernier frequently attached to the index for measuring very slight variations. The small scale, F, serves to measure the mercury in the vessel or cistern. Attention must always be directed to the vertex of the convexity of the mercury, which is formed in the ascent. In filling the barometer, care must be taken that there are no bubbles of air in the mercury, or attached to the tube, these being driven out by boiling the mercury in the tube. If these are not expelled they will rise into the top of the tube, and exert atmospheric pressure upon the top of the mercurial column, thus

neutralizing in some measure the external pressure, and causing the mercury to stand at too low a point; this undue depression will be increased, also, whenever expansion of the included air is produced by an increase of temperature. The empty space above the mercurial column of every barometer is called the Torricellian vacuum. The simplest barometers have only a straight tube, dipping directly into a separate vessel of mercury.

Since the barometer has been applied to the measurement of heights, the older construction for this purpose has been changed, and the *syphon barometer* (*fig.* 33) employed. This also consists of the tube, *b*, bent into a syphon shape at *a*, and closed at both ends. The short limb has at *c* a capillary opening which admits the entrance of air, but not the exit of mercury, so that the tube may be inverted without the contents escaping. To prevent the entrance of air into the larger limb during this inversion, Bunten has invented the construction represented in *fig.* 35. Here the mercury on inversion enters the space, *d*, so that the point of the downward projecting tube is, during inversion, constantly closed air-tight by the superincumbent mercury. It will readily be understood that in the figure only the lower part of the barometer is represented. In the syphon barometer, the quicksilver surface exposed to the pressure of the atmosphere has no fixed position, and the zero of the scale must therefore be brought to the place of the inferior surface.

In the barometer of Gay Lussac (*fig.* 34), the long limb, *b*, is bent in such a manner, that its upper part and the short limb, *a*, lie in the same straight line; the stations of the two surfaces can therefore be read off on the same scale, and then the zero is in the centre, so that the reading is of how much one scale is above, and how much the other is below 0; the sum is then the proper height of the barometer. This double observation is necessary on account of the influence of temperature upon the mercury.

The barometer of Fortin (*figs.* 36–38) is a cistern barometer, and has the advantage over others, that the mercury in the cistern, *a*, has an invariable level. The bottom of the cistern is formed by a leather pouch, *h* (*fig.* 37), against which a screw, *k*, presses, by which the surface of the mercury may be elevated or depressed. If then *g* be screwed fast to *i*, the surface of the mercury in the cistern must correspond exactly with the zero of the scale, which is at the extremity of a fine point. When the image of this point in the surface of the mercury is made to coincide with the point itself, the adjustment is made. The barometer is surrounded by a metallic tube, in whose upper part there are two opposite slits for observing the top of the mercury. The scale is attached to the metal tube. To assist the eye in determining the exact height of the mercury, there is a slider on the metal tube, which has also two slits corresponding to those of the tube, only a little broader. The slider is so adjusted that the upper edges of its slits coincide exactly with the top of the mercurial column.

Experiments and calculations instituted for the purpose, assign to a station of the barometer of 28.6 inches, an atmospheric pressure of about 14.6

pounds upon the square inch, which, upon a surface equal to that of the human body, amounts to from 30,000 to 40,000 pounds. This at first appears incredible, as it seems impossible to resist so enormous a pressure; the matter becomes more intelligible, however, when it is considered that the pressure acts on all parts, both inside and out, at the same time, so that the pressure from one direction is exactly neutralized by that from the other. This weight then is only sensible when the equilibrium is disturbed, as in a violent wind, &c. The compression or crushing of the body is resisted by the penetration of the external air into all the cavities of the body by means of innumerable fine pores as well as of larger passages, so that both inside and out, air is present in the same state of tension. This atmospheric pressure is of the greatest importance to the animal organism, as will be made evident by a single example. It is known that the head of the thigh bone consists of a ball playing in a socket of the pelvis inclosed in a capsular ligament, and possessing motion in almost every direction. If the leg be unsupported, and even if all the muscles and tendons be severed, the head of the thigh bone does not fall out of its place. If, however, the capsular ligament be pierced, or communication be made in any other way with the external air, the thigh immediately descends out of its place. It is thus evident that the pressure of the air upon this air-tight joint must play a great part in keeping it in position. In this manner may be explained the peculiar sensation of weakness and relaxation experienced at great elevations on mountains; the diminished pressure of the air takes from the whole frame its compact and well knit character.

One of the most important propositions in the theory of equilibrium of gaseous bodies, is the law discovered by Mariotte, and called after him Mariotte's law: that the volume of a gas is inversely as the pressure to which it is subjected. Thus twice the pressure is required to reduce a gas to half the volume, &c. Arago and Dulong have shown the accuracy of this law up to a pressure of 27 atmospheres, or a pressure 27 times that of one atmosphere. For this purpose they employed the apparatus represented in *pl.* 18, *fig.* 39. In the middle of an old tower, a mast, *a*, of about 100 feet in height, was erected, to which a long glass tube, *t*, was attached, composed of 13 single tubes of six feet in length. At the foot of the mast was a cast iron vessel, *v*, filled with mercury, with a forcing pump, *p*, attached at *b*, and provided with a manometer tube, *mn*, closed above, graduated, and filled with dry air. When the mercury stood at an equal height in the tubes, *t* and *mn*, the air in the latter, of known volume, experienced the ordinary pressure or that of one atmosphere. Forcing water, however, by means of the forcing pump, into the upper part of the vessel, *v*, the air in the tube, *mn*, would become compressed, and the mercury rise in the tube, *t*. The scale on the first tube gave the volume of the included air; the difference of height of mercury in the two tubes gave the corresponding pressure. *Fig.* 40 represents the manner in which the single parts of the vertical glass tube were united by strong rings, *aa′*; *c* is an upward projecting rim, filled with melted cement, to render any escape of mercury impossible. *Fig.* 41 shows how the manometer tube, *mn*, was fastened to the plate, *c*, of the cast

iron vessel, by means of the shoulder, *h*. The apparatus, *qy* (*fig*. 39), served to move along the vernier of the manometer, which was inclosed in a glass tube.

It has been mentioned above that the barometer was applicable to the measurement of heights, as the atmosphere in its lower strata exercises a greater pressure than in the upper, and that consequently the height of the barometer would be greater in one case than in the other. These measurements would be very simple if the air were not elastic, or at least very slightly compressible; for then, by obtaining a point of departure or unity by direct measurement of one height, other altitudes could be readily calculated. This, however, is impossible, as the less the pressure upon a layer of air, the less is its density; or in other words, the greater the ascent, the greater the rarity of the air. Mariotte's law renders it possible, however, to attain to accurate results. Suppose the height of the barometer at a certain elevation to be 760 millimetres, and by ascending 11.5 millimetres, the height of the mercury to be only 759 millimetres $= 760\left(\frac{759}{760}\right)$. Taking 11.5 metres as unity—then as the density of the air is proportional to its pressure, the next layer will be less dense, and, indeed, only $\frac{759}{760}$ as dense as the one below it; the height of the barometer then is there only 760 $\left(\frac{759}{760} \times \frac{759}{760}\right) = 760\left(\frac{759}{760}\right)^2$, and so on, so that for $n \times 11.5$ metres, the height of the barometer is $760\left(\frac{759}{760}\right)^n$. If now B be the height of the barometer at *a*, and B′ that at a place, *b*, higher than *a* by unity, and the quotient, $\frac{B'}{B} = q$, then, according to the preceding considerations, the height of the barometer for a place, *b*, higher by *m* units, will be $= Bq^m$, and *m* can be obtained from this equation. Thus $q^m = \frac{b}{B}$, and $m = \sqrt[q]{\frac{b}{B}}$. Here, however, must be taken into account the temperature and the vapors present in the atmosphere; the consideration of the corrections necessary on this account would carry us too far beyond our limits.

For determining altitudes where the greatest possible accuracy is not required, the easily transportable *Differential Barometer* of Kopp (*pl.* 18, *fig*. 42), may be employed to advantage. It consists of a straight cylindrical glass tube, *k*, united by means of a narrow tube with a glass vessel, *i*, closed tight above, through whose upper cap a thinner tube, *cd*, passes. In the tube, *k*, is a leather piston, which may be moved up and down. The instrument is filled with mercury, so that when the piston, *f*, is raised, in consequence of the atmospheric pressure, almost all the mercury passes from *i* into *k*, and the air contained in the vessel, *i*, communicates with the external atmosphere. A scale is attached to the tube, *cd*. Depressing the piston, the mercury is again forced into *i*, and there confines, as it closes

the lower end of the tube, *cd*, a certain quantity of air of the same density as that external to it. Continuing this depression until the mercury touches a point attached, similar to that described in the barometer of Fortin, the inclosed air becomes condensed, in a proportion dependent upon the dimensions of the instrument and the position of this point. If, for instance, the air were condensed to three fourths its original volume, the height of the mercury according to Mariotte's law, would be one third of the actual height of the barometer, and for this proportion, as well as any other, the actual height of the barometer would be obtained by multiplication into a factor developed from the construction of the instrument. If now there be another point in the instrument, standing somewhat deeper or lower than the first, it can be brought in contact with the mercury by a change in the position of the piston, where then the factor would of course be different. Making observations in immediate succession, and at the same place, with the two points, the products of multiplication by the different factors must be equal; the two points therefore control each other. There must, of course, be attached to the tube, *cd*, as shown in the figure, two different scales for the two points.

Upon the law of Mariotte depends an apparatus termed volumeter (*fig.* 93), invented also by Kopp, for determining the volume of powders. The tubes, *k* and *i*, correspond to those of the same name in the differential barometer, being likewise filled with mercury; from *i* passes a bent tube to the wide glass cylinder, *n*, whose upper broader end is carefully ground off for the purpose of placing a plate of glass upon it, and rendering it air-tight by the addition of a little tallow. Closing the cylinder, *n*, and depressing the piston, *k*, until the mercury touches the lower end of the ascending tube, a certain quantity of air will be inclosed in *l* and *n*: pressing down the mercury to the point, *a*, the included air will be compressed, a corresponding column of mercury rising in the ascending tube. If, before laying on the glass plate, any body had been placed in the cylinder, *n*, then the mercury standing at *c*, less air would be included than before, and in forcing the mercury up to *a*, it would be more compressed, so that the ascending tube would contain a greater column of mercury than before. From the height of this column of mercury the volume of the body contained in the cylinder is to be calculated. The powder to be examined is introduced in a platinum vessel, of about the shape of *n*, and nearly the same size. The volume of air included when the empty vessel alone stands in *n*, suppose it to be 15.07 cubic centimetres; and also the volume between *c* and *b*, say 2.5 cubic centimetres, to which the air is compressed, must be known. Now introduce the body whose volume is to be determined into *n*, and depress the piston again from its highest position, where *c* is closed by the mercury; a quantity of air, *x*, is inclosed, and when the mercury comes in contact with the point *a*, the air is compressed to $x - 2.5$. Let the column of mercury last obtained = 90 lines, and the actual height of the barometer = 336 lines, then the compressed air now experiences a pressure of $336 + 90 = 426$ lines, and $426 : 336 :: x : x - 2.5$; x, therefore, $= 11.72$. As now, when *n* is empty, the volume included = 15.07 cubic centimetres,

the volume of the body examined will be $= 15.07 - 11.72 = 3.35$ cubic centimetres.

Next to the barometer comes the air-pump, invented by Otto von Guericke, one of the most important instruments for elucidating the properties of the air. It serves to produce by successive rarefaction as complete a vacuum as possible, although this can never become so perfect as the Torricellian vacuum. Imagine a cylinder in which a piston moves air-tight, and closed below, then on raising the piston a vacuum will be produced. If, now, the cylinder be united with another inclosed space by a tube, so that the air can pass from the latter into the former, then, on raising the piston, the air would make this transit, but on depressing the piston it would return again. Suppose, however, a cock to be placed in the tube, by means of which the return of the air can be prevented, while its egress is allowed; then by the alternating action of the piston and turning of the cock, the air in the vessel may be reduced to a minimum, even if a perfect vacuum may not be attainable on account of the infinite expansion of air. This is the simplest construction of the air-pump; it has, however, since its invention, received various modifications and improvements.

Pl. 18, *fig.* 44, represents a small hand air-pump, according to the construction of Gay Lussac. The main part consists of a hollow cylinder or tube of brass, in which an air-tight piston plays up and down. In the latter is a valve opening upwards; thus shut during the ascent of the piston, and open during its depression. At *b* is attached the receiver, the vessel in which the vacuum is to be made, consisting generally of a plate and glass bell. The screws *a* and *f* serve to screw the air-pump to a table or board; at *d* a cock is attached, as also at *s*. If, now, the latter cock be opened and the former closed, and the piston elevated, a part of the air in the receiver will pass out through the first horizontal and then vertical canal, *ab*, into the cylinder, and the air in the receiver will become rarefied. Depress the piston after closing the cock *s*, and the air under the piston passes out through it by means of the valve in the piston head. To let the air again into the receiver, the cock at *d* must be opened.

A sectional view of a larger air-pump is shown in *fig.* 45, *pl.* 18. Here *a* is the cylinder, in which works the air-tight piston, *b*, which contains a valve opening upwards, and is moved by the piston rod, *c*. The rod *ed* opens and closes the valve for the cylinder; at its lower end is a truncated cone, *e*, fitting in a conical opening. At *h* is seen the glass bell to be exhausted, whose edge must be ground perfectly plane, in order that it may fit air-tight upon the ground plane, *pp*. In the centre of this plate is a female screw, *v*, for screwing on any other form of receiver; and from this goes a canal to the conical opening at *e*. If, now, the piston resting on the bottom of the cylinder be elevated, the valve at *e* opens until the shoulder at *d* strikes against the upper plate of the cylinder, and the air in part rushes from the receiver into the cylinder: on depressing the piston, the valve at *e* is closed, and the air in the cylinder escapes through the valve in the piston. At *r* is the barometer gauge, or contracted barometer, inclosed in a long narrow bell, and in communication with the air in the receiver by

means of the canal *ev*. The mercury at first completely fills the one leg of the bent barometer tube, but after a considerable rarefaction, begins to sink, and the difference of height of the two mercurial surfaces gives the pressure of the air in the receiver. If, for example, this difference amount to one inch, at a barometrical height of 28 inches, the air in the receiver will be rarefied 28 times. *Fig.* 46 represents a double-acting cock, *y*, in the canal between the receiver and cylinder, that is, a cock bored through in two directions: the one aperture is straight, and during exhaustion unites the receiver and cylinder; the other is bent into a knee, and opens into a lateral opening, which, during exhaustion, is closed by a metal stopper, *b*. To admit air into the receiver after exhaustion, the stopper must be extracted and the cock turned in such a manner that the air can penetrate into the receiver through the side aperture.

Air-pumps are divided into *cock* and *valve pumps*, and moreover into *one* and *two-cylindered*. *Fig.* 47 represents an air-pump provided with two cylinders. Here the two piston rods are toothed, and a piston interposed in such a manner that by the motion of a handle they can be alternately elevated and depressed, the one ascending, the other descending at any given time. In this manner the exhaustion goes on uninterruptedly, and is completed in much shorter time.

In the common air-pumps, however well they may be constructed, there is always a space intervening between the piston and its point of greatest depression and the bottom of the cylinder, which can never be exhausted. The air in it obstructs the rarefaction of air in the receiver, and sooner or later puts a stop to it. To obviate this difficulty, Babinet has constructed a cock of peculiar form, represented in *figs.* 48—50. In *fig.* 48, *a* and *d* are the two cylinders of a double cylinder air-pump, and *r* the cock attached between the two cylinders, a little below their base. This cock has four openings (*figs.* 49 and 50). The first and second, *s* and *t*, pass completely through and are perpendicular to each other; the third, *v*, is parallel to *s*, going, however, only to the centre of the cock, and opens in *t*. The same is the case with the fourth opening, *u*, which runs parallel to the longitudinal axis of the cock. From the bottom of the two cylinders pass curved canals which terminate at *b* and *e* in the openings of the cock. At first, the cock is fixed in such a manner that the opening, *t*, unites both canals; and this position, in which it exerts no particular influence, is continued until the mercury will fall no longer in the gauge. The cock is now slightly turned, so that the bore, *s*, unites the two cylinders; and at the same time the opening *v* unites the cylinder *a* with the receiver. If the piston in *a* be depressed, the rarefied air beneath it is driven over into the other cylinder; when, however, the piston in *d* is depressed, the valve in the bottom of *d* is closed, and in the cylinder *a* the space above mentioned contains only rarefied air, so that the rarefaction in *a* is much greater than before. It is only after repeated strokes of the pistons that a new limit to rarefaction is attained.

The *condensing pump* (*fig.* 51) serves to condense the air, and differs from the air-pump merely in having the valves to open and shut in a dif-

ferent direction. In depression the piston drives the air into the receiver and compresses it ; in elevation the external air opens the piston valve and presses into the tube, while the air in the receiver is retained by the valve in the bottom. The receiver must be screwed down, else it will be forced up by the compressed air. Many condensing pumps are so arranged as to be applicable to various apparatus or receivers in which the air is to be condensed. One of this kind is represented in *fig.* 52. It consists of a tube or cylinder, and a piston, *b*, without a valve. The receivers are screwed on to the lower end of the tube, either at *c* or *d* ; a valve then attached admits only the ingress, not the egress of air. The receivers *f* and *i* may be closed when necessary by the cocks, *e*, *h*, and *g*. For admitting successive portions of air into the cylinder or tube, a lateral opening in the tube, or, as in the figure, a lateral valve, may be used. The latter serves principally when a gas, not atmospheric air, is to be condensed.

The air-pump, in its application, is confined not merely to physical experiment, but is of the highest importance in the arts. It is there employed on the one hand for rarefying the air, as in the steam engine and sugar manufacture, and on the other for condensing air, as in driving of machines by condensed air, in the air-gun, &c. In the air-gun the air-vessel in which the condensed air is contained is either a ball screwed on beneath the stock or it is the piston itself. This vessel has then a valve which prevents the escape of the included air, and upon which stands a pin connected with the discharge of the gun. Thus, when the trigger is pulled and the cock descends, this pin is pressed upon for an instant with such force as to open the valve sufficiently to allow the escape of enough air to propel the ball.

To measure the pressure of gas contained in a certain apparatus, pressure valves are partly used, and partly manometers, to which latter belong the barometer gauge of the air-pump, as also the *safety tube* represented in *pl.* 18, *fig.* 53. The latter contains a liquid, standing at an equal height in the two legs when the pressure is equal to that of the atmosphere. When this is not the case the liquid cannot stand at an equal height in the two legs ; and from the difference of level, knowing the density of the fluid employed, the pressure in the interior of the inclosed space to which the tube is applied can easily be determined.

For pressure valves the relation is somewhat different, since while in manometers the internal pressure is measured by the height of the mercury or other fluid, in those it is given directly in terms of weight. The wall of the compressing vessel is provided with an aperture of determinate size, a square inch for instance, which is so constructed by opening outwards as to form the bed of a conical valve. This valve is loaded with weights, either directly or by means of a lever, upon which, as in the steelyard, a shifting weight may be placed. In such cases the valve when raised gives directly the pressure exercised by the gas, upon every square inch of surface. All these valves, however, give indications only when the

pressure is greater than that of the atmosphere; when it is less they are themselves kept closed by the pressure of the external air.

Upon the pressure of the atmosphere or compressed air depends the action of very many important and useful arrangements, apparatus, and machines, some of which will here be considered.

The pressure of the air amounts to about 14 pounds to the square inch; if then a vessel be constructed with an opening of not more than one square inch, and the pressure of the fluid therein contained does not exceed 14 pounds, then when filled it may be covered with a sheet of paper and inverted without the escape of the fluids on the withdrawal of the paper, this escape being prevented by the atmospheric pressure. Upon this principle depends the straight syphon, *fig*. 54. This is a tubular vessel, contracted above and still more below, and open at both ends. When dipped into a liquid so as to be completely filled, and the thumb placed upon the upper extremity, the tube may be elevated without the escape of the liquid, which only occurs on the withdrawal of the thumb. The *Syphon*, *fig*. 55, is a bent tube, *bsb′*, whose legs are of unequal length. If now the shorter limb be immersed in a liquid, and the entire tube filled by suction or other means, the liquid will continue to flow from the extremity of the long limb, until the opening at *b* is laid bare—provided, however, that the extremity of the long limb always occupies a position lower than that of the shorter one. For the purpose of more conveniently filling the syphon and obviating any danger of getting the fluid into the mouth, a sucking tube, as *at fig*. 56, is frequently attached. Closing the opening at *b′*, and sucking at *t*, the whole limb, *sb′*, will become filled; the escape of fluid will commence on removing the obstruction at *b′*, and continue until the fluid has run off to the level *bn*. This is sometimes called a *Poison Syphon.*

In this place belong the various forms of apparatus depending on the syphon, and called *Cup of Tantalus.* They are used principally for purposes of amusement, or to excite astonishment when a vessel filled with water empties itself spontaneously. *Fig*. 59^a represents a metallic vessel divided by a floor somewhere near the middle into two parts. Through an opening of this floor passes a glass tube open at both ends, over which a larger tube is placed, *fig*. 59^b, hermetically closed above, and with only a small opening near the floor to admit the water. On pouring water into the vessel it passes through the small aperture into the large tube, standing in this as high as in the vessel itself. On rising as high as the top of the smaller tube the water runs over into the lower division, for which purpose the latter must have a vent-hole above to admit the escape of air. The water will then run off until its level reaches the aperture in the large tube, the lower part of the vessel thus becoming full as the upper is emptying. The experiment becomes most astonishing when the mechanism is concealed by some figure. *Figs*. 57 and 58 represent vessels which, properly filled, retain the water when standing erect, allowing it to flow out, however, when inclined. The vessel in *fig*. 57, as in *fig*. 59, is divided into two parts; through the bottom of the upper one there passes the long leg of a

syphon, the short leg resting upon this bottom. Pouring water into the vessel, so that its surface is a little below the inside of the curvature *b*, then in an erect position of the vessel the water cannot flow out; in an inclined position, however, as in drinking, this will immediately take place; the long arm of the syphon becoming filled and allowing the escape of the water. The same occurs in the drinking vessel, *fig*. 58, when inclined towards the left side.

In the first vessel the syphon lies concealed in its double wall, and the short leg has a small opening only at the floor, while the height to which the water is to be filled, and the point to which the water is to be applied, are accurately indicated. In the second vessel the construction of the double wall itself forms the syphon, and in this case the point to which the tongue in the double wall rises, and which must not be exceeded in filling, must be marked on the inside of the cup. In both cups the water runs into the lower division, whence it must be removed before the experiment can be repeated.

Finally, *pl*. 18, *fig*. 60, represents a very ingenious and amusing apparatus depending in principle upon the syphon. The principal part consists of a vessel divided into three compartments by a horizontal and vertical partition, one below and two above. Through the horizontal partition pass two tubes; a third passes through the covering of the upper division to the left, and at the same time through an open cup, into which a hollow bird, *i*, inclosing a concealed syphon, dips its bill. Filling now with water through the proper apertures, the upper apartments *f* and *c*, which, however, must not reach the upper opening of the tube at *e*, this water passes from the right hand compartment through the tube *d* into the lower chamber; the air displaced escapes through the tube at *e*, presses upon the water in the upper left chamber, and causes it to pass out in a jet through *h*, and to fall into the inclosing basin. As the air in the right chamber becomes rarefied by the depression of the water, the syphon at *g* is filled with water by atmospheric pressure upon the water in the basin; this then passes as if drunk by the bird, through the tube *g* down to *c* again.

Hero's Ball, *fig*. 61, consists of a strong well closed vessel, *v*, partially filled with water, in which at *j* a piece of thermometer or other fine tube, *t*, with a fine opening, passes through the stopper *a* nearly to the bottom of the vessel. If the air in the upper part of the vessel is compressed, as by blowing in air from the lungs, or if the air above the water is expanded by heat, the pressure of the air forces out the water in the form of a vertically ascending stream.

The *intermitting spring* (*fig*. 62) consists of a water vessel, *r*, with escape tubes, *j*, *j*, and a tube, *t*, whose upper end projects above the surface of the water at *r*, while the lower, which has a small notch in it, stands in a vessel, *p*. When the notch is free, the pressure of the atmosphere upon the surface of the liquid in *r*, causes this to flow out through the tubes *j*, *j*, into the vessel *p*. As soon as the lower end of the tube *t* is covered by the water pouring into *p*, the discharge through *j*, *j* ceases, because no more air can pass through *t* into the vessel *r*. In the meantime, however, the water

passes through a small opening in the bottom of the vessel p into the lower receptacle, the inferior opening of the tube t again becomes free, and the discharge through j, j begins afresh.

Hero's fountain is essentially nothing else than a self-acting Hero's ball, in which the compression of the air which drives out the water is produced by means of a column of water. *Fig.* 63 represents the apparatus in its simplest form, which, if not blown in one piece, may consist of vessels connected together by glass tubes. To use it, the upper vessel, c, is filled with water through d, until it stands nearly up to the termination of the tube b. Filling the vessel above a with water, the water descending in a compresses a column of air in b, whose elasticity and pressure upon the surface of the water in c, force out the water through d. *Fig.* 64 represents a somewhat more complicated form of this apparatus, where the tube x answers to the tube a in *fig.* 63, and y to the tube b; the vessel z occupies the place of the lower ball, and the upper vessel that of the ball at c; and at a is the discharge pipe, which reaches nearly to the bottom of the vessel.

A *pump* (*suction pump*) in its simplest form is a tube of uniform diameter within, open at both ends, and the lower dipping into water. In this tube may be moved up and down a well-fitting and air-tight piston attached to a rod. Supposing at first the piston to stand at or near the surface of the water, and that it be elevated by means of the rod, then the water, by the pressure of the air on the surrounding liquid, will be forced into the pump, and ascend to a height of not more than 32 feet. If the water is to be not only raised but turned into a receiver, its return must be prevented, and some provision made for getting it above the piston. The lower extremity of the pump tube in this case must not be open, but must have a bottom provided with a valve opening upwards; also with a suction tube dipping into the water where it may be closed by a strainer. The piston also must have a valve opening upwards. On raising the piston, the water is forced by atmospheric pressure through the lower valve into the pump tube, the valve in the piston remaining closed; on depressing the piston, its valve is opened by the pressure of the water, which then rushes through it and occupies a place above, the return of the water through the lower valve being prevented by its closing. By repeated elevations and depressions of the piston, the water is at length lifted to the level of the top of the tube, or to an orifice in the side where it can escape. If the water is not to flow directly from the pump tube, but into some other place, or if it is to be discharged with great force, or carried to a great height, the *forcing-pump* must be employed, as represented in *pl.* 17, *fig.* 34. It consists of a pump-stock or tube in which is a massive cylindrical piston, F, moving up and down, passing air-tight through a stuffing box, E, and a grease box, D, but without touching the pump tube itself, which therefore need not be perfectly cylindrical in its box.

Upon the suction tube, C, is placed the valve lid, f, with the valves i, i, through which, on raising the piston, the water passes into the cylinder; on depressing the piston, the water is driven into the tube B, after forcing open its valve, d. On raising the piston again, the valve d falls, and the valves i, i,

which had just been closed by the depression, are aga n open, and admit a fresh quantity of water, which also is then forced into B; the operation may thus be continued for any length of time. It is necessary to mention a special contrivance which must be attached to the pumps of this construction when the water is to be forced to a great height. The water, as is well known, contains a great deal of air mixed with it, which is set free during pumping, and collects under the piston. If, now, the column of water behind *d* has a great height, as of 40 or 50 feet, the air in A has to overcome a pressure of more than one atmosphere, and thus, instead of passing out through *d*, becomes compressed by the descending piston, expanding when this is elevated, so that when the amount of this air is considerable, it becomes impossible to produce a sufficient rarefaction in A to admit of the opening of the valves *i*, *i*, and the ingress of the water. Some plan must be resorted to, then, for removing the accumulation of air from the cylinder. For this purpose a canal, *abc*, is bored through the piston, to allow of an exit for the air beneath; a screw at *a* keeps this canal closed. If, now, a quantity of air has collected, as indicated by a diminished discharge of water, the screw *a* is to be opened on the descent of the piston, and closed when it has reached its lowest point, or when water escapes instead of air through the canal *abc*.

Suction and forcing-pumps find numerous applications in the arts and manufactures, and we shall have frequent occasion to refer to them in the technical part of this work. We will here only mention their application in hydrology, as, for instance, in the water-works at Marly, where water is raised to a height of over 500 feet. Here also belong the fire-engines, the largest of which consists of two forcing-pumps, working alternately, and driving the water into a larger air-tight vessel, whence it escapes through an escape-pipe. A more particular account of various kinds of fire-engines will be presented in the tenth division of the work.

The *hydraulic press* of Bramah, represented in full on *pl.* 18, *fig.* 65, and in section of the working part in *fig.* 66, is another application of the forcing-pump. It consists of two principal parts: a forcing-pump which exerts a pressure by means of the water raised, and a piston which receives the pressure and transmits it, through a plate resting in its upper extremity, to any body upon which pressure is to be exerted. The piston, *s*, is raised by the lever, *l*, and in consequence, the water presses from the reservoir, *b*, through the strainer, *r*, raises a valve, and thus gets underneath the piston. When this piston is depressed, the water closes that valve, opens the valve *d*, and passes through the canal, *tbu*, into the cylinder, *cc'*; here it presses against the piston, *p*, and raises it with the plate *p'*, so that any body between this plate and the fixed plate, *e*, experiences a great pressure. The force with which the smaller piston, *s*, is depressed, will be to the force with which the larger, *p*, is elevated, as the area of a section of the piston *s*, to the area of a section of the piston *p*. The amount of force transmitted to the piston *p*, is regulated and measured by a safety valve, *g* (*figs.* 67–69). Thus knowing the weight, *p*, the length of the lever arms, *fx* and *fy*, and the area of the lower surface of the valve, *g*, the pressure experienced by the valve

can be easily calculated when the lever, *fxy*, becomes elevated. The weight, *p*, must be so regulated as to admit of the raising of the valve only when the pressure has reached a certain limit. *Pl.* 18, *fig.* 70, represents the part through which the piston, *s*, passes, constructed so as to prevent the escape of any fluid. *Fig.* 71 is an ingenious contrivance of Bramah, intended to supply the place of a water-tight end of the piston, *p*. It consists of a bent leather, laid in an annular channel of the piston, and against whose walls, as well as against the piston, it is pressed the tighter with an increased pressure from below.

It has been before mentioned that the force increases with the ratio of the sectional surfaces of the pistons. When the smaller piston, *s*, is depressed, every part of the inclosing walls, equal in area to the bottom of the piston, experiences the same pressure as that with which the piston *s* is depressed. The lower surface of the piston *p* is, however, a part of these inclosing walls, and every part of the surface, equal in area to the bottom of the piston *s*, must experience the same pressure, and the sum of all these pressures will represent the force with which the piston *p* is elevated. Thus, if the small piston have an area of one square inch, and that of the larger 100 square inches, the force on *s* will be multiplied a hundred fold on *p*. By means of the lever, *l*, a pressure of 600 pounds can easily be exerted by one man on *s*, and the piston, *p*, must therefore be raised with a force of $600 \times 100 = 60{,}000$ pounds, and the same pressure exerted upon any body between p' and *e*. From this some deduction must be made for friction, &c.

A proof that the law of Archimedes, established for liquid bodies, applies also to gaseous, is furnished by the *Air Balloon* or *Aerostat.* Every body surrounded by, or immersed in the air, loses an amount of weight equal to that of the air displaced, and must therefore ascend in the atmosphere whenever its weight is less than that of an equal volume of air. Owing to the great lightness of the air, this can only be attained when a hollow body is filled with some very rare matter. These conditions may be fulfilled by making a bag of paper, gold-beaters' skin, or oiled silk, and filling it with rarefied air, or with a gas lighter than the atmosphere. Vacuum balloons, whose contents would be certainly of least possible weight, are not feasible, as independently of the great difficulty of exhausting air on so large a scale, they would be immediately compressed by the external air, unless made of some very strong material, as metal, in which, to compensate for the great weight, the size must be enormously large to produce an ascent.

Independently of the material, there are two principal kinds of air-balloons characterized by the mode of filling: 1, *Montgolfier,* open below and filled with heated, and consequently rarefied air. The source of heat must be at some distance below the lower opening, and must accompany the balloon in its ascent, to continue the rarefaction, which would otherwise be of short duration. This balloon derives its name from the inventors, the brothers Montgolfier, who caused the ascent of the first balloons at Annonay in France, June 5, 1783. The second kind of balloon is the *Charlière,* filled with hydrogen gas, which, when perfectly pure, is fourteen times lighter

than air. It derives its name from Professor Charles of Paris, who also, in 1783, employed this method of filling, and with one companion ventured on the first aerial voyage in a car attached to the balloon. Balloons of this latter construction are decidedly preferable, as being less exposed to the danger of catching fire than the other; and secondly, on account of the greater lightness of hydrogen, they may be made smaller, or when of equal size, they will sustain a much greater weight, and will ascend higher in the atmosphere. Hence, when an ascent is to be made by individuals, the Charlière balloon is almost always employed. The descent of this kind of balloon is effected by the escape of gas through a valve attached to the upper part, and regulated by a cord; and the higher ascent, by the discharge of sand-bags taken along as ballast. The ascent of a balloon must of course cease as soon as it attains to a stratum of air of so slight density that the air displaced is no heavier than the balloon with its load.

Pl. 17, *fig.* 39, exhibits the construction of the valve for the escape of the gas used in the so-called *Hampton Balloon*. The balloon itself consists of forty-one strips of oiled silk, each of which is sixty-seven feet long and three feet broad; its circumference amounts to one hundred and twenty-three feet, its diameter to forty-one feet. The first constructed valves consisted of a simple door opened by a cord, in which case the aeronaut could not see how much gas escaped, and consequently sometimes let out more than he wished. The present valve consists of a hoop, A, four and a half feet in circumference, and six inches deep. At *dd* are spiral springs attached inside and inclosing the axis *cc*. The whole resembles the upper part of a drum. To the valve proper which turns about the axis *cc*, the draw cords, *bb*, are attached, of which the right opens the valve and the left closes it. The spiral springs *dd* would of themselves close the valve, the cord being attached merely by way of precaution. Over the straight part of the springs pass two rings which spring off when the valve is opened to a certain point. This latter then remains open and the gas entirely escapes. This takes place when the balloon is on the ground, otherwise the aperture may be regulated to $\frac{1}{89}$ of an inch. The cords used in this balloon are of cocoa fibres, as being stronger and lighter than common.

An appendage very frequently attached to the balloon, for the sake of descending from a considerable height, is the *Parachute*, A, *fig.* 40. Its principle depends on the resistance of the atmosphere, which diminishes the velocity of descent of every falling body, and this the more, as the surface of the body is greater in proportion to its weight, and as the velocity already attained is greater. The parachute, at the ascent of the balloon, is placed between it and the car, C, to which latter it is fastened: on breaking the connexion between balloon and car, the latter immediately falls with increasing velocity, the parachute being at first folded up, but expanding more and more until at length it sweeps over the car in the form of a great umbrella from 25 to 30 feet in diameter. The velocity then decreases to a less dangerous amount, which it retains until the ground is

reached, which is done with impunity The anchor D serves to attach it to the earth.

Pl. 18, *fig.* 73, represents an ordinary balloon, A, with its valve at C, and to which is suspended the car D by means of the network F and the cords E, E, E, E. B is the hose through which the balloon is filled. *Fig.* 74 represents the copper balloon constructed in Paris according to Marey Monge's plan for conducting physical experiments in the upper strata of the atmosphere. The segments are of copper plate, about one eighth of a line thick, and the joints well soldered. The balloon is thirty feet in diameter, weighs 800lbs., and contains about 100lbs. of hydrogen gas.

The guidance of the balloon in any given direction has up to the present time not been accomplished; a rise and fall can indeed be effected, but not a horizontal motion, this being dependent upon the currents of the wind. For practical purposes the balloon is therefore inapplicable, and except for scientific purposes, its employment by the French army in military reconnoissance was the only application ever made of it. Experiments to effect a guidance of the balloon, and a motion in a determinate direction, have indeed been frequently made, and it may perhaps be advisable to refer in brief terms to several contrivances proposed for this purpose. *Pl.* 17, *fig.* 41, represents the *Flying Machine* of Henson (air steamboat), which, however, is essentially nothing but a great parachute, and has by no means answered its intention. AA are two wings, each one hundred and fifty feet long and thirty feet broad, constructed of iron work, over which is stretched a silk or linen covering; this latter consists of three parts, which can be opened or shut by means of a rope. The wings are sustained by the iron posts, B, B, and cords stretched over them, and are immovably attached to the firm middle part. The motive power consists of the fan wheels, D, D, set in rapid motion by the steam-engine, G; to this latter is attached the car for passengers, &c. The change of direction, in a horizontal plane, is produced partly by a rudder, partly by the tail, E, which is composed of a fan-shaped frame covered with silk, and movable freely about F. The visionary nature of this arrangement, presented here only as a curiosity, is evident at the first glance; any practical value is entirely out of the question, for the reason that no balloon is employed in its construction, but the machine must begin its journey from a high tower or lofty mountain, which journey then can be nothing else than a long protracted fall without any possibility of ascent. The aerial ship proposed by the Englishman Partridge, some years ago, and by him called *Pneumodrome,* certainly promises better results. It is represented in various details on *Pl.* 17, *figs.* 42—48, *fig.* 42 being a half-side view, *fig.* 43 a half-longitudinal section, *fig.* 44 a vertical transverse section, and *fig.* 45 an end view, with the balloon partly omitted. The principal parts of the balloon consist of: 1. the balloon, A, of air-tight india rubber cloth, a square yard of which weighs about one pound, spheroidal in shape, and whose length, breadth, and height are as 7 : 4 : 2. For the filling the inventor employs pure hydrogen gas. As, however, the inclosed gas expands greatly at a height where the pressure of the atmosphere is much

less than at the surface of the earth, care must be taken to fill the balloon to only three fourths of its greatest capacity; in order, however, to cause it to appear equally stretched at all times, a second balloon is placed within the first, and about one fourth of its cubic contents. This second balloon is filled with air by a tube communicating with the car, but can be emptied through valves during the expansion of the gas. The balloon is, moreover, provided with a warming apparatus, partly by heating the gas in the large balloon to about 60° R., to distend the balloon completely, and so bring to bear the entire force of ascension; and partly to increase and diminish this ascending force at pleasure. This heating apparatus consists of a system of tubes, C, attached internally to the bottom of the balloon, and connected by a conducting tube with the heating apparatus or the boiler of the steam engine hereafter to be mentioned, so as to admit of being filled with steam or air. 2. The spar and sail work. To the balloon is fastened a light spar work, consisting of iron, covered externally with tin plate, and strengthened by braces and tight ropes. This rests against the spindle or middle column standing in the middle of the car, and consists of an iron frame covered with some air-tight material. In it is a strong spiral spring, which serves the purpose of weakening the shock caused by striking on the ground. In the spar work is a horizontal main-mast with the horizontal mainsail, D, D, whose two halves may be brought into any inclined position, to produce a change of horizontal and vertical direction according to the rules of navigation. H, H, are vertical sails for using the wind in change of vertical direction. 3. The car with the steam-engine. The former is fastened in the spar work, and provided beneath with strong spring buffers to weaken the shock in the descent of the vessel. It is divided by a floor into two compartments, the lower intended to serve as a coal, water, and freight room, the upper one for the passengers and engine; the latter a high pressure and of the rotating construction. 4. The motive apparatus to be driven by the engine, consisting of three spiral wind wheels GG, and the horizontal wind wheels, FF (*figs.* 44, 46, 47), the latter attached above the car in the middle of the plane of the mainsail. They consist of wings turning in a box, in the centre of which the wind enters, to be again driven out at some point of the circumference. The object of these horizontal wind wheels is to produce an artificial current in a determinate direction, which, acting upon the part of the horizontal mainsail presented to it, produces an oblique upward or downward motion. As far as known, this idea of the Pneumodrome has never been carried out on a large scale.

F. Of the Motion of the Air.—Pneumatics.

If any aeriform body be confined in a vessel, it must escape through a given aperture whenever it becomes more condensed than the air in the space to which the opening leads. A vessel used for containing any kind of gas, and from which the gas will stream forth with a certain rapidity on the appli-

cation of pressure, is called a *gasometer*. Such vessels are constructed in various ways, according to the use to which they are to be applied. The principle of construction in all, however, consists of a vessel filled first with water, into which the gas is then admitted, displacing the water. By the direct application of a weight, or by means of a column of water which exerts a pressure upon the gas, this is forced out through tubes attached to the vessel. *Pl.* 19, *fig.* 1, represents a large apparatus of this kind, such as is used in gas works. It consists of a cylinder, B, of tin, closed above and open below, which sits in a great water reservoir of masonry. Two tubes, D and E, rise into the cylinder from below, their upper extremities standing above the surface of the water; the one tube comes from the apparatus in which the gas is prepared, and serves to fill the gasometer; the other, D, is closed by a cock during filling, and serves for the exit of the gas. At some distance from the gasometer it divides into several branches, which carry the gas to the various points where it may be required. The tube E has also a cock, which is open during filling, and closed when the cock in D is open. It is evident that only one can be open at a time. The pressure exerted by the tin cylinder upon the gas, and which may be increased by the superposition of weights, causes the escape of the gas, and may be regulated by the counterpoise, C.

To produce a regular stream of air, bellows and blowers are employed. A common *bellows* is the simplest means of producing a strong stream of atmospheric air. This consists of an air-tight leather or wooden box, whose inclosed space may be increased or diminished; air passing in through one small opening during the former, and passing out through a second aperture during the latter. A simple bellows of this kind cannot produce an uninterrupted stream of air, as it acts only intermittingly. To produce a continuous blast, a double or compound bellows must be employed, as represented in *fig.* 6 This consists of two sections, *a* and *b*. Press down the lower plate or the section *b*, and the air enters through a valve; press the plate up again, and the air compressed in *b* opens a valve between the two, and passes into the upper division, where it is compressed by superincumbent weights, and must escape through the opening at *c*.

These bellows are only used by hand, or at most in small forges and organs. If a very powerful and intense stream of air be required, as, for instance, in smelting furnaces, &c., large blowers are employed, driven by steam or water power. These form a kind of condensing air-pump, excepting that they have an escape aperture. The most convenient and generally employed of these contrivances is the *cylindrical blower* represented in *pl.* 19, *figs.* 2 and 3. A is a cast iron cylinder, in which a piston, *cc*, fitting air-tight, may be moved up and down by a piston rod, *a*. Through the upper valve at *b*, and the lower at *d*, the inside of the cylinder is in communication with the external air, while the valves at *f* and *g* unite the cylinder with a four-cornered box, E. At all the openings are valves, of which those at *b* and *d* open inwards; those at *f* and *g*, outwards. When the piston descends, it closes the valve at *d*, while the air penetrates through the opening *b* into the upper part of the cylinder. The reverse takes place

when the piston rises. The air compressed in the box E, which serves as a reservoir, pours out through a tube attached at *m* to the fireplace.

To maintain a uniform stream of air, which is necessary in most smelting processes, *regulators* of various forms are employed. One of these, represented in *fig.* 4, depends for its action upon the pressure of water introduced, whence it is called a *water regulator*. It is very similar in its nature to the gasometer previously described: E is a box consisting of iron plates screwed together, containing from 30 to 40 times the volume of the cylinder of the blower, and into which the air pours from the cylinder through the tube D, escaping again through C. The entire box, E, is suspended equably in a cavity of masonwork or iron plates, so as not to touch its bottom. This cavity is partly filled with water, which completes the box E. In a state of equilibrium, the water will stand at the same level in both vessels; when, however, air is introduced into E from the blower through D, exit through C being for the time prevented, the surface of the water in E must become depressed to *rr*, while it rises to *vv* in A. Upon the difference of these two surfaces depends the amount of pressure experienced by the air in E, and consequently the force of escape through C; which escape is rendered uniform by the regulator. If the pressure is to be increased, all that is necessary is to increase the height of the water in A by fresh additions.

It is often necessary to observe and measure the pressure existing in the interior of the cylinder, as, for instance, the case might readily occur of an escape-valve refusing to do its duty, which might result injuriously, either in a bursting of the cylinder, or some other accident. Such results can only be avoided by being able to examine at any time the interior pressure. For this purpose, the *wind measurer*, a kind of manometer, has been invented. This is represented in *pl.* 19, *fig.* 5. It consists of a tin box, air-tight and partially filled with water, through whose bottom passes a tube, *a*, which can be attached to the blower by a male screw, and through which, therefore, a communication is established between the blower and upper part of the box. With the lower part of the latter communicates a glass tube, *b*, provided with a scale, in which, at the beginning, that is, before the blowing commences, the water poured in through an opening in the cover of the box must stand at the zero of the scale. If, now, by the action of the blower, the water in the upper part of the box becomes compressed, that in the tube ascends, and by its height indicates the pressure of air in the blower. At *d* a tube is attached for letting out the water in the manometer.

We will here only add a few words respecting the laws which come in application in the escape of air. As a general rule, the same laws apply to gaseous as to liquid bodies, namely, that the velocities of efflux are as the square roots of the heights of pressure, although the latter cannot, as in the case of liquid, be determined directly by experiment. In the case of liquids we had to do with a pressure column of the same nature and density as the escaping liquid; here, however, the pressure is produced by a column of air having neither a uniform density nor a fixed limit. In general, however, the pressure exerted upon a vessel in escaping is measured by a mano-

meter with a water or mercury column, and the amount of pressure estimated by the height of the column. Supposing air subject to the pressure of one atmosphere to pour into a vacuum, we know that the pressure of one atmosphere holds in equilibrium a column of water 32 feet or 10.4 metres in height, and that the density of air is 770 times less than that of water; consequently, a column of air having this density throughout, must be 8008 metres high to maintain in equilibrium the pressure of the atmosphere, and in this case the velocity of discharge would be $= \sqrt{2 \times 9.8 \times 8008} = 396$ metres, = nearly 1300 feet.

If the space into which the stream is to pass already contain air of a slight tension, the tendency to escape is dependent upon the difference of the two tensions. Expressing by H the height of a column of air representing the difference of these tensions, and having the density of the more strongly compressed air, the velocity of discharge will be $= \sqrt{2g\text{H}}$, where g indicates the velocity at the end of the first second (9.8 metres, or about 31 feet: see page 202) [Physics 28]. The factor, H, must be developed by a series of inferences and calculations. Suppose gas to escape into the open air from a gas-burner, the pressure in the gasometer is determined by a column of water of measured height which we may call h; it is then only necessary to ascertain how high a column of a gas like that consumed in the gasometer will be necessary to hold this pressure of water in equilibrium. If we had to deal with air of mean atmospheric pressure, then for the column of water, h, a column of air of $770h$ may be taken; as, however, the gas is more condensed, the column of air need not be so high. Now, however, atmospheric air is compressed by a column of water thirty-two feet high, which pressure may be called b, while the gas has to sustain a pressure of $b' + h$, where b' indicates the height of a column of water at the barometric pressure of the same instant. The density of air at the mean pressure is therefore to the pressure in the gasometer, as $b : b' + h$; the gas is therefore $\frac{b' + h}{b}$ denser than atmospheric air, and, instead of $770h$, we must take $\frac{770hb}{b' + h}$, this being the value of H, and consequently $c = \sqrt{2g \frac{770hb}{b' + h}}$; the quantity, M, discharged in t seconds through an aperture whose cross section is m, will then amount to $ft \sqrt{2g \left(\frac{770bh}{b' + h}\right)}$. Nevertheless, here, as in the case of liquids, a considerable deduction must be made in practice, and the above result must be multiplied by a definite fractional factor. In water this is 0.64, and is constant; in gases it is variable, and can only be obtained by trial. Cylindrical and conical escape-pipes increase the amount of discharge.

The laws of friction and of lateral pressure in the conducting pipes agree as to the rest with what has been determined for liquids; and the phenomena of suction likewise take place in the motion of gases, just as in the flow of liquids.

ACOUSTICS; OR THE THEORY OF SOUND.

a. General Observations; Wave Motion.

Before entering upon the theory of sound itself, it will be necessary to premise some observations upon the motion of waves in general, as these play a great part in this section of Physics.

Imagine a body making oscillations similar to those of a pendulum, in which, however, the relative positions of the different parts do not, as in the pendulum, remain the same; then these parts, to return to their original equilibrium, must likewise take up an oscillatory motion which differs from that of the pendulum, in that the mutual position of these particles changes every moment. Two conditions of things may here occur: either all the parts oscillate at the same instant and in the same time, or the oscillations may be propagated in different parts successively, so that one part may begin its motion when the preceding has ceased. The first case presents itself in a steel spring fastened at one end, or in a string attached at its two extremities; in the second case waves are produced, and an illustration furnished when a stone is dropped into still water. All these vibratory motions admit of various modifications in extent and rapidity; if they exceed a certain degree of velocity, their combined action produces wave movements in the surrounding medium, which are propagated to our organs of sense, and produce peculiar impressions upon them. These vibrations, within certain limits, produce waves in the air, consisting of alternate condensations and rarefactions, and are perceptible to our ears as tones; light is the impression which a vastly more rapid vibration of particles produces upon our eyes, by inducing wave motions in a peculiar elastic fluid, the ether. It will therefore be necessary, as wave motion serves to propagate vibrations, to begin with that, and first to consider water waves, whose formation and conditions may be directly observed by us.

If a stone be dropped into water, it forms concentric circular waves, which consist of alternate elevations and depressions, in whose advancing motion the individual particles of water do not take part, as is shown by the fact that a floating body, although rising and falling, yet remains in the same place on the water. When regular waves are formed, the single particles of water on the surface, during the advance of the wave, describe curves returning into themselves, which are only closed when the succeeding wave is higher or lower: in cases of great regularity the curves are circles. Let us suppose that a motion, assumed to be perfectly regular, is propagated from one side to the other over a series of water particles, twelve for instance, then, when the first particle has completed its circular motion, the twelfth will be just beginning, and each intervening particle will be just one twelfth of its course behind the preceding. By means of these different motions is produced the curvilineal form of waves, and wave arcs are formed whose summits are where the water particle has completed its cir

cuit, and begins a new one. The distance between two water particles in the same conditions of oscillation is called a wave length, and these particles have then precisely equal oscillations, while those lying on the half wave length are in precisely opposite conditions of oscillation. Other conditions occur where the motion is not perfectly regular, as then the paths cease to be circular, and frequently become elliptical, with the long diameter sometimes horizontal, sometimes vertical. If the horizontal diameter $= 0$, the particles oscillate only at right angles to the direction of the waves, and it is motion of this kind that propagates waves in a stretched cord. A cord wave, when reaching a certain point, is thrown back again, and may traverse the same route several times: two waves again may easily meet, and by their combination produce a standing wave.

Let us now examine the character of the motion of a cord or string during a standing vibration. A standing vibration of a string may be readily produced by taking one not too tightly stretched, and, drawing it out of the position of equilibrium, letting it go again. All parts will be simultaneously on one or the other side of the position of equilibrium,—they will be simultaneously at their maximum of distance from this position, the amplitude of oscillation only being different for each particle. The oscillations of a tense string when brought out of its equilibrium, or when disturbed by a bow drawn across its middle, are of precisely the same character; they are so rapid, however, as to be indistinguishable to the eye: they therefore give a tone. The standing vibrations in a string can also be shown by attaching one end, and with the other held in the hand, describing small circles, in which case the vibrations will form a great circle in the centre: accelerate the motion of the hand, and there will be in the middle of the string a point of rest, each half swinging as the whole did previously. *Pl.* 19, *fig.* 51, represents these vibrations: *a* is the point of rest; the nodes, *ab* and *ac*, are the vibrations or bellying of the string. Two nodes and three bellyings even may by a still greater velocity be produced. There is a better mode of observing these nodes than the one just mentioned: take a stretched string, *bc* (*fig.* 52), and place a rest at *a*, so that $ab = \frac{1}{3} bc$, and draw the bow of a fiddle across the smaller portion; the other portion will be set in vibration, and in such a manner that at the middle point there will be a second node, and consequently two bellies formed. The position of the node may be shown by its being the only point along the string where a small bit of paper laid across will not be thrown off by the vibrations of the string. Place the rest at one quarter of the length of the string, and there will be in the larger portion two nodes and three bellies.

It is not strings alone that vibrate in this manner: plates, bells, and smaller bodies may also be set in vibration, and exhibit certain vibration nodes. To cause such bodies to vibrate, the apparatus, *pl.* 19, *fig.* 62, is employed, in which the plate of wood, glass, or metal, is laid upon the lower small cylinder, and then firmly fastened by means of the upper screw and a piece of cork. Set the plate into vibrations, which is best done by drawing across it a fiddle-bow, and the nodal lines and vibrating portions will be rendered evident by strewing over the plate fine sand or lycopodium. The

powder is thrown up into the air when it falls upon the vibrating portions, and finally accumulates on the nodal lines, or lines of no vibration. They remain constant, therefore, and form the well known sound figures, first discovered by the eminent natural philosopher, Chladni. By taking sand moistened with gum water and finely pulverised, and placing a damp piece of paper on the plate, the figures may be removed and rendered permanent.

Different figures result with a variation of the point of support of the plate, the rapidity of the vibrations, and the point of application of the vibrating cause; of the hundreds fixed by Savart in the manner described above, we shall represent a few (*figs.* 63–74). The simple cross is produced when the plate is fastened in the middle and intonated at one corner; if the latter take place at the middle of one side of the plate, the cross (*pl.* 19, *fig.* 71) is formed, &c. Other of the four-sided figures represented, are obtained by preventing the vibrations of one or more points of an edge of the plate, in which case several nodal lines are formed; symmetrical figures, however, are always produced, as the vibration which is hindered on one side ceases also in the corresponding parts of the other three. Triangular and polygonal plates give similar results. In circular plates very different tones may be produced, and each tone has its proper figure Here may be distinguished three kinds of figures: diametral, concentric, and mixed. The diametral figures are obtained in a manner similar to the method employed for *figs.* 63 or 71, and the nodal lines are then radii. In the concentric the nodal lines form concentric circles, and are obtained by piercing the centre of the plate, drawing the hair of the bow through the hole, and thus producing the intonation. The plate then needs only to be supported in some of the points through which the nodal lines are to pass. The figures of the mixed system consist of diametral and concentric more or less curved lines, as seen in *figs.* 75–83, and are obtained by fixing the plates in the centre, and pressing the figures upon the points through which the nodal lines are to pass. Stretched skins or membranes act in the same manner as the plates, and Marx has exhibited the sound figures of these by means of his instrument, the Eoline.

Normal vibrations occur in bells as in plates; and here also nodal lines are formed, which are, however, sometimes exceedingly irregular. To render these vibrations visible, we make use of a large wine-glass with a foot (*fig.* 84), filled with water or mercury, and intonated on the edge. There are then formed two very evident diametral nodal lines, between which the fluid remains in constant vibration, sufficiently violent at times to throw up drops into the air.

In vacuo nodal lines are obtained which do not always agree with those formed in the air, particularly when the powder employed is very light, as lycopodium.

Plates, bells, &c., which do not possess equal elasticity on all sides, likewise form peculiar figures, which, however, cease to be strictly symmetrical.

b. Transmission of Sound through the Air.

By the vibration of a body a wave motion is communicated to the surrounding air, and this it is which brings the tone, arising in the vibration, to our ears. Not air alone, however, but every elastic medium, can propagate sound; in a vacuum this propagation does not take place. Of this fact we may be convinced by placing a small bell, moved by spring clock-work, and isolated by being set on a woollen mat, in the receiver of an air-pump. Cause the hammer of the bell to commence striking, and with an increasing rarefaction of the air, the sound will become fainter and fainter, until it disappears almost entirely. Re-admit the air, and the sound will be again audible, becoming more and more distinct. Saussure found, that on the summit of Mont Blanc, a pistol-shot made only an inconsiderable sound; and Gay Lussac noticed, that when at a height of about 3000 feet in a balloon, his voice became less powerful. The loudest sound does not pass beyond the atmosphere, and terrible explosions might take place on the moon without our hearing anything of them. Water transmits sound very well, since divers hear at the bottom of the water, the voices of persons speaking on the shore.

The manner in which the vibrations of sound are propagated through the air, may be best understood by supposing an open tube, *bdtt'* (*fig.* 49[a], *pl.* 19), in which, from *t'b*, a piston may be moved quickly backwards and forwards. Suppose the length of the tube to be divided into a number of parts, equal to the length of the play of the piston, about in *s, a, b, c;* then when the piston is forced into *a'*, the air between *a'p* will fall into a vibratory motion, and this motion will be transmitted to the layer *ps*, when the piston has reached *p*, and will pass over into the second half to *b*, when the piston has finished its advance and commenced its return. This motion cannot, however, be uniform, for previously mentioned reasons, and we obtain the velocity in the individual parts by describing a semicircle above *sa*, the length of the play of the piston, dividing this semicircle, as at *x'* and *y'*, into equal parts, and letting fall the perpendiculars, *xx'* and *yy'*. The motion must, from the elasticity of the air, be transmitted successively to all the strata, while, if the air were inelastic, the piston would drive out all the air before it. From these considerations we may readily understand, that during the ingress of the piston, the air in *bs* becomes compressed before the motion is transmitted to *sa*. When the piston begins its return, the compression is propagated to *sa;* the strata between *s* and *b*, however, enter upon a retrograde motion, and when the piston has reached *b* again, occupy their old position. With a new action of the piston, the first vibration passes over to *ab;* while the layers between *a* and *s* are making their retrograde motion, those, however, between *s* and *b* are compressed, &c. Sound waves are consequently formed, each of which has the duration of a forward and backward motion of the piston, and consists of a rarefied and a condensed part, which then corresponds to the wave valley and wave elevation.

The velocity with which the waves are propagated through the air is independent of the velocity of the action of the piston and of the individual strata of air; as, however, experiment has shown that the velocity of propagation of air waves is independent of the time in which each individual part completes its oscillation, and the wave length is the distance by which the wave advances while a single layer makes a complete vibration, the wave lengths must increase in the same proportion as the time of vibrations of the individual particles of air. Thus, if the piston require triple or quadruple the time to make a complete backward and forward motion, the wave lengths will be three or four times as great.

We have thus considered the transmission of air waves in tubes: in the open air they must be transmitted in precisely the same manner in all directions.

The impression produced upon the ear in this motion of air waves is very different according to their character. If the motion be produced by a single blow, and this not repeated, as in a pistol shot, where thus the air is suddenly and powerfully condensed, and then advancing as before mentioned, we hear a *report;* in regularly successive vibrations we hear a *tone;* and if the successive vibrations become more and more irregular, we have a *noise.* The tone itself will be higher as the length of oscillation or the wave length is shorter: it becomes stronger or more intense as the amplitude of oscillations in the sounding body is greater, for so much the greater is the degree of condensation and consequent rarefaction of the air waves.

The velocity with which tones are transmitted through the air is constantly the same, whether they be high or low, strong or feeble. Experiments were instituted in 1822 by the *Bureau des Longitudes,* accurately to determine this velocity, whence it resulted that sound travelled 340.88 metres, or about 1050 Paris feet in a second. During these experiments the thermometer stood at 60°F., the barometer at 756.5 millimetres, and the hygrometer at 78°. Experiments recently performed by Sir John Herschel give 1125 English feet per second as the rate of transmission at $62\frac{1}{2}$°F. Above $62\frac{1}{2}$°F., each degree adds 1.14 feet to this velocity, and below this temperature the velocity is diminished in the same ratio.

As light travels faster than sound, it will be readily understood why the flash of a gun may be seen before hearing the report, and the lightning be observed long before the thunder reaches us, the interval depending upon the distance at which the phenomenon takes place. But for the numerous corrections required by the varying temperature, density, and hygrometric condition of the air, it would be an easy matter to determine the distance by this interval.

c. Reflection of Sound.

Whenever a sound attempts to pass from one medium into another, as from air into water, or from one gas to another, it experiences a partial reflection; this, however, is strongest when the sound strikes against a solid

body; and when the body possesses very little elasticity, the reflection may be total. In this latter case, the law that the angle of reflection is equal to the angle of incidence prevails; in the former, while one part is reflected according to the same law, the remainder is transmitted.

Upon this law of reflection depends the phenomenon called the *echo.* When sound strikes the reflecting surface at a right angle, it is thrown back again, and the quickness of the return depends on the distance from this surface. If this amount to 1125 feet, then the sound will complete its advance and return in two seconds: the tone will then be again heard after this time. As many syllables will be reflected by the echo as can be spoken in this time: the number may amount to seven or eight. The number of syllables repeated by an echo does not depend, then, so much upon rapidity of utterance, as upon the distance of the reflecting surface. At sea it has been found that even clouds have served as reflecting surfaces, so that it would seem as if the surface struck need not necessarily be a solid body.

An echo often repeats the same syllable several times, this being produced by successive reflections of the same tone from different surfaces, or from two surfaces parallel to each other. Thus, from the top of the Rosstrappe in the Harz, the discharge of a pistol gives a manifold echo resembling rolling thunder.

Here belongs the echo which returns a tone to a given spot, so as to be inaudible at a very short distance from it. Suppose an elliptical dome (*pl.* 19, *fig.* 93), *aba'*, whose foci are f and f'. A word spoken at one focus will be reflected to the other, and will be inaudible in the space between f and f'; a light whisper will be understood even if the distance between these points amounts to 80 or more feet. This phenomenon depends upon the fact that if lines be drawn from f and f' to i and i', and any other points of the curve, these lines drawn to any one point will always make the same angle with the perpendicular at this point. Another phenomenon, such as occurs in the Rathskeller in Bremen, where the ticking of a clock in one corner of the arch is heard in the other, depends upon the fact that the flutings used to ornament the arches supply the place of tubes, which propagate sound better than the open air.

The construction of rooms for public speaking or music, involves to a great extent the principles of the reflection of sound; into all such constructions the parabola enters, or should enter, very largely, as a sound produced in the focus of a parabola is reflected in every direction with the greatest possible uniformity.

d. Formation of Musical Tones.

If we have a tube closed at one end, at the open end of which a sound wave enters, this latter will be transmitted to the other extremity and there be reflected. Standing vibrations may then be formed in the tube itself by the opposite action of the reflected and re-entering wave, as all the single strata in the tube begin their motion at the same time, attain at the same

time the maximum of their velocity, and likewise reach simultaneously the terminus of their path, again to recommence in an inverted order. In standing wave vibrations of this character, the air is condensed uniformly in the tube when the single strata of air pass their position of equilibrium with the maximum of their velocity: if the particles have arrived at the extreme points of their course in their oscillation towards the closed end of the tube, the greatest condensation here takes place. If, now, they begin to return after half an undulation, a rarefaction takes place at the closed end of the tube; at the open end there is neither a marked condensation nor rarefaction. When the tube has an opening in any part of its length, the formation of a standing wave experiences an interruption, since in the moment of greatest condensation the air can escape, and can enter during the rarefaction; this circumstance operates less as the aperture is nearer the open end, since here neither the condensation nor the rarefaction is so great as to exercise any material influence. Cutting off the tube at this place would produce the same effect, and the sound waves would thus be no longer than from the beginning of the tube to the orifice.

The formation of a standing air wave depends, then, upon the relation between the length of the tube and the wave length of the incident tone; it is also essential to the formation of a standing wave in the tube, that close to the bottom the amplitude of oscillation shall become almost nothing; that there the alternating condensations and rarefactions shall take place, while at the open end they must not occur. To this end the distance of the opening from the bottom of the tube must be $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, $\frac{7}{4}$, &c., of a wave length, and we then obtain in the tube vibration nodes similar to those which we have already found to exist in strings and plates.

To put the air in a closed tube into such vibration, we need only bring an oscillating body near the open end of the tube, which shall give such a tone that the length of the tube has one of the above proportions ($\frac{1}{4}$, $\frac{3}{4}$, &c.) to the wave length. If, for example, a vibrating tuning fork be placed about two inches above the open extremity of a glass tube closed below, then if the latter is of the proper length, the two will become resonant, in which case the strata of air contained in the tube will be put into a condition of standing vibrations. By this means the tone of the tuning fork is increased considerably in intensity. If the tube be too long for the sounding body, it may be shortened to the proper length by pouring in water. Instead of the tuning fork, one of the glass plates used in the production of sound figures, or a glass bell, may be intonated with a fiddle-bow before the opening. Savart constructed for this purpose the apparatus represented in *pl.* 19, *fig.* 92. It consists of two wide tubes, movable one within the other by means of a screw, by which the sound tube may be lengthened or shortened at pleasure. Before the opening of the tube stands a glass bell which can be sounded by means of a fiddle-bow. Bringing the tube to the proper length, the sound of the bell will be much increased in intensity; removing the bell from the vicinity of the tube, by sliding it along the groove in the base of the apparatus, the tone will become remarkably thinner.

The air within a tube may also be put into standing vibrations by causing a current of air, flowing past the tube, to break against the edges of the opening, waves being thus produced which are reflected from the bottom, and interfere with those subsequently created. In narrow tubes the air may be set into standing vibrations by bringing the open end of the tube against the lower lip, and blowing into it obliquely against the edge. The tones will be deeper in proportion to the length of the tube, and inversely. The so-called Pan's pipe is an illustration of this condition of things.

Upon the principles just explained depends the construction of organ pipes, which are made principally of wood, in which case they are four-cornered, or of tin, when they are made cylindrical. *Figs.* 53 and 54 represent the form of the wooden, and *figs.* 55, 56, and 57, that of tin pipes. Such a pipe consists of the foot or pedal, *p*, the labium or mouth-piece, *b*, *b'*, and the tube. The pedal is hollow and sharpened to a cone below, to place it in the sound-board from which the pipe receives the air, which is to produce in it the vibrations of sound waves; above the widest part of the pedal is placed a bridge, *l*, which contracts the opening to a very fine slit, and thus directs the entering column of air against the sharp edge of the labium. The pipes themselves are supplied with air by means of a pair of bellows, a very convenient apparatus for which is exhibited in *pl.* 19, *fig.* 58. Between the feet of a small table, *ss'*, is attached a bellows, set in operation by the foot-board, *p*, and forcing its wind into the superincumbent wind box, which sends it through the tube, *ff*, into the upper sounding-board, *cc*. As this wind box, by continued motion of the bellows, will soon become full, if little air is used, a lever connected with a valve in the wind box strikes against a pin attached to *ff*, and thus lets out the superfluous air. The rod, *tt'*, serves to give greater pressure to the wind box where a sharper current is required. In the upper floor of the sound-board are several holes, *oo'*, generally twelve, in which pipes may be inserted. These holes are always closed with valves, which may be opened by a register at *hh'*, upon which the air can enter into the pipes and cause them to sound. With a feebler wind the same pipes give a lower, and with a stronger a higher tone.

Not covered tubes alone, or those closed at the upper end, can be thus intonated, but also those open above, and in precisely the same manner. In these the short and narrow tubes will always give the higher tone. Another method of employing open tubes consists in generating hydrogen gas in the apparatus represented in *fig.* 91, letting it escape through a fine mouth-piece, and after setting it on fire, placing a tube, *ab*, over the jet.

Standing vibrations arise in an open tube, from the circumstance that a greater condensation takes place in the middle, the particles of air not being able to escape; as soon as this condensed portion comes to the open end of the tube, the particles expand, thus producing a rarefaction, which, sent back, traverses the tube in the opposite direction. As, however, at the open end a condensation and a rarefaction coincide, no vibration nodes can here occur, these necessarily existing in the inner portions of the tube; if, therefore, the deepest tone of an open tube is to be equal to that of a closed, the former must be twice the length of the latter.

If holes capable of being closed by a slide, are made in different parts of an organ pipe, it may be shown that the tone remains unchanged if the opening exists at a belly, while another tone is produced if the opening is made at a vibration node.

However little the influence exerted upon the tone of a pipe by the direction in which the current of air strikes the mouth-opening, so much the more considerable is the effect produced by the shape of the labium, and the height of the air-hole.

The walls including a vibratory mass of air exert a great influence upon the tone, and a pipe constructed of poor tin, or of soft or resinous wood, gives constantly a smothered feeble tone; even moisture upon the wood produces the effect of lowering the tone.

With regard to the musical notes produced by organ pipes, let us call that tone produced by a pipe four feet long, the fundamental note C. If we examine the pipes whose tones harmonize with that of C, we shall find that the rapidity of oscillation of notes produced by them, stands in a simple relation with that of C; the pipes will therefore be $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, &c., the length of C. A pipe of half the length gives then the octave; that whose length is two thirds, and which makes three oscillations to two of C, is the fifth; three fourths the length gives the fourth; four fifths of the length gives the major third; and five sixths the minor third. The intermediate tones are obtained by taking one of the pipes in question as the fundamental tone and finding its accord. Thus we obtain for the G accord the fifth D, if we take a pipe two thirds the length of G, and the major third H with a pipe of four fifths, and the minor third B with one of five sixths the length of G, &c.

The deepest tone in music is that C given by a covered pipe of sixteen feet in length, or an open one of thirty-two feet. We know, however, that for the deepest note of a covered pipe, its wave length must be exactly one fourth of the wave length of the tone; in the open air, therefore, the wave length of this amounts to 64 feet. Sound travels about 1050 German feet in a second, hence it follows that to produce this deepest note there must be $\frac{1050}{64}$ or 16.4 oscillations in a second (more correctly, perhaps, $\frac{1125}{64}$ or 17.5). We obtain the number of vibrations necessary to bring out the deepest tone of any covered pipe, by dividing 1050 by four times its length. Thus the C's forming the six lower octaves make respectively 16.5, 33, 66, 132, 264, and 528 vibrations in a second. The greatest number of vibrations observed in a second amounts to 24,000; the tone thus produced is, however, scarcely audible: the deepest audible tone is that produced by 7.8 vibrations. Still higher and deeper tones may perhaps be produced and rendered audible by artificial means.

The length of pipes gives a ready method of determining the number of vibrations: this is nevertheless not entirely exact, and Cagniard de la Tour has invented a special apparatus by means of which the absolute number of vibrations in a tone can be accurately determined. This instrument is repre-

sented in *pl.* 19, *figs.* 59—61, where t, t', f, f', is a round box of brass about two or three inches broad and one inch high, whose upper surface is perfectly plane and well polished; there is an opening in the middle of the bottom ff', into which the air tube, gg', is screwed. In the bottom, tt', represented from above, and laterally in *fig.* 60, a number of holes equidistant from each other are bored, their interspaces being somewhat greater than the diameter of the holes, which generally amount to ten; pp' is a movable plate, ground upon the plate tt', and provided with holes corresponding in size, number, and position, with tt', so that by turning pp' about its axis, x, on tt', all the holes may be simultaneously opened or closed. At the upper extremity of the axis x there is an endless screw, catching in a wheel, rr', of 100 teeth; ec' is a second wheel of 100 teeth, standing in such connexion with the first that it completes only one revolution while the first makes 100, an arm on the axis of the first wheel pushing the second forwards by one tooth at each revolution. The axes of these two wheels carry indices, which mark on the dials attached to the side plate (as represented in *fig.* 61) the revolutions and their fractions. To start this part of the machinery, or arrest its motion at any moment, the axis of the wheel rr' is united in such a manner with the buttons b and b', that this wheel can either be caught in the endless screw or separated from it. The apertures in the plates tt' and pp' are directed obliquely to the surface, so that the air rushing through gg' is capable of causing a rapid rotation of the plate pp'. Suppose, now, that in the movable disk there are ten holes, and in the other only one, then this would be opened and shut ten times in a revolution of the plate: there thus arise ten complete sound waves in one revolution, of which there may be 1, 10, 100, &c., in a second, so that all the tones may thus be produced. The lower plate has, however, ten holes; and as each one exerts its influence, there is produced a strong lasting tone.

To count vibrations with this instrument (called by its inventor the Siren), place upon the sound-board (*fig.* 58) a concordant pipe, as the a of the common tuning fork, and near it the siren in another hole of the sound board. Allow the air to enter, and regulate the pressure upon the wind box by the rod t, until the two are in unison; then couple the wheel of the siren, and allow it to revolve a certain time by a seconds-watch. Stop the motion of both watch and siren, and from the latter may be obtained the entire number of revolutions, and from the former the number of seconds; comparing the two will give us the number per second. We shall then find that in one second 440 revolutions have been made, which is really the number of vibrations for the tone a of the tuning fork.

The vibrations of strings are much too rapid to admit of their being counted; they are even visible only in the longest and deepest strings. It was known very early that the tone of a string was higher the more the string was stretched, or when it was shortened. It was not possible, however, to indicate by means of calculation the connexion between the tone of a string, its tension, its length, and the rapidity of its vibration. The eminent philosophers, Taylor, the two Bernouillis, d'Alembert, and Euler,

occupied themselves with the investigations of this relation; Lagrange, however, was the first fully to elucidate it. The propositions established by him are the following: 1. The number of vibrations of a string is inversely as the length, that is, half the string makes twice the vibrations of the whole, &c. 2. The number of vibrations is proportional to the square root of the stretching weights, that is, four times the weight produces twice the number of vibrations. 3. The number of vibrations of cords of the same material is inversely as their thickness, that is, a string half as thick as another makes twice the number of vibrations in the same time. 4. The number of vibrations of strings of different material is inversely as the square roots of their densities: thus, taking a string of copper whose density is 9, and a string of catgut whose density is 1, their diameters and lengths being equal, the latter will make three vibrations in the same time that the first makes one.

The *Monochord*, invented by Savart, and represented in *pl.* 19, *fig.* 50, is used for determining the laws of oscillation of stretched strings, and their tones. It consists principally of a hollow box, ss'. At c is a bridge with slits in which the strings are fixed, which then pass over the two bridges, f and m, and beyond m may be stretched by weights. A third bridge, h, may be moved along under the strings without touching them, and any point of the string may be pressed down upon it by means of a binding screw. By moving along this bridge, all the notes of an octave may be produced, and we shall find that the lengths for a fundamental note $c = 1$ are in the following proportion: $c = 1$, $d = \frac{8}{9}$, $e = \frac{4}{5}$, $f = \frac{3}{4}$, $g = \frac{2}{3}$, $a = \frac{3}{5}$, $b = \frac{8}{15}$, $c = \frac{1}{2}$, the same ratio that is found to exist in organ pipes. These ratios confirm, at the same time, Nos. 1 and 2 of the propositions just adduced; for to obtain, for instance, the octave of the fundamental note by tension, it is necessary to attach four times, and for the fifth, nine times the weight, &c.

e. Of Longitudinal Vibrations.

Strings and rods have not only transverse vibrations, such as we have already considered, but they also vibrate longitudinally, like the air inclosed in a tube. This is shown by rubbing a glass tube longitudinally with a damp finger, or drawing a fiddle-bow across it at a very acute angle. The same takes place in massive rods of glass, metal, or wood, although here it becomes necessary to make use of a piece of rag, sprinkled with powdered rosin. It is, however, more convenient to make use of a so-called sounding rod, namely, a short glass tube whose axis is made a continuation of that of the body to be set into vibration. Vibrations produced in the first by rubbing with a damp cloth, will then be communicated to the second, and the two will vibrate together. Straight rods held in the middle and free at the extremities vibrate like open tubes; and all rods of equal length, whatever be their thickness, give the same tones. Nodal lines are also

formed on the rods, consisting of the points of rest formed by the individual molecules during their motion produced by the vibrations. These nodal lines form peculiar curves, which exhibit a certain similarity to a greatly elongated helix, forming a node at each revolution. The inner surface of a vibrating tube presents nodal lines similar to those of a rod. In prismatic rods the nodal lines are more complicated.

f. Tongue Work and Reed Pipes.

Any thin plate set into vibration by a current is called a *tongue*. Thus, in *pl.* 19, *fig.* 95, *ll* is a tongue, which, by means of a small screw, is so attached to a plate that it can vibrate in the little aperture, *nbcd*, without touching the edges. The plate may be of brass or zinc; the tongue, *ll*, should be a very thin elastic slip of brass. The current must be directed against the free end of the tongue, *ll;* sounding vibrations are thus produced by the alternate opening and closing of the aperture, whose length depends upon the number of vibrations of the tongue. The accordeon is a combination of several tongues, yielding the successive notes of the scale; these are placed upon a sound-board, and played upon by air from a bellows. Each tongue has its valve, which may be opened by a stop, and the air thus admitted to produce vibrations in the tongue.

The tongue-work in an organ has a similar construction, although the attachment is somewhat different. *Fig.* 97 represents the arrangement on a large scale. The tongue-work consists of a pedal, *p*, in which is a hollow channel, which appears above as a round hole. This channel is closed by the plate *r*, in whose opening is the tongue *l*, to be vibrated by the air passing through the channel. To tune the tongue its length must be changed, for which purpose there is a tuning-wire passing through the pedal, and by its two extremities pressing the tongue against the plate. The vibration of that part of the tongue between the plate and the wire is thus prevented.

This tongue-work is combined with the pipe, *t* (*fig.* 96), in such a manner, that the air entering through the pedal of the pipe presses against the tongue, setting it in vibration. It then escapes through an opening in the head, *t'*. When the pipe is used, by way of illustration, a glass plate lies before *ab*, to exhibit the action of the tongue. Sometimes, and generally in organs, the tongue-work is placed in the pedal, and the tube is then directed upwards.

The form of the tube gives character to the tone; thus the trumpet works have tin tubes widening above, &c. In such tongue-work, however, the vibrations of the tongue depend upon the motion of the column vibrating in the long tube, and the tongue is more vibrated than if it made entirely independent vibrations.

g. On the Beats of Tones.

If two tuning-forks of very nearly the same pitch, or two strings or pipes of almost precisely the same tone, be sounded simultaneously, we shall hear a variation of the tone, consisting in an alternate increase or diminution of its intensity. This is caused by the fact, that both sounds are produced by undulations of very nearly, but not quite the same rapidity, so that at one time these will come together in the same phase of vibration, and at another time in opposite phases. In the first case the intensity will be double that of a single sound; in the latter, no sound whatever would be perceived but for the momentary persistence in the ear of the sound of the instant previous. The tone will consist then in a gradual increase or diminution between these extremes. The greater the difference in the rapidity of undulation, the more frequent will be these *beats;* when the two instruments are in unison they cease entirely. Any number of strings may thus be brought into unison by tuning until the beats are found to have disappeared altogether. When two sounds are heard, of which the vibrations stand in a simple ratio to each other, as of two to three, three to four, or four to five, and in which the coincidence of two impulses or undulations recurs with sufficient frequency, a third sound is produced by this coincidence, always deeper than the primary notes, and generally the fifth or the octave below the lower of the two. These are called tones of combination, or the accessory sounds of Tartini, and must not be confounded with harmonic notes.

h. Sound in various Media.

Sound diffuses itself through all ponderable matter, although with various velocities. Newton gave an expression for the motion of sound in the air, which was much too small, being but about five sixths of the actually observed velocity; Laplace explained the difference by showing that a motion of sound cannot take place but by compression of the molecules of the air, during which, in all cases, there must be a development of heat; and that then the heat, now become sensible, must influence the law of elasticity in such a manner as to bring about an acceleration in the transmission of sound. Consequently, temperature would influence the motion of sound, as we find to be actually the case. Laplace has given a formula for the rapidity of this motion in vapors and gases; according to him, $v = \sqrt{gmb\,(1 + at)\,k}$, where v is the velocity in a second; g, the accelerating force of gravity, 386.29 inches; m, the ratio of the density of mercury to that of atmospheric air, found by experiment to be 10.466, at a temperature of 32° F., and a barometric pressure of 29.927; b, the standard height of mercury in the barometer; a, the constant co-efficient of expansion, ascertained by experiment to be .00208; and k, the square-root of the quotient, which is found by dividing the number which expresses the specific heat of the air (or other gas) under a constant pressure, by that

which expresses its specific heat under a constant volume. The value of k for atmospheric air has been found to be 1.421, hence $\sqrt{k} = 1.192$, and substituting the various values in the above formula, it becomes $v = 916 \times 1.192 \ (1 + .00104t) = 1092 + 1.14t$, where t is the number of degrees above 32° F. The velocity of sound in the air is therefore dependent upon the temperature, and not upon the pressure of the atmosphere. From this formula the velocity of sound in other gases may be determined, whenever the value of k is known, or k may be determined from the known velocity.

Since sound depends upon condensations and rarefactions, and such media alone can propagate it as are capable of this, it follows that this velocity of sound in fluids depends upon their compressibility. This compressibility must be obtained by direct measurement, for which purpose Oersted invented the *Piezometer*. By the use of this instrument and calculation, it has been found that in water of 54° F., the velocity of sound in a second amounts to 4630 feet; direct experiments by Colladon, in the Lake of Geneva, have given results indicating a velocity inferior to the above by less than sixty feet.

The same principle holds good in general for solid bodies. Chladni and Savart have instituted very extended experiments on this subject, and have found that this velocity is universally greater than in the air, being least, however, in whalebone, where it amounts to $6\frac{2}{3}$ times, and greatest in deal, where it is 18 times greater than that in the air.

If several solid bodies be united together, sound is transmitted with great facility throughout the whole mass, and, arrived at the extremity, the sound waves partly pass into the contiguous medium, whether fluid or gaseous; they are, however, partly reflected, and form their standing vibrations with the re-entering waves. If, however, the whole system of bodies is set into vibration simultaneously with each individual point, they lose their individual character in a great measure by this union. Upon this circumstance, among others, depends the variety of musical instruments, and this is the reason why, for example, two equally proportioned pianos may exhibit a very different character with respect to sound and tone.

Although vibrations are readily transmitted over a system of uniform bodies, solids for instance, this takes place with more difficulty when the bodies are different, as from solids to fluids or gases. Here the vibrations of the sounding body must be communicated to another, for the purpose of being increased in intensity: in other words, its vibrations are strengthened by *resonance*. An example has already been given of the strengthening of sound by a tube; another is to be found in the *sounding board*, where the vibrating strings are brought into contact with a large thin surface easily set into vibration.

In a similar manner bodies may be set into vibration by a sound wave in the air, as a door, a window, and even strings themselves. Here the sound waves in the air, started by the vibrations of a solid body, or even the original vibration of the air itself, come in contact with the body, causing it to vibrate in concert. Savart has ocularly demonstrated such sympathetic

vibrations in the shape of *sound figures*, a few of which are represented in *pl.* 19, *figs.* 85—90. These were produced by stretching a membrane over a wooden hoop or glass bell, sprinkling it with fine sand, and causing in it a sympathetic vibration, by means of an approximated tuning-fork or organ pipe. The whole series of figures here answers to one and the same tone, their different forms being produced by making the tone higher or lower.

i. Voice and Hearing.

For a description of the organs of voice and hearing existing in the animal body, we must refer our readers to the section *Anthropology*, and confine ourselves here to the consideration of the more strictly physical part of the subject, how a tone is produced and modified by the larynx. The larynx consists of four cartilages: the cricoid, the thyroid, and the two arytenoid, which are intimately connected with the windpipe and form its continuation, contracting to a mere slit, the glottis. This may be opened or closed by means of muscles attached to the cartilages forming its walls. Over this glottis lie two sack-like cavities, the ventriculi morgagnii, whose upper edges form a second glottis half an inch above the first. The whole is covered by the epiglottis, which prevents solid particles of food from entering the trachea, while passing through the œsophagus to the stomach. Various individuals, as Biot, Ferrain, and Cagniard de la Tour, have instituted experiments with caoutchouc on the formation of tones by the organs of voice; the most satisfactory, however, are those of Müller, performed with separated larynges. *Pl.* 19, *fig.* 98, represents such a larynx attached to a board, *f*, the larynx terminating with the chordæ vocales, which are stretched between *a* and *b*. *a* is one of the arytenoid cartilages (the other is behind it), *b* is the under side of the thyroid cartilage, *d* the inner membrane of the larynx which ends in the chordæ vocales, which are stretched between *a* and *b*. The upper parts are not represented, for the sake of greater clearness of the figure. If such a larynx be blown through by means of the air-tube, *u*, it gives a tone precisely similar to that of the human voice, which is strength ened, not altered, by the upper parts, which vibrate and intonate at the same time. The change of tone is produced merely by the greater or less tension of the chordæ vocales, this being effected by the action of special muscles in approximating or separating the cartilages. This motion is imitated by the strings *x* and *y*, which are loaded with weights. In this manner Müller was enabled to produce all the tones of the human organ, the higher by drawing *x*, the lower by means of *y*. In animals, the organs of voice are constructed on the same plan, but with different modifications.

The organ of hearing consists of three parts: the external ear, the cavity of the tympanum, and the labyrinth. The external ear serves by means of the concha to catch external vibrations and to convey them through the meatus externus to the tympanum, which separates the outer from the inner chamber. This tympanum is a membrane stretched over a long hoop, and to its

inner surface is attached a small bone, forming one of a connected series of four—the *malleus,* the *incus,* the *os orbiculare,* and the *stapes.* The aerial undulations are transmitted from the tympanum, by means of this series of bones, to two openings, the *fenestra ovalis* and the *fenestra rotunda,* in the labyrinth. This consists of several long excavations filled with a fluid in which the auditory nerve is expanded, passing in very fine ramifications into the *cochlea.* These various parts will be found represented in the anatomical portion of the work, to which we refer our readers.

The precise function of the individual parts of the ear is not so well established as in the case of the larynx. The tympanum, however, serves essentially in hearing by its greater or less tension, and upon its sound condition depends, to a considerable extent, the excellence of hearing. The application of the hearing tube (*pl.* 19, *fig.* 94) gives a proof of this, for in its employment the hearing is better when the sound waves received in the funnel, *cc′*, are concentrated in the tube *tt′*, and by means of the aperture *mm′* are conducted towards the tympanum. By this means the latter is set into more vigorous vibrations, and the tone strengthened without the internal portion of the ear being directly affected.

PYRONOMICS; OR, THE SCIENCE OF HEAT.

a. Expansion of Bodies by Heat.

Our knowledge of heat is limited almost entirely to its effects; of its true nature we know almost nothing. It cannot lie concealed in the interior of bodies, as in this case the refinements of modern chemical analysis would obtain some indications of its presence.

The term heat, then, is to be understood as expressing an effect; when it has reference to a cause, it will be readily intelligible from the context.

One of the most remarkable properties of heat is, that it expands all bodies; this expansion, as a general rule, increasing with the increment of heat. It is greatest in elastic fluids or gases, and least in solids.

As all bodies are expanded by heat, the amount of expansion of a body may serve to measure the degree of its heat. For a moderate range of temperature, the expansion of liquids is employed; for very elevated points, however, the extension of a solid must be substituted. Heat measures of the first kind are called *Thermometers;* of the second, *Pyrometers.*

If a glass tube with a bulb at one end be partly filled with a liquid, and if the upper part of the tube be melted together, after a vacuum has been formed in the portion not occupied by the liquid, then, by heating the ball the liquid will expand, and will rise in the tube without obstruction, owing to the vacuum above. If now the tube be graduated to a certain number of equal parts, the proportional elevation of temperature can, in every case, be determined. For filling the tube either colored alcohol or mercury may

be employed. The latter is most generally advisable, on account of its retaining its fluidity at a low degree of temperature, not vaporizing but with a considerable degree of heat. In addition to this, its expansion, without the ordinary range of temperature, is in direct proportion to the increment of heat.

The *Mercurial Thermometer* (*pl.* 19, *fig.* 7) consists of a narrow cylindrical glass tube, with a bulb blown at one end, the whole, except part of the tube, filled with mercury. The space above the mercury is a vacuum; the upper end of the tube is hermetically sealed. The filling of the thermometer is effected by atmospheric pressure. Thus, the empty tube is heated as much as possible, and the open end immersed in a vessel of mercury. A partial vacuum being formed on the cooling of the tube by contact with the mercury, a certain portion of this liquid is driven into it. If a sufficient amount be not yet introduced, the mercury already in the tube is made to boil, and, after the empty space is filled with the vapor, the tube is again inserted in the vessel of mercury as before. When the tube becomes thus completely filled with mercury at an elevated temperature, its upper end is hermetically sealed by being brought into the flame of a blow-pipe. On the contraction of the mercury by cooling, the empty space left is a perfect vacuum. The height of the mercury in the tube is measured by the scale or graduated division attached to it. This scale is constructed by fixing in the first place two points of temperature corresponding to the freezing and the boiling points of water. To obtain the former, immerse the thermometer in a quantity of finely pounded ice melting into water, and after a short time mark the elevation of the mercury upon the tube by making there a fine mark or scratch. For the latter, take a long-necked vessel filled with distilled water, and after causing the water to boil, again immerse the thermometer tube. The elevation of the mercury, after a short time, must be again marked on the tube, as being indicative of the boiling point of water. The distance between these two points, the freezing and the boiling points of water, being thus obtained, the intervening space may be divided into any number of parts. In the scale of Reaumur it is divided into eighty, and in that of Celsius or the centigrade thermometer, into 100 parts, the zero being at the melting point of ice. Graduations of the centigrade thermometer over 360° above zero, and 30° or 40° below zero, are hardly available, as these degrees are too close to the boiling and freezing points of mercury, near which the expansion and contraction are not in precise proportion to the variation of temperature.

Besides these two scales, the first of which (Reaumur's) is chiefly used in Germany, the second (the centigrade) in France, there is still a third (the Fahrenheit) employed in England and America. Fahrenheit, seeking to avoid negative quantities, obtained, as he thought, the point of maximum cold, by mixing salt and ice together; this he called zero of his scale. He divided the interval between this and the boiling point into 212 equal parts, the freezing point falling at 32°, and thus gained the advantage of having fewer fractional quantities in estimations of temperature by his instrument. There are, of course, 180 degrees between the freezing and

boiling points, so that 0° of Reaumur or Celsius (R. or C.) = 32 F. It is customary in graduating for the Fahrenheit scale, to call the melting point of ice 32°, and marking off about 70° below this point equal to 70° above it.

The measurement of temperature by means of the thermometer is exceedingly simple, all that is necessary being to bring the bulb in communication with the temperature to be measured, and marking the elevation of the mercury after it has become stationary.

As before observed, solids expand much less than liquids and gases, and must therefore be employed when high degrees of temperature are to be determined. As this expansion is of very small amount, it becomes necessary to resort to some contrivance for rendering it sensible. Now, if a rod be placed in contact with the short arm of a lever, the other being much longer, and its point serving as an index to a circularly graduated scale, then a slight expansion of the rod acting on the short arm will cause a considerable traverse of the other over its graduated scale. A better arrangement for this purpose is the apparatus of Lavoisier and Laplace, represented in *pl.* 19, *fig.* 8. A rod, *a*, of the material to be tested lies horizontally upon glass bars, one end resting against a vertical glass bar, *b*, which is suspended to a horizontal iron cross-bar, whose extremities are cemented into two massive stone pillars. The other end of the rod *a* is in immediate contact with a similar glass bar, *c*, carried by a bar, *d*, movable about its axis. To the prolongation of this latter bar, *d*, a telescope is attached, directed towards a distant scale. If, by the expansion of the rod *a*, the lower end of *c* be ever so slightly moved, the telescope will be turned, and its sight line, directed to another part of the scale, will indicate the amount of rotation. A box filled with heated water or oil is placed between the four pillars, for the purpose of heating the body to be examined, when dipped into it. This apparatus answers only for indicating temperatures below the boiling point of oil, as about 300° R = 707° F.

For higher temperatures, the apparatus represented in *figs.* 9—11 is better adapted: *f* is a strong iron plate, upon which is fastened an alidade, *ab*, turning about the point *a*. This carries a telescope, *g*, while a second telescope is fastened to the iron plate itself at *c* and *d*. A rod, *mn*, is now brought in front of the two telescopes, so that its extremities fall in the centre of the field as indicated by the cross hairs. If the rod be increased in length to *m'n*, the extremity *n* remaining fixed, the alidade must be turned until the extremity *m'* again falls in the centre of the field of the telescope *g*. The amount of this rotation is measured on a circular scale attached to the plate *f*. If the proportion between *am* and *ab* be known, then, from the arc VV' is obtained the desired extension, *mm'*, for $\frac{mm'}{VV'} = \frac{am}{ab}$. The adjusting screw, *r*, serves to shift the alidade by a very slight amount, for the purpose of adjusting the telescope *g*. For temperatures below 300°R., a copper box is used, placed upon a furnace and filled with oil. The bar to be examined is placed upon an iron support, which rests on the box. The extremities of

the bar *mn* lie opposite to two lateral apertures, closed by glass plates. For higher temperatures, the bar is placed on a support, likewise iron, in a brick furnace, in which are small holes opposite to the telescopes.

As it is in our power, from the known temperature, to determine the extension of any body, so, conversely, from the known extension of a body, the temperature to which it is exposed may be ascertained. The ordinary thermometers range only to about 360°C., or 660°F., above which mercury is converted into vapor, so that it is the melting points of such bodies only as are below this degree, such as tin, tellurium, bismuth, and lead, that can be ascertained by the mercurial thermometer. All other metals have higher melting points, and from the expansion of these it has been attempted to ascertain elevated degrees of temperature. Muschenbroek, in 1769, invented the metal pyrometer, which, in its general features, agrees with the apparatns described above for measuring the extension of a metallic bar. The pyrometer invented by Wedgewood in 1782, depends upon a different principle, namely, that of the contraction of a certain kind of clay by heat. Small cylinders of this clay were carefully measured before and after the exposure to heat, and from the difference of length the intensity of heat was determined. The great defect here, however, was, that even in the most carefully constructed cylinders, the contraction was not sufficiently uniform.

Daniell's pyrometer possesses fewer defects than any yet constructed. The indications of this instrument rest upon the difference of expansion of an iron or platinum rod, in a tube of plumbago, when extended by a great heat. A metal bar, shorter than the tube, is placed in it, and over the bar is placed a shorter bar of clay, which, placed in the opening of the tube, serves as an index by being placed upon the bar in the tube, and attached in such a manner, by means of a small plate of platinum, as to move only with a certain degree of friction.

If, now, the point be marked where, in an unheated state, the clay bar meets the tube, and the apparatus be then exposed to heat, the expansion of the metal will drive out the clay to a certain point, at which, owing to the friction, it will remain on cooling. The amount by which the clay has been protruded will give the elongation of the platinum bar. The disadvantage, in this case, is, that the extension of the plumbago tube itself cannot be determined with sufficient accuracy.

From the measured linear expansion of bodies, their cubic expansion may readily be ascertained, it being necessary only to find the coefficients for the first. The coefficient of expansion for solid bodies will be three times as great as that for these linear expansions, as these bodies are extended in height and breadth, as well as in length.

The expansion of solids by heat, and their contraction by cold, are powerful forces; for if a weight of 1000lbs. be necessary to compress a body as much as it is contracted by a diminution in temperature of one degree, then this diminution will push or pull an obstacle with a force of 1000lbs. Use has been made of this force to restore walls, by means of the contraction of iron braces, to a perpendicular from which they had swerved.

It is necessary, also, in certain circumstances, to anticipate by timely precautions acts that would arise from this property of bodies. Thus, if on a railroad the rails be laid in cold weather, with their ends in absolute contact, the summer heat will cause them to elongate, and, having no room to yield in length, to warp. The bars or rails must therefore be laid at the highest temperature, or with an interval sufficient for the greatest possible elongation. Similar cases occur in tubes for conducting steam, gas, or water, where it becomes necessary to employ special compensation pipes. The influence of temperature on pendulums and its compensation has already been referred to (p. 208), [Physics, 34]. Here belong the compensation bars, whose construction depends upon the fact, that different solids possess different expansibilities. If, for instance, two strips, one of zinc and the other of iron, be soldered together, forming a straight bar at a temperature of 20°R., then, at a temperature above this, the compound bar will become curved, and the zinc will occupy the convexity of the curve; at a lower temperature the case will be reversed, the zinc now occupying the concavity. The cause of this lies in the fact, that at equal temperatures, zinc both contracts and expands more than iron.

Upon the arrangement of compensation strips depends the construction of *quadrant* or *metal thermometers* (*pl.* 19, *fig.* 12). The strip *fgh*, consisting of copper and steel, is attached at *f*, and curves at *g* towards *h*. Against it rests, at *h*, the short arm of a lever, movable in its axis, the longer arm, *b*, being provided with the rack, *dd'*. The latter catches in a pinion moving on the central axis, whose motion is magnified still more by the needle *li*, turning on the same axis. With an increment of temperature the strip, *fgh*, becomes more curved, and the rack becomes turned in a direction from *d* towards *d'*, and with it likewise the needle serving as index. If the curvature be diminished by a bending in the opposite direction, a special spring wound about the axis produces a corresponding retrograde motion of the index. The compensation strip is so calculated, that the needle, at an increase of temperature of 80°R., shall make a complete revolution. The dial plate must be graduated separately for each instrument, by comparison with a good mercurial thermometer, and, if possible, degree by degree; as in the former the degrees are not equal, and cannot, therefore, as in the case of the latter, be described mechanically.

The most sensitive metal thermometer is that of Breguet (*pl.* 19, *fig.* 13). It consists of a spirally wound compound band of metal, formed by soldering together three thin strips of silver, gold, and platinum. This is fastened at its upper extremity to a brass arm, the lower end being free. At this lower extremity is a very light horizontal needle, whose point traverses a scale on the upper edge of a ring, supported in three feet. For protection against external influences, the apparatus is covered by a glass bell. The needle is made to turn by the unequal expansion and contraction of the silver and platinum, with change of temperature; the use of the gold is merely to unite the two other metals.

The expansion of liquids is not uniform at high temperatures, the most

even do not expand uniformly between 0° and 100° C. The elaboration of these, as well as of the actual absolute expansions, consequently always presents difficulties.

The density of a body must always be connected with its expansion, for an increase of volume always implies a decrease in density. Water, however, forms an exception to this law, for, although according to a proposition previously given, water should be of greatest density at 0° R., or 32° F., that is, at the freezing point, accurate experiment has shown, that when heated at this point it contracts, and continues to do so until the temperature has risen to 4° R., or 39°.1 F., when it is in its state of maximum density. Above this degree it expands according to the usual law. The vast importance and almost absolute necessity of this peculiarity of water will be referred to hereafter.

It has been before mentioned that mercury is most desirable for filling thermometer tubes, owing to its uniform expansion between 32° and 212° F.; to show the difference produced by irregular expansion, we have given in *fig.* 16 the rate of expansion of mercury, water, and alcohol, at temperatures between 0° and 100° C. The lowest curve represents the expansion of mercury, and appears a straight line, owing to the uniformity of expansion. The middle curve is the expansion of water. It exhibits first a contraction (to 4° C.), at 8° C. is as at 0°, and expands then in a very progressive ratio, so that at 70° C. the ratio between W and *q* is almost 2 : 1. The upper curve exhibits the expansion of alcohol. To A, or 50° C., the expansion is uniform, and consequently the curve is a straight line; then, however, the curvature increases more and more. The figure shows that water is not applicable to the filling of thermometers, and that for any other liquid than mercury, a great length of tube would be required.

We have seen that the expansive effect of heat on solids and liquids is different according to their force of cohesion, being inversely as this cohesion. In gaseous bodies, therefore, in which the cohesive force is zero, no obstacle is presented to the expansive force of heat. This must therefore be the same for all gaseous bodies, and proportional to the increment of temperature; experiments instituted for the purpose have verified this conclusion. An *air thermometer*, therefore, may be constructed by employing air perfectly free from moisture, which may be done by passing it over chloride of calcium. For this purpose a thermometer tube is prepared, on which is accurately marked the ratio between the contents of the bulb and the volume of the divisions on the tube itself, produced by the graduations. The tube is now filled like a thermometer tube, the mercury boiled, and the tube placed in a vertical position with a tube open below, and filled with chloride of calcium, fastened to its open extremity. The mercury will escape from the tube, and in its place there will enter a quantity of air, purified from moisture, by passing through the chloride of calcium. The further entrance of air must cease while there is yet a small quantity of mercury in the tube, which must remain for two purposes, to prevent the escape of the air, and to serve as an index. The point at which the mercury stands when the tube is placed in melting ice, gives the volume of

the air at zero, when the ratio between the volume of the tube included between any two divisions and the volume of the bulb is known.

The instrument is now introduced into a box filled with water heated to a temperature t (*pl.* 19, *fig.* 14), so that the tube with the index may project above the side of the box. The index will then be driven to a certain point, and the increase of volume for the temperature t may be determined. In this manner the coefficient of expansion for dry gases is found to be 0.375, which Rudberg, by means of another apparatus, corrected to 0.365. This coefficient of expansion increases with increasing pressure.

In referring previously to the specific gravity of bodies, the temperature was left out of account. This could very well be done, as the slight differences of temperature usually occurring during such determinations, exercise little influence on the density of solids and liquids. The case is, however, very different with regard to gases, where the least change of temperature produces a material difference in the density. In investigations of the density of gases, a hollow ball is employed, provided with an arrangement by means of which it can be screwed on the plate of an air-pump, there to be exhausted. A tightly-fitting stop-cock prevents the entrance of air when the ball is removed. The exact capacity of the ball must be known, which is best obtained by filling it with distilled water and then weighing it. The ball is then emptied, dry air admitted and weighed, and then again weighed after exhaustion of the air. If the experiment be performed with a perfectly exhausted ball, at a barometric pressure of 29 inches, and at a temperature of 0°C., or the results corrected to these conditions, the density of dry air, or its specific gravity, will be found to be 0.001299. Any other gas may be substituted for atmospheric air, and its density ascertained in the same manner. For this purpose, the *Pneumatic apparatus* figured in *pl.* 19, *fig.* 15, may be employed. This consists of a receiver, c, provided with a cock, d. This receiver is placed in a trough filled with mercury, a hand air-pump screwed on at d, and by the exhausting of the air, filled with the mercury. When entirely filled with the mercury, the cock is closed, and the air-pump replaced by the exhausted ball, y. The gas, as generated, is admitted through the tubes a and b into the receiver, and thence, opening the stop-cocks d and e, into the ball.

b. Effects of Heat in Changing the State of Aggregation of Bodies.

The state of aggregation of a body depends entirely upon heat, that is, whether it is to be solid, liquid, or gaseous. By heat many solids become liquid, and liquids gaseous; and conversely, by withdrawal of heat, gases may be changed into liquids, and these into solids. Sometimes the same body can be made to assume all three states in succession. Even if, in the case of certain bodies, this process has not been observed, we are fairly entitled to conclude that it is owing to the difficulty of attaining the extremes of temperature necessary for the purpose. It is thus certain that upon heat it depends whether a body shall be solid, liquid, or gaseous,

although some bodies before fusion experience chemical changes. The melting point of a body, or the temperature at which it becomes liquid, is invariable for one and the same body; as also, with certain restrictions, is the boiling point, or the temperature at which a liquid begins to vaporize. During liquefaction the temperature does not alter, however great a degree of heat may be applied; the excess of heat, therefore, becomes latent.

The opposite to melting in a body is its solidification, or the transition of a body from the liquid to the solid state. This generally takes place at the same temperature of a body as melting, all the combined heat being given out. We may be convinced of this by causing water to boil in a glass tube, and, when this is filled with steam, melting it together at the open end. If, now, the tube be cooled to about 15°F., the water will remain liquid; at the least agitation, however, it will become converted into ice, and a thermometer placed on the tube will ascend immediately from 15°F. to 32° F. As much heat, formerly latent in the water, will therefore be set free as sufficed to elevate its temperature 17° F.

The solidification of bodies takes place in different forms, according to the circumstances. If it be carried on slowly, a crystallization characteristic of each body takes place; if the cooling or solidification be accelerated, the particles have not time to arrange themselves properly, and irregular, confused formations are produced.

c. The Formation of Vapor.

If a fluid be in contact with the air, its amount gradually decreases by evaporation, or its conversion into vapor. The Torricellian vacuum is best adapted for exhibiting the laws of vaporization. In a broad vessel, VV′ (*pl.* 19, *fig.* 17), place three barometer tubes close to each other, the height of the mercury being the same in all. If some water be introduced into one of these tubes, as b', it will rise to the top, and the mercury will be sensibly depressed. This can only be produced by the giving off of a vapor which exerts an expansive force like the gases. The depression of the mercury gives the measure of the tension of the vapor. If some other fluid, as sulphuric ether, be introduced into the third tube, b'', there will be observed a much greater depression of the mercury, owing to the tension of ether vapor being much greater at the same temperature than that of water.

The elasticity or tension of vapor is increased by compression, just like air; there is, however, a certain limit or maximum of compression, above which the vapor becomes converted into a liquid. This maximum varies with the temperature, increasing with its increase. In this circumstance is a characteristic difference between vapors and gases. Suppose, in the apparatus, *fig.* 23, the upper barometer tube be filled for a few inches with mercury from which all air has been removed by boiling, and the rest with ether; now let the tube be inverted and immersed in the vessel *cn*, and the

ether will immediately rise to the top, there becoming partly converted into vapor. The mercury will by this means be depressed, the depression being produced by the tension of the ether vapor, and being in all cases greater than what would prevail in the presence of a vacuum above the mercury. If the tube be depressed still more in the mercury of the lower tube, the height of the mercury will remain unchanged, while if air were present it would increase, owing to its continued compression of the gas. The more the tube is depressed, the more the quantity of fluid ether increases, and the vapor is consequently condensed, not compressed; and this may be continued so far as to exhibit an entire condensation of the vapor, provided that no air be present. If the pressure be diminished by elevating the tube, the vapor will again be formed.

If vapor be contained in any space unequally heated in different places, the tension of the vapor in the whole space will be the same as in the coldest part, as may be shown by means of the apparatus represented in *fig.* 18. Let the bulb, *a*, be half filled with ether, and this brought to boil; after ebullition has continued long enough to drive all the air out of the bulb and the tube connected with it, quickly immerse the lower open end of the tube, *b*, in a vessel, *c*, filled with mercury. On cooling the bulb a part of the vapor will become liquid, and the mercury will ascend in the tube, until the bulb has attained the temperature of the surrounding air. If the bulb be cooled to a still lower point, the mercury will rise higher, and, in fact, to such a point, as if not only the bulb, but even the entire tube had been greatly cooled.

Various forms of apparatus have been employed to determine the expansive force of the vapor of water. This, however, at elevated temperatures and tensions, becomes very difficult. For moderate tensions, as those under 212° F., a form of apparatus may be employed, consisting of a vessel of mercury, in which are two glass tubes, the longer of which is a complete barometer, while in the shorter there is contained some water above the mercury, which is vaporized in the vacuum. The whole apparatus may be dipped in a vessel of water, and the latter heated, by degrees, from 32° F. to 212° F. Both barometers will have the same temperature, and the elasticity of the watery vapor thus formed, may be obtained for any degree of temperature, from the ratio of depression in the vapor barometer, to the height of the mercurial column in the complete barometer. When this depression is reduced to 0° we have the true tension of the vapor.

It is much more difficult to obtain the tension when the pressure exceeds several atmospheres. Quite recently, Arago and Dulong have instituted an extensive series of experiments, to obtain the elasticity of vapor at the highest pressures likely to occur. For this purpose they employed the apparatus represented in *pl.* 19, *fig.* 19, where *c* is a strong steam-boiler of plate iron, in which the steam is generated; *f*, the furnace; *y*, the grate; *t*, the tube through which the steam escapes. In the cover two gun barrels, *e* and *r*, are let in, open above and closed below, both being filled with mercury. The one descends below the water in the boiler, the other

does not reach its surface, so that the former has the same temperature as the water, the latter as the steam. A thermomete· is sunk in each barrel, with its upper end bent horizontally; this horizontal portion, as represented more clearly in *fig*. 20, is maintained at a constant temperature by a stream of water. From the boiler rises a vertical tube, *b*, in which the steam ascends, and at *u* presses against the top of a column of water which fills the tube *udb*, and the upper part of the cast iron vessel, *vv'*. This pressure of the vapor is transmitted to the surface of the mercury in *vv'*, and produces a compression of the air in the manometer tube, *mm'*, by means of which the tension of the vapor may be ascertained. To determine the varying height of the mercury in the vessel, *vv'*, a glass tube, *nn'*, is employed, communicating with both the upper and under part of the vessel; in this tube the height of the mercury may be ascertained by means of a movable slide in the graduated rod, *z*.

Observations with this apparatus are conducted in the following manner:—Water is poured into the boiler, until the gun barrel containing the smaller thermometer stands just above the surface. This is kept boiling for fifteen or twenty minutes, with the safety-valve and the vertical tube, *b*, remaining open, in order to expel all the atmospheric air. When this is effected, fuel is placed in the grate of the furnace, and all the openings in the boiler closed. Both thermometers, and the mercury in the manometers, then quickly rise to a maximum, which being attained, the height of the mercury in the above-named instrument is ascertained by two observers, and carefully noted down.

To determine from experiments already made, degrees of tension which have not been observed, or in other words, to interpolate the series, it becomes necessary to develope certain empirical formulæ for the purpose, whose results shall agree in the closest possible manner with the observations already made. In these formulæ the force of tension, E, and the corresponding temperature, T, must occur, of which one or the other must be known. Such a formula, with which the observations made by Arago and Dulong agree closely, is that of Tredgold, available to a pressure of four atmospheres, where $\log. E = \frac{23.94571T}{800 + 3T} 2.2960383$. For higher tensions, even up to fifty atmospheres, we have the formula $E = (1 + 0.7153T)^5$ where T indicates the temperature above 212° F.

Hitherto investigations have been instituted principally with reference to the degree of elasticity of the vapor of water; quite recently, however, experiments have been made with the vapors of alcohol, sulphuret of carbon, and sulphuric ether, by Ure, Schmidt, and Muncke. Bunsen has investigated the tension of some condensed gases, particularly of sulphurous acid, cyanogen, and ammonia.

The *density of watery vapor* is best ascertained by means of the apparatus invented by Gay Lussac (*pl.* 19, *fig.* 21). Upon the furnace, *f*, stands the cast iron vessel, *c*, containing mercury; in this a graduated tube, *g*, is

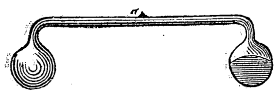
placed, about a foot in length, surrounded by the glass covering m, itself filled with an appropriate fluid. Upon the horizontal ground edge of the vessel, c, lies a small board, t, through which passes the divided vertical rod, r. Before introducing the tube, g, into the vessel, it must be entirely filled with mercury, so that after immersion it may remain filled with mercury, and contain no air-bubbles. Now introduce a small glass bulb, filled with water, and with the opening melted together, into the tube, g; it will rise to the top, and on the mercury being heated, will burst by the expansion of the water. Vapor of water will immediately form in the upper part of the tube, g, and the mercury in it will sink. When, by continued application of heat, all the water becomes vaporized, the weight of the vapor will be known, provided that the volume of water in the bulb had previously been ascertained. The volume of the water is ascertained by the divisions on the tube, g; its temperature by the thermometer; and then its tension by the graduated rod, r. This latter is pushed down until its lower extremity touches the mercury in the vessel, c; the slide, v, is brought to an equal height with the surface of the mercury in the tube, by which means the height of the latter is ascertained. This, deducted from the barometer pressure, gives the tension of the vapor.

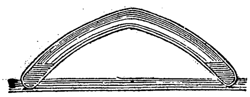
From the now known weight of a given volume of steam, which at a known temperature exerts a known pressure, the weight of any volume of vapor can be ascertained. As we have previously ascertained the density of the air to be $= 0.001299$, we can ascertain the weight of equal volumes of air and watery vapor at equal temperatures and equal pressures, and thus determine the ratio of density of the two. According to Gay Lussac, the density of steam is five eighths of that of the air. To determine the density, d, for other temperatures than those investigated, the following formula by Gay Lussac may be employed: $d' = d \frac{P}{760} \frac{(1 + 100\,\alpha)}{(1 + \alpha T)}$, where d is the density at 212°F., and a barometric pressure of 29 inches; P, the pressure, and α, the coefficient of expansion, amounting, according to Gay Lussac, to 0.00375. It is, however, assumed that vapors, like gases, follow the law of Marriotte to the maximum of tension.

The density of the vapors of various other liquids has been investigated by Dumas, Gay Lussac, and others.

Vapors are condensed by pressure and by cold; nevertheless a vapor can be compressed without being at the same time partly condensed, only when it is not saturated. Hence we are led to the conclusion that even the so-called permanent gases are really vapors which are far from their point of saturation. Davy, and particularly Faraday, have succeeded by means of great cold and pressure in condensing into liquids, and even solids, gases which had previously been considered permanent. The method employed consisted in condensing the gases by their own pressure, for which purpose an instrument was used similar in its principle to Wollaston's cryophorus for producing artificial ice, but rather more simple.

Wollaston's Cryophorus.





Faraday's Condensation Tube.

In the one side of the tube are placed the materials from which the gas is to be generated, as, for instance, cyanide of mercury, &c.; and this part being carefully heated over a spirit lamp, the gas will pass over into the other side of the tube, and there be compressed more and more, by the arrival of successive portions, until condensation ensues by placing the extremity in a freezing mixture.

d. Mixture of Vapor with Air.

When vapors and gases, or aeriform bodies in general, exercising no chemical influence upon one another, become mixed together, they do not, like liquids, separate according to their specific gravities, but each gas diffuses itself uniformly throughout the entire space, just as if the others were not present. If this were not the case, the watery vapor from streams, &c., would, on account of its lightness, speedily become elevated above the atmosphere, until, finally, all the water on the earth's surface would become converted into vapor and disappear from it. The coexistence of two gases may be readily exhibited by producing a communication between two glass vessels, as in *pl.* 19, *fig.* 22, the one containing hydrogen, and the other carbonic acid gas. The tension of the mixture, which is diffused uniformly through the whole space, is in every case equal to the sum of the tension of the individual gases, each one being supposed to fill the entire space exclusively.

That vapors resemble gases in this respect may be shown by the apparatus represented in *fig.* 23. Fill a barometer tube with mercury, allowing a small portion of the tube to remain free, and immerse it in the mercury of the vessel *cn*, upon which the air contained in the tube will expand, and occupy five times, for example, its original space. If some sulphuric ether be introduced in the manner previously explained, the mercurial column will sink still deeper; by depressing the tube, however, the space above the mercury may be brought to the same amount as before the introduction of the ether. Since the air is diffused through the same space as before, and this space contains as much vapor of ether as if no air were present, it follows that the tension of the mixture must be equal to the sum of the tensions of the air previously present, and the saturated vapor of ether for the existing temperature. This is completely verified by examining the height of the mercury above the level in *cn*.

The conversion of liquids into vapors or gases is called vaporization; it

takes place either by boiling, in which case vapor is formed throughout the whole mass of the liquid, or by evaporation, where the surface only is affected. In the first case, two conditions must be fulfilled: firstly, the heat must be sufficient to enable the tension of the vapor to resist the pressure of the liquid on the vesicles of vapor, on which account the boiling point depends upon the amount of this pressure; secondly, there must be enough heat to admit of a sufficiency being absorbed in the formation of steam. For this reason the rapidity of boiling will depend upon the amount of heat applied within a given time. Under the receiver of the air-pump, water of moderate warmth, as at 86° F., will begin to boil as soon as the air is sufficiently rarefied.

A curious experiment, relating to this subject, may be performed by means of the apparatus represented in *fig.* 24. A glass balloon, *a*, with a long neck, is half filled with water, and this is made to boil: when, by the ascending steam, all the air is expelled, the mouth is closed by a cock, *b*, and the balloon inverted as in the figure. Now, if cold water be poured on the upper part of the balloon, the water in this vessel will begin to boil violently, owing to the condensation of the vapor above the water, and the consequent diminution of pressure.

Since the height of the boiling point of any liquid depends upon the atmospheric pressure, the boiling will not only vary under different pressures at one and the same point, but the boiling point itself will be different in different countries, and at different heights above the level of the sea. Boiling water will therefore not be equally hot everywhere, as at Quito water boils at 194° F., while in the latitude and level of New York, 212° F. are required.

As by diminishing the pressure, the boiling of a liquid may be accelerated, so, also, by increasing this pressure, it may be retarded. *Papin's digester* (*pl.* 19, *fig.* 25) depends upon this principle, and is an instrument in which water may be heated far above the usual boiling point without boiling. It consists of a cylindrical vessel, *abcd*, of metal—best of brass or copper—whose sides can sustain a very great pressure, and which, after being filled, may be closed by a cover, pressed down firmly by the screw passing through the bow, *m*. The single opening in the cover is closed by a safety-valve, which may be loaded so heavily as to require a very great pressure to elevate it. If this vessel be filled with water and strongly heated, the water cannot boil, on account of the pressure exerted by the vapor which forms, and is prevented from escaping.

The lower layers of fluid, as is well known to our readers, have to sustain the pressure of all the superincumbent ones, in addition to the entire weight of the atmosphere; for this reason boiling should commence later at the bottom than at the top of the liquid. Nevertheless, the lower layers, expanded by heat, and becoming consequently specifically lighter, rise continually through those above them; the bubbles or vesicles of vapor which are formed, increase in size as they approach the surface, that is, as the pressure becomes less. This arrival at the surface takes place, however, only when the upper strata have attained the same temperature as the

lower; until this time the vesicles become condensed before they reach the top, giving out their latent heat to the upper strata.

Substances only mechanically united with water do not change the temperature at which boiling takes place; the case is different, however, if solution takes place, the boiling point being elevated. The steam formed is, nevertheless, pure watery vapor, and its temperature is precisely the same as if generated from pure water.

The generation of steam, both in respect to quantity and rapidity, depends entirely upon external circumstances, particularly upon the more or less suitable application of fuel, upon the material and form of the boiler, and upon the amount of surface coming in contact with the flame.

As boiling is a formation of vapor, taking place throughout the entire liquid, so there is still another formation of vapor, which takes place only at the surface, namely, exhalation or evaporation. This phenomenon occurs over the whole surface of the earth at all temperatures. The vapor thus formed has a certain tension, which, however, is not sufficient to overcome the pressure of the atmosphere. A chemical mixture here takes place, as between two gases, and the principal condition is, that the air be not saturated with vapor, else the exhalation ceases. For this reason evaporation does not take place so readily in a calm as during windy weather. As to the rest, evaporation is constantly in proportion to the amount of surface exposed to the air. In the section devoted to meteorology, we shall have occasion to refer more particularly to this phenomenon, and its influence in organic nature.

When a liquid evaporates, heat combines with the vapor, or becomes latent, as is shown by the fact, that whatever be the amount of heat applied, the temperature of the water never rises above the boiling point. The vapor must therefore take up the heat, even although its own temperature does not rise above the boiling point. This phenomenon may be illustrated by pouring upon the hand a few drops of ether or other quickly vaporizing liquid. A sensation of cold will be experienced, which is owing to the abstraction of heat from the hand during the production of vapor, this heat becoming latent in the vapor. The amount of heat latent in the vapor may be ascertained by allowing the vapor of a known amount of water to pass into a quantity of water, also known, and determining the temperature to which this water is elevated. Now, knowing how many units of heat, that is, how many times the temperature necessary to raise one pound of water, one degree in temperature, are required to raise the water to as many degrees as has been done by the steam, we can calculate the amount of heat which was rendered latent.

In the process of *distillation*, the steam raised from the liquid is conducted through a tube lying in cold water, and there condensed by becoming cooled. The heat given out in this process elevates the temperature of the circumambient water very considerably. The small apparatus of distillation (*pl.* 19, *fig.* 47) exhibits this very clearly. The steam generated in the small balloon passes through the straight tube into the wide one, provided with a funnel and an escape tube The water poured in through

the funnel, enters the tube cold, and passes from it heated. In the larger cooling vessels, as in *fig*. 48 (exhibiting a sectional view), the steam-tube is carried in a spiral through the vessel, in order that the steam may remain as long as possible in contact with the cooling water, and become completely condensed. The upper strata of water become very soon heated, and if the process is to be continued any considerable length of time, must be renewed. This is done best by allowing the cold water to enter below, and as heated, to pass out above through an escape-pipe.

In reality, any cooler might be employed as a means of measuring the amount of latent heat, provided that it were known how much moisture was condensed in a certain time, and how much was given off into the cooling water. Brix, however, has invented a special apparatus for the purpose, represented by *fig*. 49. The cylindrical vessel, C, of about three inches in breadth and height, served as the cooler, and the steam generated in the retort, R, entered, not into a cooling tub, but into the cylinder, EG, which had an aperture in the middle, also cylindrical. The steam entered at M, while the inside of the condenser was in communication with the open air, by the tube, L, so that the air in the condenser could escape. The cooler, C, was filled with a given weight of water, whose temperature could be ascertained by a thermometer attached to the apparatus. In the space between the vessel, EG, and the cylinder, C, was placed a metallic disk, B, which could be moved up and down by means of a wire, so as to keep the water in constant agitation, and thus maintain it at an uniform temperature. The condensing apparatus was protected from the heat radiant from the heating apparatus and the retort. The liquid passed over was determined by the quantity remaining in the retort. Brix, in this manner, found the latent heat of watery vapor to be 540 units; that of alcohol in vapor, 214 units; of the vapor of sulphuric ether, to be 90 units, &c. From this it followed, in connexion with other experiments, that the latent heat of the vapor of different liquids is nearly in the inverse ratio of the densities of these vapors.

If a liquid boils in the open air, it retains the same temperature, owing to its continually receiving from the walls of the vessel heat enough to replace that rendered latent in the formation of vapor. The case is different, however, when ebullition takes place under the receiver of the air-pump: here the temperature continually sinks, as the latent heat derived from the water itself cannot be renewed. If we place under a shallow receiver on the air-pump, a small flat metallic capsule containing water, above a dish filled with sulphuric acid, and exhaust the air, the water will undergo a rapid evaporation, which is immediately absorbed by the acid. The rapid abstraction of heat from the water during the evaporation, will reduce its temperature to such an extent as finally to cause it to freeze. In Wollaston's cryophorus (see p. 269) [Physics, 95], water is likewise caused to freeze by its own vaporization. A small quantity of water is introduced into one of the bulbs aad brought to boil. When the other bulb and the tube are filled with steam, a small aperture left open is closed by melting the glass over it. If, now, all the water be collected in one bulb, and the other be immersed in a freezing mixture,

the vapor arising from the water will be condensed so rapidly as quickly to convert the water into ice.

e. The Steam-Engine.

The steam-engine serves in general to convert the vapor of water into a motive power. As early as the year 1687 Papin constructed an apparatus, which may be considered the earliest steam-engine on record. It is represented on *pl.* 19, *fig.* 26. It consists of a glass tube with a bulb blown at one end containing some water. A piston, *p*, moves air-tight up and down the tube. If while the piston is depressed the bulb be heated, the steam will force it up to the top; then, if dipped in cold water, the steam will become condensed, and a vacuum being produced, the piston will be depressed by the incumbent pressure of the atmosphere. Papin employed an iron cylinder instead of a glass tube. Savery made the first practical application of the steam-engine: he employed it in removing water from the bottom of mines; which was also the application of Newcomen's atmospheric engine. This latter was constructed according to the principles of Papin's engine, except that he admitted cold water into the cylinder to condense the steam. Watt made the great improvement of attaching a receiver, separate from the cylinder, to condense the steam. To him we also owe a great number of other important improvements; and with justice he is considered as the inventor of the steam-engine in its present perfected form.

A sectional view of Watt's steam-engine is exhibited in *pl.* 19, *fig.* 27. Here A is the cylinder, air-tight below and above, in which the piston, C, moves. The steam, generated in a boiler, enters through a pipe, Z, and thus is introduced into the cylinder alternately at the upper and lower ends at E and O. If it enter above, as in the figure, the steam beneath the piston escapes at O, and enters the condenser, I, through the pipe H, where it is condensed. There is thus a rarefied space beneath the piston, which must consequently descend when pressed on by the steam above. The condenser, I, stands in a cistern partly filled with cold water; there is a pump, K, to remove the water from the condenser, and likewise the air which rapidly accumulates there. This is called the air-pump. It brings the water from the condenser into the receiver, R, whence it flows through the pipe S to be partly employed in feeding the boiler. The water required for the boiler is brought through the pipe M to a pump, and, by means of this, through the pipe M′ to the boiler. This latter pump, called the hot water pump, like the air-pump, is kept in motion by the engine itself; thus, the pump rod, L, is attached towards one end of the great beam or lever set in motion by the piston, C, and is elevated or depressed with the elevation or depression of this end of the beam. During elevation the suction valve opens, and during depression the valve *n*. On the other side of the beam, not visible in the figure, is a pump rod, by which cold water is raised in the pipe T′, and brought through the tube U into the cistern in which is placed the condenser.

By means of the piston rod an alternating upward and downward motion

is communicated to one end of the beam, and of course an opposite motion to the other end. This upward and downward or rectilineal motion is converted into a circular by the connecting rod P, and the winch Q; the axis of this winch is the principal axis of the machinery to be set in motion. About it turns also the great fly-wheel, X, which serves to maintain uniformity in the motion of the engine. This, however, is not quite sufficient. A continual diminution of the resistance to be overcome by the engine, with the same head of steam, must gradually produce an increasing, and finally exceedingly dangerous acceleration in the velocity of rotation of the fly. To set a certain limit of safety to the velocity of rotation, it becomes necessary to attach a valve to the steam-pipe, Z, whose increase or diminution of the aperture may restrain to a greater or less extent the admission of steam to the cylinder. The turning of this valve is effected by the engine itself, by means of an apparatus termed the regulator or governor. An endless string, *i*, is passed round the axis of the fly-wheel, and a vertical pulley, so that the motion of the former is communicated to the latter. To the axis of the pulley a conical or bevel-edged wheel is attached, whose teeth play in those of a similar conical wheel placed horizontally. The axis of the latter is prolonged into a rod, whose upper end carries the conical pendulum (or centrifugal regulator) V. The pendulum consists of two heavy balls, which are attached to the upper end of the vertical rod, hanging by two short rods, which are again connected by means of other rods with a collar, *h*, surrounding the vertical rod. If, now, this rod rotate rapidly, the two balls separate in consequence of the centrifugal force; by this separation the collar *h* is elevated, and with it the connected angular lever, *r*S*a*, turning about the axis, S. This motion draws the horizontal rod *ab* towards the right, which turns the angular lever, *bcd*, about the axis *c*, and this lever, being connected with the vertical rod *ed*, draws it downwards. Now, as *e* is the extreme end of a lever arm, by whose turning the valve in the pipe Z is turned, this valve will be closed during the depression of the rod *de*. Less steam enters, therefore, than before, and the rate of the engine is retarded. The converse takes place when the engine goes too slowly: the balls fall, and by means of the connecting lever work, open the throttle valve for an additional supply of steam. This system of levers is in our figure represented only by lines as, being placed on the front side of the engine, it is not really visible in a section.

The alternate admission of the steam into the upper and lower parts of the cylinder, may be effected in various ways, among which the *cross-cock* (*pl.* 19, 2 *figs.* 29), is perhaps the simplest. This is a cock with two perforations: the tube, K, leads to the boiler, C to the condenser, O to the upper and U to the lower part of the cylinder. When the cock has the position of the upper figure, the steam enters from the boiler into the upper part of the cylinder, and at the same time escapes from the lower part to the condenser. When the piston reaches the bottom of the cylinder, the cock is brought into the position of the lower figure by a quarter turn, by which means the steam can enter the lower part of the cylinder, and escape from the upper.

More frequently, however, a *sliding-valve* is made use of for this purpose, as in our representation (*fig*. 27); it is delineated on a larger scale in *figs*. 30 and 31. The steam from the boiler enters through the pipe Z, into a space separated into two parts by a slide, and communicating by the pipes, D and E, with the cylinder. The middle space, *m*, into which the steam enters from the pipe Z, is entirely shut off from the upper part, *d*, and the lower, *a:* the two latter are in communication with the condenser as well as with each other by means of the cavity under the slide. If now the latter have the position represented in *fig*. 30, the steam will enter from *m* through D into the lower part of the cylinder, and the steam above the piston is drawn out through E towards *d*, through the slide towards *a*, and finally into the condenser. In the other position (*fig*. 31) the steam enters from *m* into the upper part of the cylinder through E, and the steam under the piston flows through D to *a*, thence to the condenser. *Pl.* 19, *fig*. 32, exhibits the slide-valve as seen in the direction of Z.

In all cases the arrangement for admitting steam into the upper and lower part of the cylinder, must be kept in operation by the engine itself, whether a slide-valve or a cross-cock be employed. This is done by means of the *governor*, the most important part of which is the excentric circular disk represented at *y* in *fig*. 27. This is attached to the axis of the fly-wheel, the centre of the disk not coinciding, however, with the centre of the axis (*figs*. 33 and 34). About the periphery of the disk is laid a ring, prolonged on one side into a rod, whose end fits at T into the arm of a lever working about a fixed axis. As the central point of the excentric disk is always at an equal distance from the point T, then, during a half revolution of the principal axis, the lever arm at T must pass from the position in *fig*. 33 to that in *fig*. 34, and back again when the revolution is completed. Thus the point T describes an arc, whose chord is equal to the diameter of the circle described by the central point of the excentric disk (during each rotation of the principal axis). As shown by *fig*. 32, the fixed axis, F, of the lever arm passes through the whole breadth of the machine. To this axis are attached two perfectly equal and parallel lever arms, N, on either side of the receiver, containing the slide-valve. *Fig*. 32 represents these foreshortened; *figs*. 33 and 34 exhibit only one of them, but in its true shape. A vertical rod, M, is attached to each of these lever arms; these rods being connected above by a horizontal cross-head bar, Q, to the middle of which is attached the rod R; to the latter the slide-valve is fastened. It is evident that the motion of the lever arms, N, must produce a rise and fall of the cross-head, Q, by means of the rods, M, and thereby elevate and depress the valves themselves.

Other applications of the steam-engine are to steamboats and locomotives. As, however, the principle is the same in all, being only modified for the special purpose, it is unnecessary to consider them here, especially as we shall have occasion to describe them minutely in another part of our work.

f. Specific Heat of Bodies.

One substance, when compared with another, has a greater or less capacity for heat, according as a greater or less amount of heat is necessary to produce a given change of temperature in it; the amount of heat thus necessary is called the *specific heat* of bodies. In some substances the capacity for heat varies. Thus, for instance, it requires more heat to elevate the temperature of platinum from 212° to 213° F., than to elevate it from 32° to 33° F. As, however, the capacity for heat possessed by water is constant, this is taken as the unit for all determinations. To determine the specific heat of a body, the following three different methods may be employed:—

1. *The method of melting of ice,* in which the calorimeter of Lavoisier and Laplace (*fig.* 43) is employed. The instrument, represented in section, consists of three vessels of sheet iron, one inside of the other. The interval, *a,* between the outer and middle vessels is filled with pieces of ice (not pounded finely), as also is the interval, *b,* between the middle and inner one; the water formed in melting flows off through the cocks *d* and *e.* If the body to be investigated be brought into the inner vessel, it becomes cooled to 32° F., the heat given off serving to melt the ice in *b.* The specific heat of the body is estimated from the mass and original temperature of the body placed in *c,* and the amount of ice melted. The ice or snow in the external space, *a,* serves only to keep off the surrounding heat.

2. The *method of mixtures* consists in heating a given weight of the body to be examined to a certain temperature, and then immersing it in water, whose temperature is elevated by the cooling of the body; from the quantity of the water, and the elevation of temperature produced in it, the specific heat of the body may be ascertained.

3. *Method of cooling.* A body cools, other circumstances being equal, the slower as its specific heat is greater. On this principle Dulong and Petit determined the specific heat of many bodies by means of the apparatus represented on *pl.* 19, *fig.* 44. Here *a* is a leaden receiver which may be exhausted of air; in the middle of its cover is a metallic nut, *c,* in which the thermometer, *d,* is fixed; the cylindrical mercury vessel of the latter is placed in a small silver vessel, *e* (shown in the figure between *figs.* 27 and 37), which is suspended by strings, and contains the substance to be examined. If the latter be a solid body it is reduced to powder and tightly pressed in the silver vessel. This, with the body inclosed, is now heated from 15° to 20° C., and introduced into the leaden receiver, *a,* which itself is immersed in a water-bath of given temperature. The receiver, *a,* is now exhausted of air, and observation made of the length of time necessary for the thermometer to fall 50° from a temperature exceeding that of the water by 10° C. From this interval of time, and the amount of the body, its capacity for heat may be ascertained. This method, however, gives no very trustworthy results.

From the experiments upon the specific heat of bodies, many remarkable results have been ascertained, among which not the least important is the law discovered by Dulong and Petit, that the specific heat of bodies is inversely as their atomic weights, or in other words, that the product of the specific heat and the atomic weight of certain bodies is always a constant quantity. There may be here and there slight differences, yet the products fall within narrow limits, being for elementary bodies between 37.849 and 42.703. The specific heat of a body experiences some change with its density. With respect to the specific heat of compound bodies, Avogadro, Neumann, and Regnault have determined, that in all such bodies of equal atomic and similar chemical composition, the above law equally holds good.

The specific heat of gases has been investigated by De la Roche and Berard. The apparatus used by them in their experiments is represented on *pl.* 19, *fig.* 46. The vessel, *a*, filled with air, has an air-tight cover, through which a perpendicular tube is raised, opening into a vessel, A, filled with water, so that the water can enter the vessel *a*. Through the air-tight cover of the vessel A, there passes into the water a tube open at both ends, so that when the water passes out of A, bubbles of air can enter A, through the lower end of the tube. From the vessel, *a*, pass two tubes, afterwards uniting into one, to the balloon, *c*. One of these tubes reaches nearly to the bottom of *a*, and is closed by a cock; through the other pass the upper portions of air from *a* to *c*. In *c* is suspended a bladder, *l*, filled with gas to be examined, from which the gas passes by the pressure of the air compressed in *c*, through the tube, *m*, into the worm of the calorimeter, *s*. It is previously heated in its passage through *e*, by the steam rising from boiling water. The gas, after passing through the calorimeter, is conducted through the tubes *n* and *p*, into the empty bladder, *e*, placed in the balloon, D. From this balloon there is conducted a tube, *q*, entering the vessel, *d* (filled with water), by two branches, one of which, provided with a cock, leads to the upper part of the vessel, the other goes nearly to the bottom. When the air passes through this latter arm from D to *d*, the water flows from *d* through a cock. If the bladder, *l*, be empty, and *c* filled with gas, then *a* must be filled with water, and *d* with air; all the cocks hitherto open are closed, and those closed opened. The air in D and *d* is immediately compressed by the water coming from B, and the gas driven out of the bladder, *e*, through the tubes *p* and *v*, towards the heating part, *e*, thence to the calorimeter, from whose worm it reaches the bladder, *l*, through the tubes *n*, *w*, and *m*; the air from *c* is forced into *a*, and the water in *a* flows out through the cock, *h*. If the bladder, *l*, be filled afresh with gas, the circuit begins anew. One thermometer indicates the temperature with which the gas enters the calorimeter, a second its temperature at the exit, and a third the temperature of the water in the calorimeter. A screen separates the calorimeter from the rest of the apparatus, to keep off accidental changes of temperature.

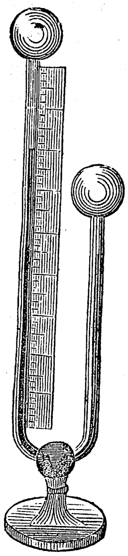
The heated gas passing through the worm of the calorimeter communicates to the surrounding water a certain amount of heat, so that

finally it assumes a constant temperature when it receives as much heat as it gives off. In this manner the excess of stationary temperature of the calorimeter above the surrounding medium may be determined for the various kinds of gases, and as in equal times equal volumes of gas pass through the apparatus, it is evident that the values of the specific heat of various gaseous bodies must, for equal quantities, be in direct proportion to the above-mentioned excess of temperature. Then taking the specific heat of air as unity, the proportional values for other gases may readily be determined. The philosophers above-mentioned, referred the specific heat of air, and consequently that of other gases, to water. As De la Roche and Berard have determined the capacity for heat of gases at a constant pressure, Laplace has determined the same for constant volumes.

g. Transmission of Heat.

Heat is transmitted partly by radiation, partly by conduction. Heated bodies send off heat on all sides, as it were heat rays, which traverse the air. If we imagine a source of heat at any point, then the intensity of heat at different points will be inversely as the squares of the distances. At a distance $=1$ the intensity $= 1 \times 1 = 1^2$; at a distance $= 2$ the intensity $= \frac{1}{2} \times \frac{1}{2} = (\frac{1}{2})^2 = \frac{1}{4}$, &c. In radiant heat, however, there is no uniform heating of the strata of the air, for although near a fire we may experience a piercing heat, this becomes immediately stopped on interposing a screen. Placing two large spherical or parabolic concave reflectors of polished brass (*pl.* 19, *fig.* 35) at a distance of sixteen or eighteen feet apart, and putting in the focus of one a piece of tinder, and in that of the other a red hot iron ball, the tinder will become inflamed. If, instead of the red hot ball, one merely hot, at a temperature of about 300° for instance, be employed, and instead of the tinder a thermometer, the latter will quickly rise. If a vessel containing hot water be placed in one focus, the ordinary thermometer will not exhibit any appreciable change of temperature; we are not to suppose from this, however, that the vessel of water radiates no heat. The truth of the matter is, that while a radiation does take place, the ordinary thermometers are not sufficiently sensitive to exhibit it, for which reason it becomes necessary to employ a more delicate instrument. Such thermometers are:

1. The *differential thermometer* of Rumford (*pl.* 19, *fig.* 36), consisting of two glass bulbs, *a* and *b*, connected by a bent glass tube. In this a drop of alcohol or sulphuric acid serves as an index, upon which the air presses from both sides. The position of this index or drop of fluid, when both bulbs are of the same temperature, is taken as the zero of the scale, which is placed on the horizontal part of the tube. If one bulb be heated more than the other, the drop, by the expansion of the contained air, will be driven towards the colder one; and the distance to which it is driven will be in proportion to the difference of temperature of the two bulbs.



2. The *differential thermometer* of Leslie (see accompanying figure) consists of a curved tube with bulbs blown at both ends, and standing on a foot. The tube is filled with a colored liquid. If the one bulb be placed in the focus of a concave mirror and the other out of it, at the least heating of the first bulb, the liquid in the tube will change its position; and the amount of this change may be read off on the scale.

3. *Melloni's thermo-multiplier* (*pl.* 19, *figs.* 37 and 37[a]). This consists of a sensitive multiplier and a thermo-electric pile, composed of twenty-five to thirty fine needles or bars of antimony and bismuth, connected alternately at their extremities, and separated laterally by some non-conductor, the whole united into a compact bundle. Each of the terminating elements of the pile is connected with one of the projecting pins, which thus form the poles of the pile. The pile is lamp-blacked at both ends, and, with its covering, placed on a foot at *p*. The bonnets *a* and *b*, of which *b* is conical, serve to keep off from the pile all lateral radiations. In addition to this, *b* serves to concentrate the rays of heat coming from that side. The copper wire, twenty-two to twenty-four feet long, forming the galvanometer, is wound upon a metal frame. The carefully compensated magnetic needles are, as shown in *fig.* 37[b], united together and suspended by a fibre of raw silk, hanging in the middle of a glass bell. By turning the button *f*, the fibre with the needles may be slightly elevated or depressed. The extensible spiral wires, *g*, serve to connect the poles of the pile or battery with the extremities of the multiplier wire. The entire apparatus is so placed and adjusted upon a sufficiently firm table as that the thread shall hang in the middle of the graduated circle, and the needles point to the zero of the graduation. The least change of temperature between the two extremities of the pile produces an immediate deviation of the needle, which may be measured on the graduated circle.

If, in the focus of a mirror, any one of the above-mentioned pieces of apparatus be introduced, and in the other a body whose surface amounts to one third to three fourths of an inch, the apparatus will show that this body constantly radiates heat, even if its temperature be but little higher than that of the surrounding bodies. Thus, in a cold room, melting ice will radiate heat, and thereby elevate the temperature in the other focus. If the temperature of the room be above 32°, the thermometer in the focus of one mirror will sink if ice be placed in that of the other. This, however, is not an instance of radiation of cold, but simply an inversion of the usual operation; the thermometer is now the radiating body, giving off its heat to the ice. If Melloni's thermo-multiplier be employed, a mirror is not necessary, for by attaching the conical hood, *b*, the rays are concentrated by it so strongly, that even if the hand be held against the opening of the hood, at a distance of several steps, the radiation from the former will be sufficient to produce a very sensible deflection of the needle.

Heat rays, on striking any body, are either absorbed, reflected, or trans-

mitted. That an absorption of heat rays must occur is shown by the heating of a body placed in one focus of the above-mentioned system of concave reflection, whenever a heated body is placed in the other. Although this power of absorption is common to all bodies, it is yet not the same in all, the less dense the body the greater being its absorbing power. Of the reflecting power of bodies we have an illustration in the above-mentioned concave metallic reflector, the mirrors themselves experiencing no elevation of temperature when a heated body is placed in one of their foci. The powers of absorbing and reflecting heat in bodies may be considered as complementary to each other; both taken together explain what becomes of the heat reaching any body. Thus, a body reflects what it cannot absorb, and the greater the absorption the less the reflection, and vice versâ. The angle of reflection of heat rays is equal to the angle of incidence. From the surface of plates not well polished, rays are dispersed irregularly, or diffused in all directions; and the same is the case with heat rays. Of this we may readily become convinced by directing a small beam of light against the wall of a dark room. By presenting the thermo-electric pile towards the light spot, a deviation of the needle will be observed in whatever part of the room it may be placed; it returns immediately to 0, however, whenever the aperture admitting the beam of light is closed. There is of course no other heat present than that diffused by the beam of light.

Solid bodies may transmit heat rays just as transparent do light. These are called by Melloni *diathermanous*, and those which intercept heat, *athermanous*. Melloni, in his experiments on the passage of heat rays, employed the apparatus represented in *pl.* 19, *fig.* 37. As sources of heat he employed: 1. a Locatelli lamp; 2. a spiral of platinum wire kept red hot by the flame of alcohol; 3. a blackened copper plate, *l* (*fig.* 39), heated to 752°F. by an alcohol lamp; 4. a hollow cube of brass plate (*fig.* 40) filled with hot water, maintained at an equal temperature by a lamp. These sources of heat were successively placed upon the stand *e*. The screen, *o*, composed of two brass plates, and turning on a hinge, could be brought between the source of heat and the thermo-electric pile, to keep from the latter any heat rays. The plate of the body to be investigated was placed at *r*. If the source of heat be placed at such a distance that the needle experiences a certain deflection (30°), and a plate be interposed at *r*, it was found that the needle returns more or less to its original position; and also that plates of equal thickness and transparency do not transmit an equal number of heat rays, and even that some bodies transmit heat better than others of much greater transparency. The thickness of the plates employed averaged from one third to two thirds of a line. Plates of rock salt were found to be most diathermanous (92$\frac{0}{0}$), and plates of ice the least (6$\frac{0}{0}$).

It was also found that the difference of radiants involved a difference in the number of rays transmitted through the same plate. In the lamp of Locatelli the transmission was greatest, in the brass plate the least, although the original deflection (30°) was the same in all. Plates of rock salt

transmitted the heat of all equally well; plates of ice only that of the Locatelli lamp. For all other sources the power of transmitting was zero.

That heat rays are capable of refraction like those of light, may be shown by the apparatus represented in *pl.* 19, *fig.* 41. Upon a stand is placed a prism of rock salt, and at some distance the Locatelli lamp. The direction is now observed in which the rays of light emerge from the prism with the least deviation from their original direction, and the thermo-electric pile placed in it: the needle will become immediately deflected. The same will be the case if, for the Locatelli lamp, the platinum spiral, the cube of hot water, &c., be substituted. The deflection ceases immediately on slightly moving the pile, whence it follows that the rays from the different sources are refracted by the rock salt.

In this great difference in the transmitting power of diathermanous bodies the question suggests itself, whether in the athermanous bodies the power of absorption and diffusion be not different. Melloni has instituted the investigations necessary to answer this question. He cut out disks of equal diameter from the same copper plate, blackened them on one side, and coated them on the other with the substance whose power of absorption was to be ascertained. He then introduced the plates, one after the other, into the apparatus, so that the blackened side was directed towards the pile, and the coated side towards the source of heat. This side became heated by absorption, and this heat, being communicated to the opposite side, was brought to bear upon the pile. He thus discovered a great difference both in the absorbing power of the bodies themselves, as also in respect to the different sources of heat. Lamp-black exhibited the maximum power of absorption, only $13\frac{0}{0}$ of which was exhibited by a bright polished surface.

Melloni and Forbes have also indicated a polarization of heat rays, by processes similar to those by which the same change is produced in light.

Dulong and Petit have instituted the most accurate experiments upon the laws of *cooling* by means of the apparatus represented in *fig.* 42. Here *a* is a copper vessel filled with water kept at a uniform temperature; *b* is a balloon of copper plate, blackened internally and sunk in the water; it is sustained by the frame *c*. Upon the broad ground edge of the balloon is placed a level plate, *d*, of thick glass, and upon this (like a receiver on the plate of an air-pump) a broad glass tube, *e*. This is provided above with a cock, and is connected by a leaden tube, *g*, with an air-pump, of which the figure represents only the plate *h*. The tube *k* is filled with chloride of calcium, which serves to dry the gas coming from the gasometer *l*, in case experiments are to be made upon cooling in different gases. The bodies whose cooling is to be observed in this apparatus are large thermometers with spherical bulbs, fastened by a cock in the glass plate *d*, and capable of being raised with it. When such a thermometer has been heated to the proper temperature, it is quickly introduced into the balloon, the tube *e* placed over it, and the air pumped out. The depression of the mercury is to be observed with the assistance of a good watch.

It has been found by experiments with this apparatus that the rate of cooling is not uniform, that is, that bodies do not cool equally in each successive minute. The greater the excess of heat possessed by bodies above that of surrounding bodies, the more rapidly does cooling take place. The loss of heat of a body is, however, only proportional to the excess of temperature when the latter amounts to about 100°—115°F.

h. Conduction of Heat.

Heat passes from one body to another, not only by conduction, but also by immediate contact; all bodies do not possess, however, the same conducting power. Some bodies allow heat to pass with great facility from one particle to another; these are called *good conductors.* Others may be inflamed at one point, while in another quite near to it, the temperature may be but slightly increased. Such are *bad conductors.* Metals form the best conductors; spongy or very porous bodies the worst.

If several rods of different material, but of the same size, be coated at the upper end with wax, and set on a hot plate, the relative rapidity of melting which will be observed in the wax, will indicate the relative conducting power of the different materials.

If an elongated body, as a metallic rod, be connected at one end with a source of heat, this heat will gradually diffuse itself throughout the entire mass: it will, however, be greatest in the vicinity of the source, and decrease inversely as the square of the distance from it. In similar rods of different metals, the conducting power is as the square of that distance from the source of heat, at which, other things being equal, equal excesses of temperature have been observed.

In liquids and gases heat is diffused principally by currents. As the heated strata become specifically lighter, and therefore rise to the surface, the displaced strata occupy their place and become heated in turn. Liquids, and still more gases, are much poorer conductors than metals; hence it follows that porous bodies, powdered substances, and even metals in a state of minute division, conduct heat much worse than those which are dense, on account of the pores being constantly filled with air or other gases.

i. Sources of Heat.

The principal source of heat is the sun, and next to this, chemical combinations, combustion particularly, that is, the rapid combination of bodies with the oxygen of the air. The heat produced in such combustion is estimated by the degree to which equal quantities of the combustibles elevate the temperature of equal quantities of water. The most satisfactory experiments on this subject have been instituted by Rumford, Lavoisier, Laplace, and Despretz.

The animal heat is constantly different from that of the surrounding air; in the temperate and frigid regions it is generally warmer, in the tropical colder than the air. The animal body consequently possesses an independent heat which is continually renewed. The heat of the human body appears to be nearly the same in all parts of the body, and under the most diverse circumstances (cases of disease excepted), varies only from 96.5° to 102° F. The animal heat of birds is higher than that of any other animal, and that of insects perhaps lower. The source of animal heat is a peculiar combustion taking place in the body between the oxygen inspired through the lungs and the carbon of the body. Other causes there are, in all probability, but this is unquestionably the principal.

Heat may also be produced by mechanical means. Thus a very considerable elevation of temperature may be produced by the compression of air, as shown in the little apparatus for inflaming tinder, where the tinder fixed underneath an air-tight piston, is inflamed by the sudden depression of the piston and consequent compression of the air. Even in solid bodies a considerable compression, as in coining money, produces a sensible elevation of temperature. Finally, friction is the principal source of the mechanical production of heat, for the pivots of a wheel become heated to redness if the friction be not diminished by some anti-attritient. Wood also may be set on fire by rubbing two pieces together.

OPTICS; OR, THE THEORY OF LIGHT.

a. Propagation of Light.

Bodies are divided, with regard to light, into luminous and non-luminous, of which the former emit light peculiar to themselves, while the latter do not. Now, luminous bodies are again divided into transparent, or those which transmit light; and opake, or those which totally intercept its passage. Light is propagated in perfectly uniform media, in straight lines; and in curved when the medium is not of this character. In passing from one transparent medium to another, it experiences a deviation or break in its path; that is, the rays of light undergo refraction. This, for instance, is very evident in its passage from water into air. Take a vessel, *v* (*pl.* 21, *fig.* 1), and place in the bottom of it a piece of money. Assume such a position with regard to this vessel, that the money shall be just concealed by the edge, *b*, of the vessel. Fill this with water, and the coin will appear as if elevated, and in plain sight. It appears to lie at *n*, though its position is not changed in the slightest degree; the illusion is produced by the bending of the ray, *mio*, coming from the object to the eye at *o*. Upon this same principle is explained the fact, that the stars are visible before their real rising, and after they have actually set. See Astronomy, section 47.

Light is most intense at its source, and experiences a gradual diminution in its intensity as the distance from this source is increased, as is shown by the fact of a body becoming less illuminated as it recedes from any radiant. The law of this diminution is the same as in the case of heat; the intensity decreases as the square of the distance from the radiant. A body which experiences a certain intensity of light at a distance of one foot, will receive at the distance of two feet only one fourth of this amount, and at the distance of three feet one ninth, &c.

When light coming from a single luminous point strikes upon an opake body, there arises behind this, on the side opposite to the radiant, a dark space called a shadow, bounded by a conical surface. If the luminous body be of considerable extent, it becomes necessary to distinguish the full shadow, or that space receiving no light at all, from the half shadow, or the space receiving light from some parts of the luminous body and not from others. In *pl.* 21, *fig.* 2, let A be a large luminous sphere, and B a smaller opake one, then both the full and the half shadows will be conical spaces, only of opposite positions; for while the diameter of the full or central shadow diminishes with the distance from the luminous body, ending finally in a point at S, that of the half shadow increases more and more with this distance. *Fig.* 3 represents the appearance which would be presented by their shadows, if received at *m'n*, on a screen. It will be seen that the central shadow is smaller, and the half shadow larger, with the distance from the body producing the shadow, until the former vanishes entirely, and only the latter remains. This increases in size, but at the same time diminishes in intensity until it also disappears.

If the light from a luminous or illuminated body falls upon a screen with a small opening, the light passing through forms a well defined beam, producing upon a second screen a bright spot on a dark ground. If an aperture of this character be made in the window shutter of a perfectly dark room, an inverted image of external objects will be found upon the opposite wall (*fig.* 4). A beam of solar light under such circumstances presents a round image, even though the aperture be angular, as a circular image is formed by every point of the aperture, and the combination of these innumerable round images must necessarily give a single image that is round.

The velocity of light is extraordinarily great. It passes from the sun to the earth in eight minutes and thirty-six seconds, and in each second traverses not less than 192,000 miles. It has been a problem in Astronomy to determine this velocity by observations on the motions of Jupiter's satellites (see page 116). The calculations were first made by Olaus Römer and Cassini.

b. Reflection of Light.—Catoptrics.

When a ray of light strikes a very smooth level surface, a polished glass or metallic plate for instance, it is reflected, and the angle formed by

the incident ray with a perpendicular to the surface at the point of incidence, will be equal to the angle formed by the reflected ray with this same perpendicular. Thus, in *pl.* 21, *fig.* 5, suppose a ray to come in the direction *dl*, forming an angle, *dlp*, with the perpendicular *lp*, the reflected ray will be *lr*, making the angle $dlp = plr$. The former is called the *angle of incidence*, the latter the *angle of reflection*. Rays reflected in this manner are said to be regularly reflected. There are, in addition, rays that are irregularly reflected, or scattered in all directions from the radiant beam. The intensity of this scattered light is in proportion to the want of polish of the reflector.

To prove the preceding proposition respecting regularly reflected light, the following method may be employed. Take a vertical graduated circle, C, (an altitude circle) *fig.* 6, about whose axis a telescope, *l*, moves. Have also an artificial horizon of mercury or linseed oil, in a wooden vessel; then sight with the telescope, first at a star and then at its image reflected in the artificial horizon. On measuring the angles which the sight lines *oe* and *o'i* form with the horizontal line *cf*, it will be found that they are equal; whence, as *eo* is parallel to the incident ray *ci*, both coming from an infinitely distant star, it follows that the incident ray, *ci*, and the reflected ray, *io'*, make equal angles with the horizontal line, and consequently with the vertical or plumb line, *pi*. The three lines, *ci*, *io'*, and *pi*, evidently lie in one and the same plane, or the plane in which the telescope rotates.

A plane mirror shows the images of objects lying before it, which images must be symmetrical with the object, in relation to the reflecting plane. In *fig.* 7, let *m'm* be a plane mirror, and *l* a luminous point before it, which sends to the mirror the ray *li*. This is reflected in the direction *ic*, and produces an impression upon an eye at *c*, as if it had come from a point, *i*, in the direction *ic*, and behind the mirror, so that $il' = il$. An eye at *c'* will observe the point *l* in the same point *l'*. Draw *ll'* cutting *mm'* in *k*, *ll'* will evidently be perpendicular to *mm'*, and be bisected at *k*. We thus find the image of a luminous point in a plane mirror by letting fall from the luminous point a perpendicular to the mirror, or the mirror produced, and taking on this perpendicular, behind the mirror, a distance equal to that of the point in front of it. As this proposition holds good for every point of an object emitting light, the image of such an object may be readily constructed. Thus, in *fig.* 8, *ab* is the image, in the mirror VW, of the arrow AB, and it is evident that the image and object are perfectly symmetrical, with respect to the plane of the mirror. The construction lines, A*k* and *ka*, B*l* and *bl*, exhibit the position of the image, while the other lines show the correctness of the figure with reference to the reflection of the rays of light.

The intensity of the reflected light, whose direction may be ascertained in the most exact manner, depends on the one hand upon the medium in which the light moves and in which it falls, and on the other hand upon the angle of incidence: the more acute the angle the greater the number of rays reflected.

If two plane mirrors be placed together at any angle, an object between

them may exhibit many images. In *pl.* 21, *fig.* 9, let VW and ZW be two plane mirrors, at right angles to each other, with a luminous point placed between them. An eye at O sees, besides the point or object itself, its two images, a, a', reflected from the two mirrors. But the rays reflected from one mirror are partly reflected back again by the other, on which account the images, a, a', may themselves be considered as objects or radiant points: the two will form a third image in the same point, a''; more than these three images cannot exist at this angle. The number of images always depends upon the inclination of the mirrors; if this amount to 60°, there will be six images, including the object, &c.; and, in general, this number (including the object) will be represented by $\frac{360}{\alpha}$, where α is the angle of inclination of the two mirrors. The number therefore increases with a diminution of the angle; when this is zero, or the mirrors become parallel, this number becomes infinite.

Upon this principle depends the construction of the instrument invented by Brewster in 1817, and called by him the *Kaleidoscope* (*figs.* 105, 106). This consists of a cylindrical or conical tube with a cap at one end, in which is a hole to look through. In the tube two plane mirrors are fixed, so as to form with each other a certain angle, 60° for instance. Instead of the mirrors usually employed, glass plates blackened on the back may be used. A false bottom of glass is placed at a short distance from the extremity opposite to that in which the eye-hole is situated, and over the extreme end is fitted a second plate of glass by means of a cap. Pieces of colored glass, feathers, and other brightly colored objects are placed in the space between the two plates just mentioned. On looking through the eye-hole, towards the light, various hexagonal symmetrical images will be formed by the reflection of the objects in the mirrors, which will be changed by every change in the relative position of the objects. Other polygonal images besides the hexagonal will be formed by varying the inclination of the mirrors. It must not be forgotten, however, that by too frequent reflections the light is enfeebled, and part of the image will be very faint. The dodecagon should be the maximum, in which case the angle of inclination must be 30°. The kaleidoscope is of great use in drawing patterns for various fabrics, for which purpose it has been variously modified, so as to produce other figures than the rosette.

Wollaston's *reflecting Goniometer* depends for its principle upon the reflecting of a ray of light. A goniometer is any instrument used for measuring the angle formed by any two sides of a crystal, and may be of various constructions, some of which will be found elsewhere. With regard to the goniometer of Dr. Wollaston, let (*pl.* 21, *fig.* 10) *abcd* be the section of a crystal, *ab* and *ac* the surfaces, appearing here as lines whose angle is to be measured. If now the edge, *a* (projected into a point in the figure), be horizontal, an eye at *o* observes in the surface, *ab*, the image of a distant horizontal line, to which the edge, *a*, is parallel also as a horizontal line. Let the eye be held in such a manner that the image of

the line, a window-bar, for instance, be visible by reflection from the surface, *ab*, at a certain readily determined part of the floor; turn the crystal about an axis parallel to the edge *a*, or about this edge itself, until the same object is seen in the same place by reflection from the second surface, *ac;* this will be accomplished when the angle, *fac*, has been described, and the surface, *ac*, is in the same position formerly occupied by *ab*. The angle of rotation, *fac*, can be measured if the axis of rotation be the axis of a graduated altitude circle; subtract this from 180°, we shall have the angle required, *bac*. *Figs.* 11 and 12 represent a reflecting goniometer, the latter of which is a sectional view: *ab* is the section of the graduated circle; *i* that of the part containing the vernier. The disk of the circle turns about a graduated axis reaching to *ef*, turned by means of the milled wheel, *ef*, which carries in addition the disk, *cd*. For fixing the latter disk, as also the circle itself, the pressure screw, *l*, is employed, and for fine adjustments of the circle the screw *o*. The axis of the circle itself is hollow; in it, by means of the head, *gh*, turns another axis, *mn*, by whose rotation is turned the right-angled arm, *nqp:* to this is fastened a similar arm, *prs*, turning about *p*. If the circle be fixed by means of the screw, *l*, the axis, *mn*, can be turned separately; if *l* be loosened, then the axis turns with the circle. The crystal is fixed to the rod, *tu*, by a little wax, and the whole instrument so adjusted as that the plane of the circle shall be perpendicular to that of the window. To measure by means of the instrument, adjust the circle to the zero, fix it there by means of the screw, *l*, and arrange the crystal in such a manner that the edge of intersection of the two planes whose angle is to be measured, shall fall in the prolongation of the axis, *mn*, until the image of the window-bar appears at the given part of the floor. Then loosen the screw, *l*, and turn again until the image of the window-bar is seen at the same part of the floor from the second surface. The angular value of this rotation may then be read off.

For large and heavy crystals the *goniometer of Gambey* (*fig.* 13) is better suited: it may also be employed in measuring the angle of a prism. For this latter purpose the prism is so adjusted that the image of any distant object appears in the cross-hairs of the telescope. The prism is then turned about its vertical axis until the same image reflected from the second surface appears in the cross-hairs, upon which the angle by which the prism has been turned is to be read off.

The *reflecting Sextant* is a very important application of the reflection of light: its principle is illustrated by *fig.* 14. Here A is a small mirror whose upper half is not silvered, so that an eye at *o* can see through the uncovered portion of the glass plate. B is a second mirror, which may be turned about an axis perpendicular to the plane of the figure. When the mirrors are mutually parallel, the eye at *o* will see a distant object situated in the direction *o*A, directly through the uncoated half of the mirror, and its reflection in the other half, while the ray, *e*B, coming from the object and passing near the mirror, A, is reflected from B to A, and thence to *o*. If the mirror B be turned, an image, visible through the uncovered part of the mirror A, will not be seen in the silvered portion, but the image

visible will be that of an object from which the ray, fB, comes. The angle which the two sight-lines, Be and Bf, from the two objects make with each other, is precisely twice the amount of the angle by which the planes of the two mirrors are now inclined to each other. It would be very easy to show that the angle eBf is twice as great as gBh. *Pl.* 21, *fig.* 15, represents a reflecting sextant of the simplest construction. For full particulars respecting this instrument in its various forms, as also for a more complete illustration of its theory, we must refer our readers to that part of our work where the sextant is treated of at length.—(Pp. 66 and 165.)

If a ray of light impinge upon any polished curved surface, it will be reflected as from a plane tangent to the surface at the point of incidence. A luminous point at the centre of a sphere emits rays which are all reflected back again to this centre. If the luminous point lie in one focus of an ellipsoid, its rays will be reflected to the other focus, and then back again by reflection to the first. If the luminous point be placed in the focus of a paraboloid, the rays will be reflected parallel to the axis: if a number of rays be incident parallel to the axis, they will be reflected to the focus. Spherical mirrors are either concave or convex. A spherical convex mirror is a part of a sphere polished externally; a spherical concave mirror is part of a sphere polished internally. The centre, c (*fig.* 77), of the sphere is called the centre of curvature; the line ca, connecting it with the centre of the mirror, is called the axis of the mirror; the angle mcm', formed by lines drawn from the centre of curvature to exterior points diametrically opposite to each other, is called the aperture of the mirror. If a luminous point be placed at the centre of curvature, all its rays will be reflected back to it again. If the radiant be at a very great distance from the mirror, its rays striking the mirror may be considered as parallel to each other. Rays falling upon the mirror parallel to each other (*fig.* 16) are reflected to a common point, c, called the focus of parallel rays, situated half way between the centre of curvature and the centre of the mirror (*fig.* 17). This is strictly true, however, only of those rays which are very near and parallel to the axis: the more they are removed from the axis, the nearer to the mirror is the focus. The focus of parallel rays, striking the mirror at a distance of 60° from the axis, will lie in the centre of the mirror itself. If all the parallel rays impinging upon a mirror are to be reflected to the same point, its aperture must not amount to more than from 8° to 10°; in this case all the rays may be considered as central. If the luminous point be not at an infinite distance, but a point, m, of the axis itself (*fig.* 16), the focus will be nearer the centre of curvature than the centre of the mirror; if placed at the centre of curvature, the focus will be there also. If the radiant be placed between the focus of parallel central rays and the centre of curvature, the focus will be further from the mirror than this centre, and will recede more and more as the radiant approaches the centre of parallel rays. In this focus the radiant will emit rays which will be reflected in lines parallel to the axis and to each other, and there will be no convergence to a focus at all. If the radiant be between the focus of

parallel central rays and the mirror, the rays will be reflected diverging, as if they came from a point behind the mirror (*fig.* 18).

All that has just been said applies equally to rays reflected from points not in the axis, as an imaginary axis may be drawn through the centre of curvature and the radiant, provided the mirror be sufficiently large.

On the principles just enumerated, it becomes easy to determine the nature of images formed in concave mirrors. If an object, AB (*fig.* 19), be placed between the centre, C, and the focus F, the mirror exhibits an image, *ab*, inverted and magnified, and situated at a greater distance from the mirror. For an object at *ab* the image will be inverted, diminished, and nearer to the mirror. The further the object from the mirror, the nearer is the image to the focus of parallel rays; if this distance becomes infinite, as in the case of the sun or stars, the image will be in the focus. An object at the centre of curvature will have an image there also, and inverted. Objects at the focus, and between this and the mirror, will exhibit no image whatever in front.

The images formed in concave are very different from those of plane mirrors. The latter appear as if proceeding from a point behind the mirror, thus diverging, while the former converge. The images formed by a concave mirror may be thrown on a screen of white paper or ground glass.

The radius of curvature of a concave mirror may be readily determined by observing the place before the mirror at which the image of the sun is formed on a screen. This image will, of course, be in the focus of parallel rays, and twice the distance thus formed will be the radius of curvature.

Although no image is formed in front of a concave mirror by objects placed between the focus and the mirror, yet an apparent image will be formed behind it. If in *fig.* 20 AB be the object, the normal ray, A*n*, will be reflected back in the direction *n*AC. A*e*, however, which is parallel to the axis, will be reflected to F; *n*AC and *e*F, produced backwards, will intersect at *a*, where will be the image of A. Obtain *b*, the image of the other extremity, B, of the object, and *ab* will be the image required. It will be observed that this is larger than the object, lies behind the mirror, and is erect.

A *spherical convex mirror* (*fig.* 21) has no actual focus, as the reflected rays do not unite; they diverge, however, after reflection, as if they came from one and the same point behind the mirror. When the rays are parallel to the axis, this point, *v*, will be half way between the centre of curvature and the mirror, thus corresponding to the focus of parallel rays in the concave mirror. The focus of parallel rays in the convex mirror is called the *virtual or apparent*, to distinguish it from the *real or actual* focus of the concave mirror. A convex mirror exhibits a direct, but diminished image, *ab*, behind the mirror (*fig.* 22), of which we may become easily convinced, by comparing the explanation of *fig.* 20, and considering F as the focus.

When the rays proceeding from a luminous point, and reflected from a

curved surface, do not unite in a focus, a *caustic curve* is produced. This is formed by the intersection of two contiguous rays in the same plane. *Fig.* 23 exhibits a caustic curve produced by a curved reflecting strip.

c. *Refraction of Light.—Dioptrics.*

When a ray of light passes from one medium to another, it experiences a change of direction, or becomes broken, i. e. refracted. When the media are perfectly homogeneous, the refraction takes place suddenly; as, however, in most cases there is a stratification of media, this refraction, strictly speaking, takes place in a curve, as has been already referred to in Astronomy. This curvature is generally so slight as to be scarcely sensible, and but little error is involved by considering refraction to take place in straight lines. If, in *fig.* 24, the horizontal line passing through *i* separate two different media, as water and air, then the angle formed by the incident ray, *il*, with the vertical line, *ni*, is called the *angle of incidence.* The *angle of refraction* is that angle formed by the ray, *ir*, after entering the second medium with the same vertical line produced on the opposite side. The plane of incidence passes through the incident ray and the vertical; the plane of refraction through the same vertical and the refracted ray. Generally, the incident ray is refracted into but one line; there are cases, however, in which this ray becomes split into two, as will be seen when we come to the subject of polarization.

For simple or single refraction, to which we here restrict ourselves, the following laws present themselves :—1st. The plane of refraction coincides with the plane of incidence. 2d. For the same media, the sine of the angle of incidence bears a constant ratio to the sine of the angle of refraction. Suppose in *pl.* 21, *fig.* 25, *l* to be a ray of light, incident at the same point as, and in the same plane with a vertical, *dd'*, and there to suffer a refraction. If it were desired to determine the angles of incidence and refraction on a graduated circle, we may suppose a circle to be described about the point of incidence, cutting the two rays. There *ad* would be the sine of the angle of incidence, and *cd* that of refraction. If the angle of incidence were found by direct measurement to be $= 15°$, then the angle of refraction would be $11° \, 15'$; if the former, again, were $60°$, the latter would be $40° \, 30'$; and the sines of these angles are respectively, 0.259, 0.194, 0.866, 0.649. Constructing the above proportions we have $\frac{\text{sin. } 15°}{\text{sin. } 11° \, 15'} = \frac{0.259}{0.194} = \frac{4}{3}$, and $\frac{\text{sin. } 60°}{\text{sin. } 40° \, 30'} = \frac{0.866}{0.649} = \frac{4}{3}$; that is, the sine of the angle of incidence is to the sine of the angle of refraction : : 4 : 3.

The *index of refraction,* four thirds, answers for the case where the ray passes from air into water; for other media other indices are required. Even in water a change of temperature will produce a different index. If the ray pass from water into air, the rays change names, but retain the same values; and if *n* be the index of refraction in the first case, of a ray

passing from a rarer into a denser medium, it will be $\frac{1}{n}$, when the ray traverses the same media in the reverse direction. As this minimum angle of incidence $= 0°$, that is, when the ray falls perpendicularly to the coinciding surfaces of the media, the angle of refraction must, in that case, be $0°$, or the ray will pursue its course unbroken. The greatest value of the angle of incidence will be $90°$; and as sin. $90° = 1$, $n = \frac{1}{\text{sin. } r}$ (when r is the angle of refraction), or sin. $r = \frac{1}{n}$. This value of n is called the *limiting angle.*

For the media air and water, $n = \frac{4}{3}$; thus $\frac{1}{n} = \frac{3}{4} = 0.75 = \text{sin. } 48° \ 35'$, and this value is the limiting angle in this instance. Then a ray of light, passing from air into water, cannot have an angle of refraction greater than $48° \ 35'$; if a ray pass at this angle from water into air, its refraction will amount to $90°$, or the refracted ray will be parallel to the surface of refraction. All rays, then, proceeding from water to air, which strike the refracting medium at an angle less than the limiting angle, will not pass out, but will be entirely reflected back again, as illustrated in *fig.* 78, where the ray loses nothing of its original intensity by reflection. *Fig.* 26 represents a particular instance of such total reflection. Dip an empty glass tube, melted together at the bottom, into a vessel filled with water. By giving it a position something like that in the figure, and looking at the tube from above, it will appear as if filled with mercury. By pouring water into the tube, the metallic lustre will vanish as far as the water reaches. The phenomenon is easy of explanation, as the rays coming from *a* strike the tube at such an angle as not to be capable of entering into the air of the tube; consequently they are reflected. This reflection must, however, cease as soon as water is poured into the tube.

The amount of deviation, or the angle of deviation, may always be obtained by subtracting the angle of refraction from the angle of incidence. This deviation does not increase proportionally, as it increases with the increase of the angle of incidence much more rapidly than of the angle of refraction.

A *prism*, in Optics, is a transparent medium, bounded by two inclined sides. The line in which these two sides intersect, is called the refracting edge, and the side opposite to this the base. The angle of the two surfaces is called the refracting angle; the intersection of the prism, by a plane at right angles to the edge, is called the principal section. The three-sided prism is generally employed, bounded by three rectangular parallelograms (*fig.* 79); the principal section of such a prism is a triangle. In optical experiments, the prism is usually fastened upon a small brass stand (*fig.* 27). The rod, *t*, may be moved up and down in the tube in which it is placed, and the prism may be placed in any direction required, by means of a hinge at *g*. If the prism be fixed with the refracting edge uppermost, all objects seen through it will appear considerably displaced and raised

from their true position; in any other position of the refracting edge, they are displaced towards it, and likewise exhibit colored borders. If a beam of solar light, coming in the direction *vd* (*fig.* 28), through a small aperture in the window-shutter of a darkened room, be received on a prism with its refracting edge uppermost, an elongated space, crossed transversely by the various colors of the rainbow, will be observed. This colored space is called the *solar spectrum*. Without the prism there would have been seen at *d*, above *r*, a white and circular image of the sun.

To follow the course of the rays in a prism, it becomes necessary to consider their direction in the plane of a principal section. In *fig.* 29, let *as* and *a's* be the refracting surfaces, *s* the refracting edge of a glass prism, *li* the incident, *ii'* the refracted ray (refracted *towards* the perpendicular), and *i'c'* the ray emerging from the prism (now refracted *from* the perpendicular). For air and glass the limiting angle is $40\frac{3}{4}°$; an emergence of a ray from the prism is then impossible, when the ray, *li*, strikes the prism in such a manner, that the angle of refraction is less than the amount by which the refracting angle of the prism exceeds that limiting angle. In a prism whose refracting angle is twice as great, or still greater than the limiting angle, an emergence of the rays from the prism is impossible. If a ray of light pass in such a manner through a prism, as to make equal angles with both refracting surfaces, the total deflection produced on the ray by the prism is a minimum, that is, less than in any other position of the refracted ray. Suppose the ray, *li* (*pl.* 21, *fig.* 80), to be refracted in such a manner, that the refracted ray, *ii'*, shall make equal angles with the surfaces *sa* and *sa*, then will $ni'i =$ the angle of refraction $nii' = x$, and the angle of deviation, *d*, of the ray at $i =$ that at i'; the total deviation thus $= D = 2d$. If the direction of the incident ray be changed, so as, for instance, to fall along *l'i*, then the refracted ray will be *im*, and the angle, *nim*, less than x; the angle made by *im*, with the perpendicular through *m*, will be just so much greater than x: the deviation thus increases on one side and diminishes on the other. If the decrease $= \alpha$, then the deviation $= d - \alpha$; as, however, it must have increased at *m* just so much more than x, as already seen, we may indicate the deviation at *m* by $d + \alpha + \beta$; the total deviation here, then, is $D' = d - \alpha + d + \alpha + \beta = 2d + \beta$, thus greater than D. The same may be proved by any other case of the kind. If the refracting angle of the prism be of small amount, then, in the case of the minimum of deviation, this is proportional to the refracting angle. If an object be observed through a prism, the direction in which the deviation is the least is easily found. If this minimum of deviation, *d*, be known, and the refracting angle of the prism, the index of refraction of the material of which the prism is composed, may be ascertained for air from the formula

$$n = \frac{\sin. \frac{1}{2}(d+g)}{\sin. \frac{1}{2} g}.$$

To obtain the index of refraction of any body, it becomes necessary then to form it into a prism. To give a liquid the prismatic shape, a hole is to be bored through two sides of a glass prism, and a smaller one through the

base. Upon the two first surfaces lay plates of ground plate glass, which may be kept firm by a brass clamp; fill the hollow prism thus formed with the liquid in question, through the small hole, and in it insert a stopper of ground glass. *Fig.* 30 represents a prism of this character, consisting of two hollow prisms close to each other. Another form of the hollow prism is shown in *fig.* 81. A three-sided prism of brass, or still better, of glass, is bored through, either, as in the figure, by a quadrangular, or by a round aperture; upon the two refracting surfaces plates of glass are laid, which may be pressed upon the surface of the hollow prism by means of four screws. Above is the aperture through which the prism may be filled, and which is then to be closed.

If a ray of light pass through a plate, as of glass, with parallel sides, or through several superimposed plates of different materials (*fig.* 82), it emerges in a direction parallel to the original one, though somewhat displaced from it.

The refractive power of a body is equal to $n^2 - 1$, or the square of the exponent of refraction, with respect to a vacuum minus unity; the quotient of the refracting power, divided by the density, or $\frac{n^2 - 1}{d}$, is called the *absolute refracting power*.

Arago, Biot, and especially Dulong, have instituted very accurate experiments with regard to the refractive indices of gaseous bodies; they have discovered that the refractive powers of gases are proportional to their densities. Dulong's experiments had particularly for their object the comparison of the refractive powers of gases at equal pressures and temperatures. To give them such a density as to produce precisely the same deviation, he employed a prism whose refracting power amounted to 145°, standing in connexion with a reservoir, *r* (*pl.* 20, *fig.* 31), and which could be exhausted on one side by connexion with an air-pump, and filled with gas on the other. He filled the prism first with dry air of the pressure and temperature of the atmosphere, and sighted then with a telescope set up at some distance, towards the image of a distant point refracted by the prism. The prism was then exhausted without disturbing it, and filled with another gas. By changing the pressure he could bring the refracted image of the same point of sight into the same part of the field of the telescope as before. Now, supposing carbonic acid gas to be compared with dry air, and that the pressure amounted to 18.9 inches, it is evident that as the pressure under which an equal deviation took place in air amounted to twenty-nine inches, under the circumstances the indices of refraction and the refracting power itself must be the same in air, that is, $18.9 : 29 :: 1 : x$; hence we obtain $x = 1.53$ as the index of refraction of carbonic acid at an atmospheric pressure of twenty-nine inches.

The refraction of light through lenses is of especial practical interest. Of these lenses the most important are the spherical, bounded either by portions of spheres, or by these and plane surfaces combined. Six kinds of spherical lenses are distinguished in optics, all of them represented in *fig.* 32:

bi-convex, *a;* plano-convex, *b;* concavo-convex, or meniscus, in which either the convexity is of least radius of curvature, *c*, or the concavity is of least radius, *f;* bi-concave, *d;* and plano-concave, *e*. In general, all lenses that are thicker at the middle than at the edges, are called convex or collecting lenses; and those which exhibit the greatest thickness at the borders are concave or separating lenses: *a*, *b*, and *c* belong to the former, *d*, *e*, and *f* to the latter.

The axis of a lens is that straight line which connects the two centres of the sphere, portions of which form the surface of the lens. Lenses are theoretically referable to the prism for their principle. In *fig.* 33, let *abcd* be an elongated rhomb, upon which are placed, above and below, equal parallel trapezia. Upon the trapezium *abfg*, a triangle, *fgh*, is superimposed, a similar one being placed on the lower trapezium. The two sides not parallel of the trapezium might, when produced, form an isosceles triangle, whose angle at the vertex is half the size of the angle *ghf*. If the figure thus produced be rotated about the axis MN, a lens-shaped body will be produced, which consists of several zones, and whose centre forms a plane disk. If a ray of light impinge upon this body, passing from a point of the axis MN, the deviation produced may be determined according to the laws of refraction in prisms. If the point S be so situated that a ray emitted from it and striking the surface *ag* in *i*, shall experience the least possible deviation in its passage through *abfg*, then it will cut the axis in a point, R, equally distant with S from the lens. A ray of light passing from S, and experiencing the minimum of deviation in passing through the triangle *fgh*, will, if the refracting angle of the upper prism be half that of the lower, be diverted twice as much as in *abfg* from its original direction. Hence it follows that the lower ray, S*i*, forms half as great an angle with the axis MN as the upper one; both rays, however, are refracted to R. If we suppose the broken lines *dbfh* and *cagh* to be replaced by curves whose centres lie in the axis MN, we shall obtain an actual (bi-convex) lens. We may therefore assume that there is a point, S, of the axis, all the rays coming from which and meeting the lens, are concentrated in one and the same point, R, situated at the same distance as S from the lens. The curvature of the lens from the centre to the circumference must, however, be very slight (as will be assumed in what follows), else the above condition would be impossible.

If a bi-convex lens be met by a number of rays parallel to the axis, or which come from an infinite distance in this direction, they will all be refracted to a point in the axis called the focus. The distance from the focus to the lens is the focal length (*pl.* 21, *fig.* 34). The focus is always half the distance of the points S and R from the lens. If the luminous point lie at a finite distance from the lens, on the axis, there is equally a point of union of the rays; this, however, is further from the lens than the focus of parallel rays, and will be further as the radiant point approaches nearer. It will be at an infinite distance when the radiant is in the focus of parallel rays. If the luminous point lie within the focal distance (*fig.* 83), the rays falling on the lens will not unite, but will diverge even after emerging

from the lens; less, however, than after refraction from the first surface.

In a bi-convex lens whose two surfaces are of equal radius of curvature, the focal length is equal to the radius. Plano-convex and convex meniscus lenses have likewise foci: in a plano-convex lens of glass (when the index of refraction for air and glass is assumed to be $\frac{3}{2}$) the focal length will be twice as great as the radius of the curved surfaces.

Concave lenses have no true focus, but rather a focal point of divergence. If the rays incident on such a glass are parallel to the axis, they diverge after emergence as if they came from one and the same point called the negative focus. If the incident rays be divergent, as if coming from a point on the axis at a greater or less distance from the lens, they will be made still more divergent; and the focal point of divergence will be nearer the glass the nearer the luminous point. If the incident rays be convergent (*pl.* 21, *fig.* 84), all these cases will be possible. If they converge towards the focal point of divergence, they will emerge parallel on the other side; if they converge still more than this, they emerge convergent. If they converge less, they diverge after emergence, as if they came from a point before the glass.

The preceding observations apply in general to rays coming from a point elsewhere than in the main axis of the lens, provided the line drawn from this point through the centre of the lens (the secondary axis) forms but a small angle with the principal axis. All rays proceeding from this point and incident upon the (convex) lens, are united in a point of the secondary axis, which is at the same distance from the lens as if the luminous point were situated in the principal axis.

We shall now be able to examine the formation of images of objects by lenses. In *fig.* 37, let AB be an object placed before the convex lens VW, and at a greater distance from it than the focus F. In this case, an actual but inverted image, *ab*, will be formed, which will be of the same size as the object, or greater, or less, as the distance of the object from the lens is equal to, greater, or less than twice the focal distance. In fact, image and object are always to each other in the ratio of their respective distances from the lens. If the object lie within the focus of the lens (*fig.* 38), no actual or convergent image will be formed, but an eye situated on the other side of the lens (to the right in our figure) will see the object, AB, magnified in *ab*; *ab* is therefore to be considered the image of AB. Concave lenses afford images of this latter kind; they are, however, diminished instead of being magnified (*fig.* 39). It thus appears that convex lenses alone magnify: concave lenses always diminish.

In order that all rays coming from a luminous point shall unite actually in one point, the aperture of the lens must not exceed 10°—15°. If the aperture be larger, as in the lens VW (*fig.* 40), only three rays near the axis will unite in the focus of parallel rays: the exterior ones will unite at points nearer to the lens.

Fig. 42 represents a *Fresnel* or *Polyzonal Lens*, by means of which the light of a light-house may be cast to a distance of many miles. It consists

of vision. This is the angle (*pl.* 21, *fig.* 85) formed with each other by the two lines, A′*a*, B′*b*, drawn between the corresponding extremities of the object and its image on the retina. Two objects of different magnitude, as AB and A′B′, may appear of the same size when their actual size is proportional to their distance from the eye. When the angle is less than a certain limit, the object becomes invisible.

An image of an object is formed in both eyes; we *see* but one, however, as soon as the eye has been adjusted properly to the distance of the object. When the eye is arranged for a distance greater or less than the true one, the object will be seen double. In *fig.* 51, let L and R be the two eyes, A and B two objects at different distances from them. If the eyes be fixed upon the nearer object, A, the optical axis will be directed towards A, so that its image falls in the middle of the retina, at *a* and *a*′. The object, A, is seen single; B, however, appears double, its image falling out of the centre of the retina at *b* and *b*′. The case is reversed when the eyes are directed to B.

Several objects may be seen single by both eyes when their images fall on corresponding parts of their retinas. In *fig.* 52, let L and R again represent the two eyes, A, B, and C, three objects before them. All three will be seen single, and at the same instant, as their images follow each other in the same order in both eyes.

By *irradiation* is meant the fact, that a bright object on a dark ground appears to us magnified, while a dark object on a bright ground seems to be reduced in size. The apparatus represented in *figs.* 53 and 54 is intended to illustrate this phenomenon. *Fig.* 53 represents a piece of pasteboard, whose upper half is covered with a piece of white paper, and the lower with black. The former is bisected by a narrow strip of black, about two lines in breadth, the latter by a strip of white of the same breadth, and in the same line with the black strip. On placing the pasteboard near a window, the white strip will, at a certain distance, appear decidedly broader than the black.

The following experiment shows that irradiation is not equally strong for all persons. Paint upon a piece of white pasteboard two equal, rectangular, black spaces, so that the border, *al* (*fig.* 54), shall be about half a line to the right, and the border, *gh*, about the same distance to the left of the vertical central line of the pasteboard. If this be observed at a certain distance, the edges, *al* and *gh*, will appear to lie in the same straight line; the precise distance necessary for this result will, however, vary considerably for different persons.

Very small objects on a white ground, vanish entirely when looked at under certain conditions, the principal of which is the falling of the image on the so-called *punctum cœcum*, that part of the retina at which the optic nerve enters. To illustrate this disappearance of an object, lay upon the white horizontal surface, *nn*′(*pl.* 21, *fig.* 86), two small dark disks, from one to four lines in diameter, and about three inches apart. Bring, now, the right eye vertically over the left disk (or the left eye over the right disk), and at a height about five times as great as the distance between the